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Robust Synthesis Technique for Single-Stage Servo Systems in HDD

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Abstract—**This paper presents a new technique for track-following control in hard disk drives, in order to achieve high tracking precision of magnetic read-write heads uniformly for a large number of disk drives. The main contribution of this paper is to propose a new technique to specify the uncertainty bound for voice-coil-motor and an iterative design procedure to design an optimal controller with robust stability.**

I. Introduction

As the storage capacity of magnetic hard disk drives (HDDs) increases dramatically, the demand for positioning control accuracy of a read/write head of HDD is becoming more stringent. To achieve this goal, higher control bandwidth is necessary to attain sufficient positioning accuracy. However, it is difficult to design high bandwidth controllers, and at the same time guarantee robust stability. Robust controller design is widely used to reach the balance between performance and stability. The selection of nominal plant, performance weight and uncertainty weight are all-important and related. Guideline for choosing these weights is a largely un-solved problem for which only few results are available [1] – [8]. In this paper, it is assumed that the nominal plant and the performance weight have been selected, and focuses on choosing an uncertainty weight to maximize performance without compromising stability.

The most commonly seen uncertainty in hard disk drive is the voice-coil-motor (VCM) bode variation from drive to drive. In this paper, it is assumed that a finite numbers of VCM bode, $P(\omega)$, from multiple drives, which are dense enough to cover all the design uncertainty, are available. And if a given controller can stabilize every member in $P(\omega)$, then the controller achieves robust stability.

To design such a controller using robust synthesis technique, a classic way to specify uncertainty weight is the maximum perturbation radius (MPR) approach [1]. In practice, even though MPR weight is simple to calculate, it will introduce conservatism since the over-bounding operations guarantee only sufficient conditions for robust stability. Motivated by this fact, a critical perturbation radius (CPR) approach [2]–[4] was proposed for interval system. Unfortunately, the method can only be applied to system with parametric uncertainty, which limits its usage, especially to HDD industry. Further study also reveals that even though CPR approach gives necessary and sufficient stability condition, it cannot be applied directly into design procedure, since there is no guarantee that the design can converge. This paper examines how to apply these approaches to HDDs, and proposes the critical perturbation frequency (CPF) and critical cone radius (CCR) concept, with which an innovative iterative robust synthesis approach is explored, which proves to be simple and effective.

The paper is organized as follows. In Section 2, the robust track-following problem is formulated mathematically; the MPR and CPR [1,2] methods are reviewed, and the applications of these methods to HDD are explored. The CCR and CPF concepts are introduced in Section 3. Section 4 presents an iterative robust control design methods that solve the formulated robust track-following problem. The convergence issue is also studied. Section 5 gives a simple example for the synthesis of a single-stage track-following controller using the proposed techniques. Readers are referred to [2]–[4] for more background on the CPR approach.

II. Problem Formulation

Consider the track-following design problem depicted in Fig 1. Here, $P_0(j\omega)$ is a given nominal plant, $W_p(j\omega)$ is a given performance weight, $Uncertain(j\omega)$ is the uncertainty weight to be selected to design a controller $C(j\omega)$ to optimize the robust performance for a finite number of VCM bodes depicted in Fig 2 (this paper will use this group of VCM bodes as the design example).

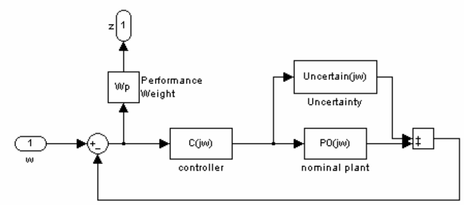

Figure 1: Block diagram of the track following design system

The finite numbers of VCM bodes can be represented as

$$P(\omega)=\left\{P(j\omega)\middle|P_i(j\omega)=P_0(j\omega)+\delta_i(j\omega):\delta\in C, i=1...n\right\} \quad (1)$$

Here, at a specific frequency ω, $P(\omega)$ are n complex numbers, representing the VCM bode responses. $\delta(j\omega)$ belongs to an uncertainty set d, which is a group of complex values reflecting the distance of the given VCM bodes from nominal plant response, $P_0(j\omega)$. Since there are only finite numbers of VCM bodes, $\delta(j\omega)$ will have finite numbers of

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samples at a specific frequency, which makes it bounded. Fig 3 depicts the relationship of $P(\omega)$, $P_0(\omega)$, and $\delta(j\omega)$ at one frequency point.

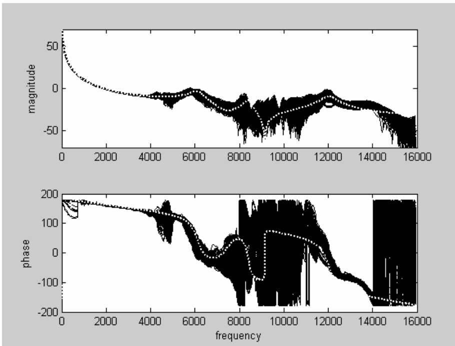

Figure 2: Bode plots of VCM dynamics. Dotted line is the nominal plant.

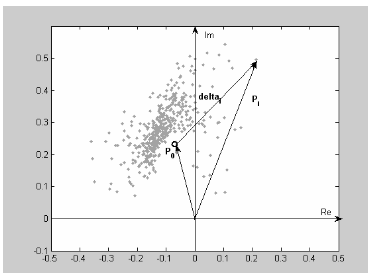

Figure 3: Nyquist plot of $\delta(j\omega)$, $P_0(\omega)$, and $P(\omega)$ at one frequency point. The solid circle is the nominal plant, and the gray dots are n samples of $P(\omega)$.

Even though the design problem is presented with additive uncertainty, it can be shown that all the results in this paper can be easily applied to multiplicative uncertainty.

A. Maximum Perturbation Radius (MPR) Uncertainty Weight

The MPR uncertainty is defined in [1] and illustrated in Figure 4.

The largest perturbation radius at a specific frequency ω is defined as

$$MPR(j\omega) = \delta_{\max}(j\omega) = \max_{d} |\delta(j\omega)| \qquad (2)$$

Searching MPR for the linear uncertain system in (1) is very simple for there are only n values to be considered at each frequency. The traditional MPR approach will use a constant weight, which is the largest $MPR(\omega)$ over all frequencies,

$$MPR(\omega) = \max_{\omega} |\delta_{\max}(j\omega)| = \|\delta_{\max}(j\omega)\|_\infty . \qquad (3)$$

To reduce conservatism, it is common practice to select a minimum-phase stable system, which covers the MPR at each frequency point, as the MPR additive uncertainty weight. Since MPR weight uses the smallest disk centered at nominal plant to cover uncertainty at each frequency, it does not care about its phase. So a minimum-phase stable MPR weight always exists, since it is always possible to convert right-plane zeros and poles to left-plane without affecting magnitude.

Figure 5 showed how to select MPR weight based on the

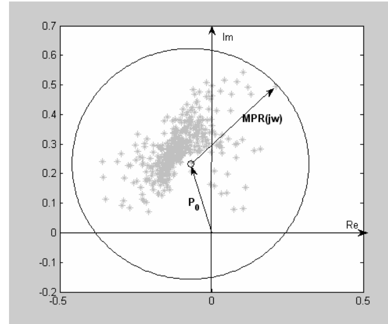

Figure 4: Nyquist plot of MPR at a specific frequency. The solid circle is the nominal plant $P_0(\omega)$, and the gray dots are n samples of $P(\omega)$. MPR will use the smallest disk centered at $P_0(\omega)$ to cover $P(\omega)$.

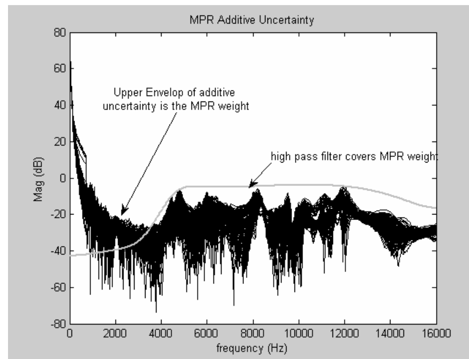

Figure 5: Frequency response of additive MPR uncertainty weight and VCM Plants. The dark lines are magnitude of $\delta(j\omega)$. The gray line is the frequency response of the selected MPR weight.

magnitude of uncertainty set $\delta(j\omega)$ for HDDs. In the example given in Fig 2, there are n VCM bodes, hence for a selected nominal plant $P_0(\omega)$, Fig 5 depicted the magnitude of the n $\delta_i(j\omega)$ in black lines. The upper bound of $\delta(j\omega)$ along the frequency defines the exact value of MPR. In Fig 5, a high pass filter was selected to cover the exact MPR weight, except at lower frequency. A high pass filter was chosen as MPR weight is because, firstly, lower frequency is the typical range for performance requirement; and, secondly, a high pass filter only adds two orders to the controller designed using a standard H_∞ synthesis package.

It is obvious from Fig 4 that MPR weight only provides sufficient conditions for robust stability. The CPR method was proposed which is a necessary and sufficient frequency domain robustness analysis technique.

B. Critical Perturbation Radius (CPR) Uncertainty Weight

The CPR uncertainty is defined in [2] and [4] through the critical direction theory. It makes use of the following objects:

1. The nominal open loop frequency-response $OL_0(j\omega) = P_0(j\omega) * C(j\omega)$, where $P_0(j\omega)$ is the nominal plant frequency-response and $C(j\omega)$ is the controller frequency-response.

2. The critical line, defined on the directed line which originates at the nominal point $OL_0(j\omega)$ and passes through the critical point –1+j0.
3. The critical direction
$$d(j\omega) = -\frac{1+OL_0(j\omega)}{\left|1+OL_0(j\omega)\right|}$$
is a unit vector that defines the direction of critical line.
4. The uncertainty value set
$$\tau(\omega) = \{OL(j\omega) | OL(j\omega) = OL_0(j\omega) + \delta(j\omega) : \delta(s) \in d\}$$
5. The critical uncertainty value set
$$\tau_c(\omega) = \{OL(j\omega) | OL(j\omega) = OL_0(j\omega) + \alpha d(j\omega)\},$$
for some $\alpha \in \Re_+$.
6. The critical Perturbation radius (CPR) family
$$\rho_c(\omega) = \max_{\alpha \in \Re_+} \{\alpha | z = OL_0(j\omega) + \alpha d(j\omega) \in \tau_c(\omega)\}$$

At every frequency, $\rho_c(j\omega)$ represents the element of $\tau_c(\omega)$ that is closet to the point –1+j0 along the critical direction, which is shown in Fig 6 [6]. The solid curve in Fig 6 described the nominal system $OL_0(j\omega)$ within a pre-specified frequency range. The shaded polygon outlines the uncertainty family at a specific frequency point.

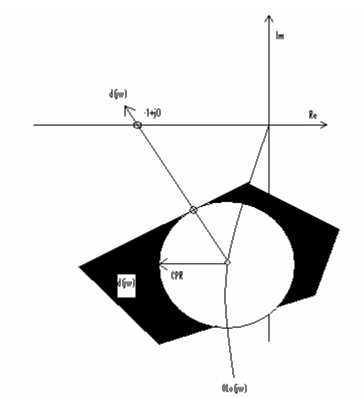

Figure 6: Nyquist plot of CPR at a specific frequency.

Reference [2] shows the numerical calculation of $\rho_c(j\omega)$ can be done when a description of the boundary of the value set is available. This information is not easily obtained in many applications, including HDDs. Another thing to be noticed is that the calculation of CPR requires the knowledge of controller frequency response, which makes any design approach using CPR to be done iteratively. The contribution of CPR is that it points out to remove the conservatism of MPR weight, it is important to pay attention to the phase of uncertainty system. Yet including the controller magnitude in uncertainty weight selection makes the design iteration vulnerable when it comes to convergence issue, since the magnitude of CPR is not bounded by the uncertain system itself any more.

To be able to apply CPR concept to a wider range of applications that do not have parametric uncertainty, the paper proposes a new concept, critical cone radius (CCR).

III. Critical Cone Radius (CCR) and Critical Perturbation Frequency (CPF)

A. Critical Cone Radius (CCR) Uncertainty Weight

To be able to apply the critical perturbation radius without parametric uncertainty model, this paper proposes the concept of critical perturbation cone (CPC),
$$\nu_c(\omega) = \left\{\hat{P}(j\omega) \middle| \hat{P}(j\omega) = P_0(j\omega) * \angle C(j\omega) + \alpha d(j\omega) * e^{j\theta} : \alpha \in \Re_+, |\theta| < \Theta\right\}$$
which originates at the adjusted nominal point $\hat{P}_0(j\omega) = P_0(j\omega) * \angle C(j\omega)$, and span Θ degree away from the critical line, as illustrated in Fig 7. The solid circle in Fig 7 represents the adjusted nominal response $\hat{P}_0(j\omega)$ at a specific frequency point ω. For the same frequency point ω, the responses of the adjusted uncertainty family $\hat{P}(j\omega) = P(j\omega) * \angle C(j\omega)$ are plotted in light gray dots. The critical cone radius (CCR) is defined as,

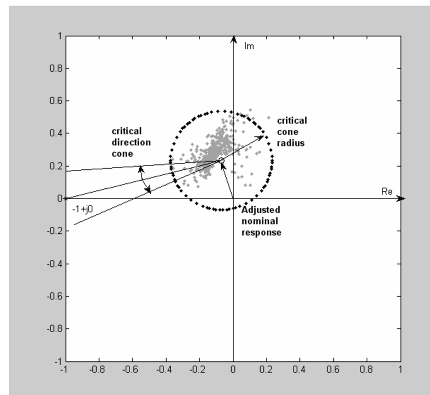

Figure 7: Nyquist plot of CPC and CCR weight at a specific frequency

$$\rho_c(\omega) = \max_{|\theta| < \Theta, \alpha \in \Re_+} \{\alpha | z = P_0(j\omega) * \angle C(j\omega) + \alpha d(j\omega) * e^{j\theta} \in \nu_c(\omega)\}.$$

At every frequency, $\rho_c(j\omega)$ represents the element of $\hat{P}(j\omega)$ within $\nu_c(\omega)$ that is closet to the point –1+j0. If none of the member from $\hat{P}(j\omega)$ falls in the CPC, then the CCR is zero.

Unlike CPR, CCR is defined on the plant bode families adjusted by controller phase, not on open loops. In other words, the magnitude of controller will not affect CCR, which makes CCR to be bounded by the magnitude of the uncertain family itself. This difference also means the necessary property of CPR does not apply to CCR weight.

Another difference is, CCR is defined not along a line, but within a cone originated from the adjusted nominal plant, and centered along the critical line, whose angle is tunable. It is obvious that if the angle of the cone, Θ, is selected to be 180 degree, then CCR weight becomes MPR weight. The function of Θ is two-folded. First it makes the CCR weight less sensible to measurement noise than CPR weight, especially when uncertainty system is non-convex. Second it makes the CCR applicable to non-parametric uncertain system, since the result does not depend on the precise location of one point along the boundary of uncertainty family, but on an area. For problem (1), as long as the VCM bode samples are dense enough, CCR weight can be used to substitute CPR.

Calculating CCR weight for the linear uncertain system in (1) is very simple since there is only a subset of n values to be considered for each frequency. Figure 8 shows an example of CCR weight, calculated for the same design family as Figure 5, using a specific controller. Θ was set to be 10 degree.

Comparing CCR weight in Fig 8 to MPR weight in Fig 5, it is obvious that CCR weight reduces conservatism at many frequency bands, for the selected controller. But at lower frequency (less than 2kHz), the CCR weight is higher than the high pass MPR weight. It is common knowledge that performance weight and uncertainty weight are related. For example, in the frequency zone that asks for high performance, the uncertainty weight should not be too high. The selected MPR weight in Fig 5 does not cover the MPR bound at lower frequency in order to design a controller that can satisfy the selected performance weight at this range. To be able to further tighten the CCR weight, to resolve the conflict between performance weight and uncertainty weight, the critical perturbation frequency (CPF) concept is proposed.

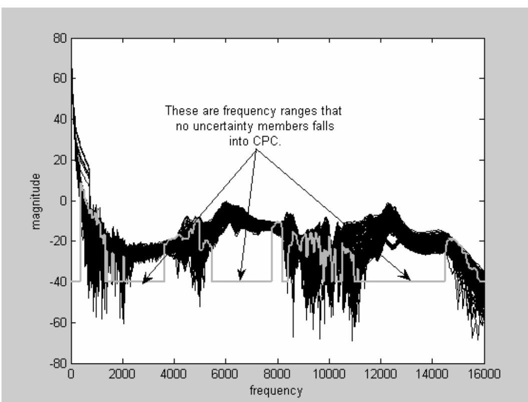

Figure 8: CCR Additive Uncertainty. The dark lines are magnitude of $\delta(j\omega)$. The upper envelope of $\delta(j\omega)$ is the MPR weight. The gray line is the CCR weight.

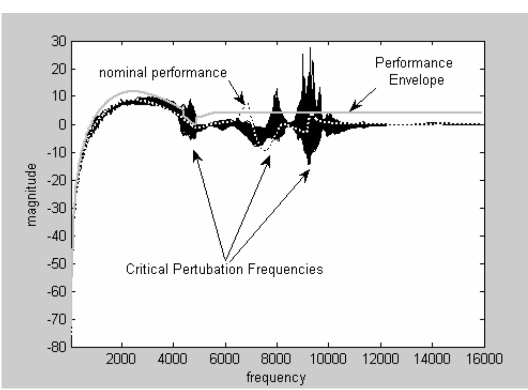

Figure 9: CPF & Performance Envelope. The dark lines are magnitude of achieved performance for a given controller. The dotted line is the nominal performance. The gray line is the performance envelope. Every frequency point where the magnitude of achieved performance exceeding the performance envelope is a CPF.

B. Critical Perturbation Frequency (CPF)

The necessary property of CPR means that the magnitude of controller is also important to the removal of conservatism in uncertainty weight. To preserve this property, Let's define an acceptable performance envelope around nominal performance, as depicted in Fig 9. The dotted line is the nominal performance for a specific controller. The black lines are the magnitude of achieved performance; evaluated using the n VCM bodes in Fig 2, with the same controller as in Fig 8. The solid gray line is a user specified performance envelope, describing how much divergence from nominal performance is acceptable during the design procedure. For a given frequency, if the performance of every member of the uncertain family is within the performance envelope, then there is no need to specify uncertainty weight. Otherwise, the uncertainty weight is needed for this frequency, and this frequency is a critical perturbation frequency. Thus, CPF is defined as,

$$\omega = \{\omega | if \exists \forall i \le n, s.t. |Perf_i(j\omega)| > |Envp(j\omega)|\}$$

where $Envp(j\omega)$ is the acceptable performance envelope, and $Perf(j\omega)$ is the performance of the uncertain family (1) for a specific controller.

Performance envelope is typically defined around nominal performance, so unacceptable divergence from the nominal performance can be detected, and uncertainty weight can be adjusted at these frequencies in the next design iteration. CPF reflects the frequency range that the mismatch between the design family $P(j\omega)$ and nominal plant $P_0(j\omega)$ is too significant to be omitted in uncertainty weight. The definition of CPF also implies that all the frequencies that are unstable in closed loop are CPFs. Like CCR, CPF is also controller dependent. In next section, we will see how to use CPF to further remove the conservatism in CCR weight.

IV. Optimal Robust H-Infinity Design

A. An iterative approach for optimal robust controller synthesis

With the concepts of CCR and CPF, this paper proposes an iterative design procedure that can balance between optimal controller design and robust stability.

<u>*Algorithm for optimal Robust Controller Synthesis*</u>

Step 1. <u>Design optimal controller:</u> Set the initial uncertainty weight, $Uncertain(j\omega)$, close to 0, such as –40db, for all frequencies. Then design a controller using optimal design package. The nominal performance of this controller reflects the best performance can be achieved with the given (nominal plant, performance weight) set.

Step 2. <u>Check if design iteration is done:</u> Using the latest controller to calculate CPFs, based on a selected performance envelope.

- If CPF is empty, then a robust controller with optimal performance has been reached. The iteration will be stopped. The uncertainty weight used to design the latest controller is the "optimal uncertainty weight".

- Else, if CPF is not empty, continue to step 3.

Step 3. Update uncertainty weight: At the CPFs from step 2, calculating CCR using the latest controller. The cone angle, Θ , determines the converging speed. The bigger the Θ , the quicker the convergence, but more reservation could be introduced in the design. Set CCR to 0 for none-CPF frequencies. Then update uncertain weight with

$$Uncertain_{i+1}(j\omega) = \max(|Uncertain_i(j\omega)|, |CCR_{i+1}(j\omega)|).$$

Step 4. Design robust controller: The result of step 3 is the outline of the new uncertainty weight. Choosing a minimum-phase stable system to cover the outline yields the new uncertainty weight. Design a robust controller using the standard robust synthesis package. Continue to step 2.

It is critical that when calculating the CCR, only the controller phase information is used to adjust the nominal plant and uncertain family. This ensures that the resulted uncertainty weight is always bounded by MPR uncertainty weight.

B. Convergence Issue

It is obvious that the iterative procedure described will converge to a controller that can stabilize the uncertain family, since the CCR uncertainty weight is increasing monotonically during the iteration, and has an overall upper bound, MPR weight. In other words, the controller will become more and more stable during iteration, hence the critical perturbation frequencies will become less and less until the halting condition is met.

Theorem 1: Consider the uncertain system (1) and assume that a robust stable controller can be found using MPR uncertainty weight $MPR\_Uncertain(j\omega)$ for a given nominal plant $P_0(j\omega)$ and performance weight, then the iterative design procedure in 4.1 is guaranteed to converge to a robust controller that can stabilize system (1).

Proof: From the property of CCR weight, we know, for every frequency ω

$$|CCR\_Uncertain_i(j\omega)| \le |CCR\_Uncertain_{i+1}(j\omega)| \le |MPR\_Uncertain(j\omega)|$$

If we assume the approach in 4.1 cannot converge, then it must be true that $CCR\_Uncertain(j\omega)$ cannot converge to $MPR\_Uncertain(j\omega)$. In other words, after certain iterations, $D_i(jw) = |MPR\_Uncertain(j\omega)| - |CCR\_Uncertain_i(j\omega)|$ will remain the same at every frequency in further iterations, and the unstable CPF is not empty.

Without losing generality, assuming iteration i has reached such state, then the following statement must be TRUE for a specific unstable CPF ω_0,

$$|CCR\_Uncertain_i(j\omega_0)| == |CCR\_Uncertain_{i+1}(j\omega_0)| < |MPR\_Uncertain(j\omega_0)|$$

In other words, there exist members from uncertainty family that locate outside the critical perturbation cone that can destabilize the closed loop system using the existing controller. This contradicts with the Nyquist stability theory.

Hence the assumption that design procedure 4.1 cannot converge to a stable controller is not TRUE.

□

V. DESIGN EXAMPLE

To illustrate the effectiveness of the proposed design procedure, a robust controller has been designed using the VCM dynamics from twenty 7200 rpm, 135kTPI, 3.5" HDDs, whose bode plots are depicted in Fig 2. Fig 5, Fig 8 and Fig 9 are results for the same drives.

Consider the VCM bode dynamics and nominal plant depicted in Figure 2. A constant –40db uncertainty weight is used to design the initial optimal "robust" controller. Figure 9 in previous sections shows the performance of the controller, comparing to nominal performance and performance envelope.

The performance envelope was selected to ensure an acceptable robust stability. Re-calculating the uncertainty weight using CCR at CPFs yielded the new uncertainty weight in Figure 10. CCR uncertainty weights in Figure 8 and Figure 10 were calculated using the same controller, except Figure 10 only included CPFs to further reduce conservatism

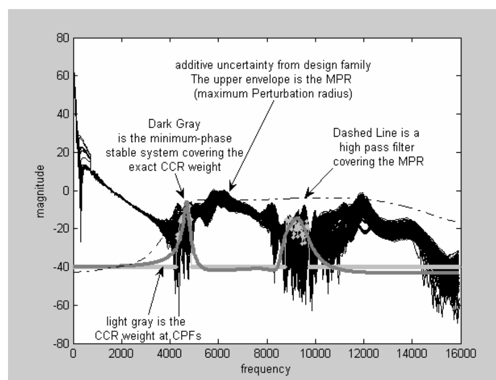

Figure 10: CPF_CCR Additive uncertainty weight. The dark lines are magnitude of $\delta(j\omega)$ over frequency. The upper envelope of $\delta(j\omega)$ is the MPR weight. The lighter gray line is the outline of CPF_CCR weight. The darker gray line is the response of a 4th order minimum-phase stable system covering the outline of CCR weight, which will be used to design controller in the next iteration.

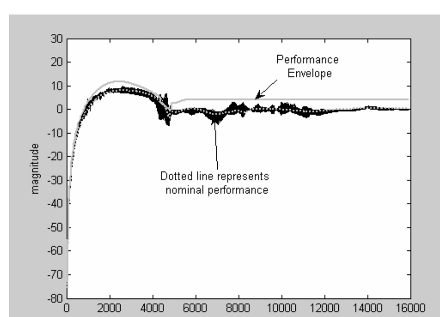

Figure 11: Achieved performance of proposed design procedure. Black lines are the performance of the controller designed using uncertainty in Figure 10. Dotted line is the corresponding nominal performance. Gray line is the performance envelope.

in CCR weights, Let's call it CPF_CCR weight. It can be noticed that the CPF_CCR weight in Figure 10 does not have high magnitude at frequencies less than 2000Hz, which are the frequency ranges that require higher performance. Thus, CPF_CCR weight effectively resolves the conflict between performance weight and uncertainty weight without manual modification, unlike what has been done to the MPR weight in Figure 5.

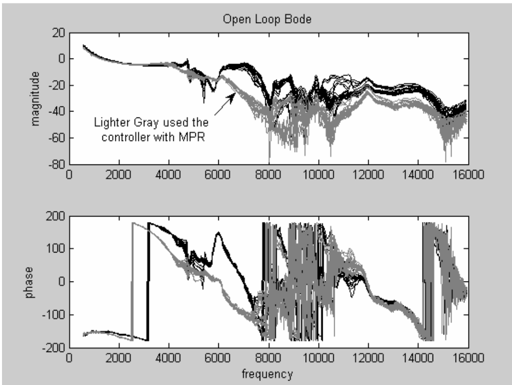

Figure 12: Bode plot of measured open loop. The black lines are the responses from the proposed method. The gray lines are the responses from MPR method.

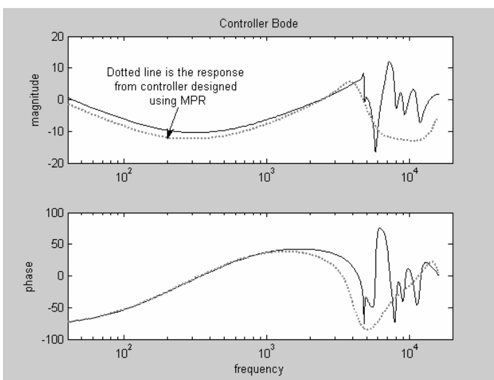

Figure 13: controller bode designed using CPF_CCR method vs. MPR

The projected result of the controller in iteration 2 is illustrated at Figure 11. Comparing to the initial optimal controller performance in Fig 9, it can be noticed that the adjustment to the uncertainty weight at CPFs, thus around 4000 Hz, and 8000 Hz to 10000 Hz, has effectively cleaned up the peaks in the achieved performance at these zones. Thus the proposed design procedure has found the optimal robust controller after only two iterations. The darker gray line in Figure 10 is the corresponding "optimal" uncertainty weight. Comparing the CPF_CCR weight in Fig 10 to MPR weight in Fig 5 or CCR weight in Fig 8, it can be seen how much reservation can be removed by the proposed method.

Controllers designed using traditional MPR (maximum perturbation radius) approach and the proposed approach have been tested on a 7200rpm, 135kTPI, 500GB hard disk drive. The results are shown in Figure 12 to Figure 13.

From open loop bode, it is obvious that the new approach achieves higher performance by not notching the system mode, and also ensures stability through a more precise uncertainty weight tailored for the given (nominal plant, performance weight) set. Table I further confirms the advantage.

In the given example, we use additive uncertainty. It is straightforward to show that the same result applies to multiplicative uncertainty.

TABLE I
OPEN LOOP RESPONSE OF DIFFERENT DESIGN METHOD

	GM (dB)	PM (deg)	BW (Hz)
MPR Approach	4.24	25.5	1293
New Approach	4.88	28	1463

VI. Conclusion

The main contributions of this paper are 1) to introduce the critical cone radius concept, which not only allows the application of CPR to a much wider design family without parametric uncertainty and without limitation to interval plant, but also adds in additional phase margin in the design process, which makes the approach insensitive to convex or non-convex problems; 2) to introduce the critical perturbation frequency and performance envelope concepts, which further reduces the conservatism of CCR uncertainty weight to only necessary frequency bands, which in turn improves the performance of the resulted controller without compromising stability; 3) to propose an innovative iterative controller synthesis approach, which not only provides an "optimal" robust controller, but also proves to be easy to use and practical in hard disk drive industry.

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