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Frequency estimation of a saturated signal with a hybrid observer

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Abstract— **A hybrid observer to estimate the frequency of a saturated signal is proposed. The saturated single-frequency sinusoidal signal is assumed to be generated by an hybrid model with unknown parameters, related to the signal angular frequency and bias, estimated on-line globally by a hybrid observer. Global exponential convergence of the estimation error and robustness against unknown saturation level is discussed.**

I. INTRODUCTION

The problem of estimating the unknown frequencies and bias of the signal

$$y(t) = E_0 + \sum_{i=1}^{n} E_i \sin(\omega_i t + \phi_i),$$

with known $n \geq 1$ and unknown bias E_0, angular frequencies $\omega_i > 0$, amplitudes E_i and phases ϕ_i, for $i = 1, .., n$, is of great interest in different engineering fields as telecommunication, image processing, identification and control.

The first solutions have been proposed from a signal processing point of view [1], using classical Fourier analysis performed off-line on batch processing data. Such an approach is not suitable when on-line estimation of the unknown frequencies is needed, as in control applications, where compensation or simply identification of disturbances may benefit from a frequencies estimator with suitable asymptotic properties (see [8]). As an example, the *extremum seeking* feedback scheme in [3], where the probing sinusoidal signal with unknown frequency is reconstructed simply filtering the system output, would considerably benefit from a robust frequency estimator.

The first on-line global estimators for $n \geq 1$ make use of $3n$ dimensional adaptive observers and resort to the adaptive identifiers structure [9], [6]. A different approach is proposed in [7], where a linear adaptive observer of dimension $5n-1$, exploiting a filtered transformation of coordinates, has been proposed. Recently, a minimal dimension, i.e. $3n - 1$, observer design has been proposed in [14].

The above solutions provide certain degree of robustness with respect to measurement noise. However, in some cases the measured signal may be deformed due to nonlinearities introduced by the sensor, for example when the signal amplitude exceeds the sensor limits or leaves certain operative ranges in which the sensor operates linearly. Note that the latter case could be transformed into the former one, if the signal is appositely cut-off to eliminate the sensor distortions. To cope with these issues, we propose a hybrid observer to estimate the angular frequency ω_1 and the bias E_0 of the signal

$$y(t) = sat_\sigma\left(E_0 + E_1 \sin(\omega_1 t + \phi_1)\right), \qquad (1)$$

where σ is the known saturation level and sat is the saturation function defined as $sat_\sigma(x) = \max(-\sigma, \min(\sigma, x))$. We approach the problem as follows: first, the saturated signal (1) is modeled as the output of a hybrid model belonging to the class of the hybrid systems defined in [15], then within the framework introduced in [11], [12], we develop a hybrid version of the observer in [14] such that suitable estimation properties are guaranteed, even if only a lower bound on the saturation level σ is known. To our best knowledge this is the first time that such problem is addressed and solved.

The paper is organized as follows. The hybrid model to represent the saturated signal with unknown frequency is presented in Section II, followed by the definition of the hybrid observer and the main results. Numerical simulations are shown in Section III and conclusions are given in Section IV.

II. GLOBAL FREQUENCY ESTIMATOR OF A SATURATED SIGNAL

We consider the problem of estimating the angular frequency ω_1 of the signal

$$y(t) = sat_\sigma(E_1 \sin(\omega_1 t + \phi_1)), \qquad (2)$$

with bounded unknown E_1, $\omega_1 \geq 0$, ϕ_1, and known saturation level $\sigma > 0$. In addition, we discuss the case in which only a lower bound of σ is known. The approach we present relies on some basic definitions of hybrid models, given in [15] and recalled hereafter. The reader should refer to [15] for further detail.

Definition 1: A *compact hybrid time domain* is a set $\mathcal{T} \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ given by :

$$\mathcal{T} = \bigcup_{j=0}^{J-1} ([t_j, t_{j+1}], j\,)$$

where $J \in \mathbb{Z}_{\geq 0}$ and $0 = t_0 \leq t_1 \cdots \leq t_J$. A *hybrid time domain* is a set $\mathcal{T} \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ such that, for each $(T, J) \in \mathcal{T}$, $\mathcal{T} \cap ([0, T] \times \{0, \ldots, J\})$ is a compact hybrid time domain. □

Definition 2: A *hybrid trajectory* is a pair (dom χ, χ) consisting of a hybrid time domain dom χ and a function χ defined on dom χ that is differentiable for almost all t on (dom $\chi) \cap (\mathbb{R}_{\geq 0} \times \{j\})$ for each $j \in \mathbb{Z}_{\geq 0}$. □

Definition 3: For the hybrid system $\mathcal{H}$ given by the open state space $O \subset \mathbb{R}^n$, and the data (F, G, C, D) where $F : O \to \mathbb{R}^n$ is continuous, $G : O \to O$ is locally bounded, and C and D are subsets of O, a *hybrid arc* χ : dom $\chi \to O$ is a *solution of the hybrid system* $\mathcal{H}$ if $\chi(0, 0) \in C \cup D$ and the following hold.

1) For all $j \in \mathbb{Z}_{\geq 0}$, and for almost all $t \in I_j :=$ dom $\chi \cap (\mathbb{R}_{\geq 0} \times \{j\})$, we have $\chi(t, j) \in C$ and $\dot{\chi}(t, j) = F(\chi(t, j))$.
2) For all $(t, j) \in$ dom χ, such that $(t, j + 1) \in$ dom χ, we have $\chi(t, j) \in D$ and $\chi(t, j + 1) = G(\chi(t, j))$. □

Hence, the hybrid system model that we consider is of the form:

$$\begin{aligned} \dot{\chi}(t, j) = F(\chi(t, j)) \qquad & \chi(t, j) \in C, \\ \chi(t_{j+1}, j + 1) = G(\chi(t_{j+1}, j)) \quad & \chi(t_{j+1}, j) \in D. \end{aligned}$$

$F(\cdot)$ and $G(\cdot)$ are usually called flow map and jump map, respectively. In the sequel (as in [15]) we omit the time arguments when possible and write:

$$\begin{aligned} \dot{\chi} &= F(\chi) \qquad \chi \in C, \\ \chi^+ &= G(\chi) \qquad \chi \in D\ , \end{aligned} \tag{3}$$

where we denoted $\chi(t_{j+1}, j + 1)$ as χ^+ in the last equation.

The next definitions are needed to state the observer properties.

Definition 4: A hybrid arc $\chi(t, j)$ *uniformly globally* converges to a set $\mathcal{A}$ if there exists a function[1] $\beta \in \mathcal{KLL}$ such that, for each initial condition $\chi(0, 0)$ and each corresponding solution,

$$|\chi(t, j)|_{\mathcal{A}} \leq \beta(|\chi(0, 0)|_{\mathcal{A}}, t, j),$$

for all (t, j) in dom χ, where $|s|_{\mathcal{A}} = \inf_{a \in \mathcal{A}} ||s - a||$. $\chi(t, j)$ *uniformly globally exponentially* converges to the set $\mathcal{A}$ if β can be taken to have the form $\beta(s, t, j) = Ms \exp(-\lambda(t + j))$ for some $M > 0$ and $\lambda > 0$. □

Definition 5: A hybrid arc $\chi(t, j)$, with domain $\mathcal{T} \triangleq$ dom χ, *uniformly globally converges to a set* $\mathcal{A}$ *with restriction* $\mathcal{T}_r$ *on the hybrid time domain* $\mathcal{T}$, $\mathcal{T}_r \subset \mathcal{T}$, if there exists $\beta \in \mathcal{KLL}$ such that, for each initial condition $\chi(0, 0)$, and each corresponding solution,

1) $|\chi(t, j)|_{\mathcal{A}} \leq \beta(|\chi(0, 0)|_{\mathcal{A}}, t, j)$, $\forall\, (t, j) \in \mathcal{T}_r$;
2) for all $(t, j) \in \mathcal{T}$, $|\chi^+| \leq |\chi|$ and $\chi(t, j)$ is bounded.

$\chi(t, j)$ *uniformly globally exponentially converges to a set* $\mathcal{A}$ *with restriction* $\mathcal{T}_r$ *on the hybrid time domain* if β can be taken to have the form $\beta(s, t, j) = Ms \exp(-\lambda(t + j))$ for some $M > 0$ and $\lambda > 0$. □

The item 2) of the new Definition 5 could be relaxed requiring less constraining properties at jumps (see also [16], [17]) to deal with more general cases.

Select $\chi = [\zeta, x, q]^\top \in \mathbb{R}^2 \times \{0, 1\}$ as the solution of the hybrid system (3) with flow map $F(\chi)$ given by

$$F(\zeta, x, 0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ F(\zeta, x, 1) = \begin{bmatrix} x \\ -\theta_1 \zeta \\ 0 \end{bmatrix}, \tag{4}$$

where $\theta_1 = \omega_1^2$, and jump map $G(\chi)$ given by

$$G(\zeta, x, 0) = \begin{bmatrix} \zeta \\ -x \\ 1 \end{bmatrix},\ G(\zeta, x, 1) = \begin{bmatrix} \zeta \\ x \\ 0 \end{bmatrix}. \tag{5}$$

Let $\mathcal{S} = \{(\zeta, x, q) \in \mathbb{R} \times \mathbb{R} \times [0, 1]\}$ and the flow set C and the jump set D defined as

$$C_0 = \{(\zeta, x, q) \in \mathcal{S} : q = 0, |\zeta| = \sigma\}, \tag{6}$$

$$C_1 = \{(\zeta, x, q) \in \mathcal{S} : q = 1, |\zeta| < \sigma\}, \tag{7}$$

$$C = C_0 \cup C_1, \tag{8}$$

$$D = \{(\zeta, x, q) \in \mathcal{S} : q \in \{0, 1\}, |\zeta| \geq \sigma\}. \tag{9}$$

The flow sets C_0 and C_1 are shown in Figure 1. Note that $F(\zeta, x, q) : C_q \to \mathbb{R}^3$ and $C_0 \cap C_1 = \varnothing$ imply $F(\cdot)$ continuous on C. The hybrid model we consider has multiple solutions since $C \cap D \neq \varnothing$, and in general it may have even Zeno solutions. However, note that it admits a specific solution $\chi(t, j)$ such that the signal $y(t)$ in (2) could be evaluated[2] as $y(t) = \zeta(t, j)$, for any (t, j). From now on, consider this is the case.

[1] See [15] for for the definition of class $\mathcal{KLL}$ functions.

[2] It is possible to modify the model (4)-(9) such that it admits only one solution $\chi(t, j)$ satisfying $\zeta(t, j) = y(t)$, for any $(t, j) \in \mathcal{T}$, and no Zeno solutions are allowed. However the proposed observer is succesfully even with this general model as long as the Zeno solutions are detectable.

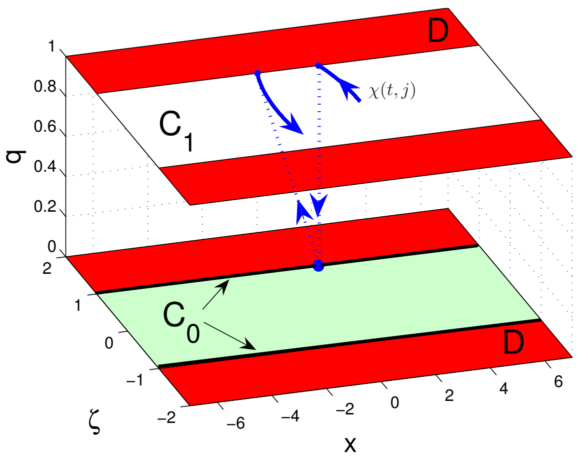

Fig. 1. Flow sets, jump set, and an example of hybrid trajectory for $\sigma = 1$.

An example of a hybrid arch χ that satisfies $\zeta(t,j) = y(t)$ is shown in Figure 1, whereas its projection χ_q^π on the plane (x,ζ), with $q = \{0,1\}$, is shown in Figure 2. In the example, χ starts in the interior of C_1 ($\chi_1^\pi(t,j-1)$) and keeps flowing up when $\zeta(t_j,j-1) = \sigma$, then it jumps to C_0 and remains constant , $\chi_0^\pi(t,j) = \hat{\chi}_1(t_j,j-1)$, up to time t_{j+1}, when it jumps back to C_1 ($\chi_1^\pi(t_{j+1},j+1)$) and starts flowing driven by $F(\cdot)$.

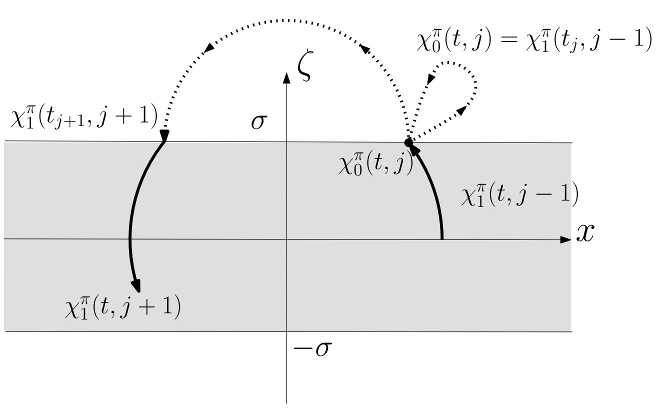

Fig. 2. The projection χ_q^π on the plane (x,ζ) of a hybrid trajectory.

To design the frequency estimator we partition the hybrid time domain as

$$\mathcal{T}_l = \left\{ \bigcup_{j=0}^{J-1} ([t_j,t_{j+1}],j) : \chi(t,j) \in C_l \right\}, \qquad (10)$$

with $l = \{0,1\}$ and define the estimation error z as

$$z_x = k_1\zeta + \xi_3 + \gamma_1\xi_1\theta_1 - x, \qquad (11)$$

$$z_\theta = \gamma_2\zeta\xi_1 + \xi_2 - \gamma_1\theta_1, \qquad (12)$$

with k_1, γ_1 and γ_2 positive constants. Define the mapping $F_o(\zeta,\xi,q)$ as

$$F_o(\zeta,\xi,1) = \begin{bmatrix} -k_1\xi_1 - \frac{\zeta}{\gamma_1} \\ -\gamma_2\xi_1\left(\xi_3 + \xi_1(\gamma_2\zeta\xi_1 + \xi_2)\right) + \frac{\gamma_2}{\gamma_1}\zeta^2 \\ -k_1\xi_3 - k_1^2\zeta \end{bmatrix},$$

$$F_o(\zeta,\xi,0) = 0, \qquad (13)$$

and the mapping $G_o(\zeta,\xi,q)$ as

$$G_o(\zeta,\xi,0) = \begin{bmatrix} -\xi_1 \\ \xi_2 + 2\gamma_2\zeta\xi_1 \\ -\xi_3 - 2k_1\zeta \end{bmatrix}, G_o(\zeta,\xi,1) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}, \qquad (14)$$

and assume that t_j's are known[3]. We are now ready to state one of the main results of the paper.

Proposition 1: Consider the signal (2), with known saturation level $\sigma > 0$ and unknown bounded $\omega_1 \geq 0$, E_1, and ϕ_1. Consider a hybrid system of the form (3), with $\chi(t,j) = [\zeta, x, q, \xi^\top]^\top$. Let the flow set be $C^a = C_1^a \cup C_0^a$, with $C_1^a = C_1 \cup \mathbb{R}^3$, $C_0^a = C_0 \cup \mathbb{R}^3$, where C_1 and C_2 are defined in (7) and (6), respectively. Let the jump set be $D^a = D \cup \mathbb{R}^3$, with D as in (9). Let the flow map be $F_H(\cdot) = [F^\top(\cdot), F_o^\top(\cdot)]^\top$, with $F(\cdot)$ and $F_o(\cdot)$ defined as in (4) and (13), respectively, and let the jump map be $G_H(\cdot) = [G(\cdot)^\top, G_o(\cdot)^\top]^\top$, with $G(\cdot)$ and $G_o(\cdot)$ defined as in (5) and (14), respectively. Finally, let

$$\hat{\theta}_1 = \frac{1}{\gamma_1}(\gamma_2\zeta\xi_1 + \xi_2),$$

$$\hat{x} = k_1\zeta + \xi_3 + \gamma_1\xi_1\hat{\theta}_1.$$

Then the following holds.

1) $\xi(t,j) \in \mathcal{L}_\infty$, the hybrid arc $\tilde{x}(t,j) = \hat{x}(t,j) - x(t,j)$, with domain dom $\chi(t,j)$, is such that $\lim_{t\to+\infty} \tilde{x}(t,j) = 0$ exponentially,

$$\hat{\theta}_1(t,j) - \theta_1 \in \mathcal{L}_\infty,$$

$$\xi_1(t,j)z_\theta(t,j) \in \mathcal{L}_2,$$

and the hybrid arc $\xi_1(t,j)z_\theta(t,j)$, with domain dom $\chi(t,j)$ globally converges to zero.

2) In addition, if $E_1 \neq 0$ and $\omega_1 > 0$,

$$\lim_{t\to+\infty} \hat{\theta}_1(t,j) - \theta = 0,$$

[3] We assume that they can be detected experimentally when the signal $y(t)$ enters or leaves the saturation tresholds (Proposition 2 deals with quantization issues).

exponentially. □

The key of the proof is to note that $G_o(\zeta,\xi,0)$ in (14) ensures that $|z_x^+| = |z_x|$ and $|z_\theta^+| = |z_\theta|$, whereas with standard techniques for linear systems the exponential convergence to zero of the extimation errors when $t \in \mathcal{T}_1$ can be easily proved. Note that since the value of q is univocally related to the value of the measured signal $y(t)$, we do not need to design a discrete "location observer" (see [16], [17]) to detect $q(0,0)$.

Remark 1: It is trivial to verify that the results in Proposition 1 hold also in the case of asymmetric saturation and deadzone by properly redefining the flow and the jump sets. □

In practical cases, where only a lower bound $\sigma_{min} \in (0,\sigma]$ on the saturation level may be available, it is still possible to achieve asymptotic estimation of the angular frequency ω_1, as stated in the next proposition.

Proposition 2: (Robustness to saturation level) Consider the signal (2), unknown bounded σ, $\omega_1 \geq 0$, E_1, and ϕ_1. Consider a known $\sigma_{\min} \in (0,\sigma]$. Let $\chi_{\sigma_{\min}}(t,j)$ be the solution of a hybrid system of the form (3), with hybrid time domain $\mathcal{T}_{\min}$. Let the flow and jump sets C^a and D^a, with maps $F_H(\cdot)$ and $G_H(\cdot)$, and the estimates $\hat{\theta}_1$ and $\hat{x}$ be as in Proposition 1, with σ replaced by $\sigma_{\min}$. Then, claims 1 and 2 of Proposition 1 hold with restriction $\mathcal{T}_{\min}$ on the hybrid time domain[4] $\mathcal{T}$. □

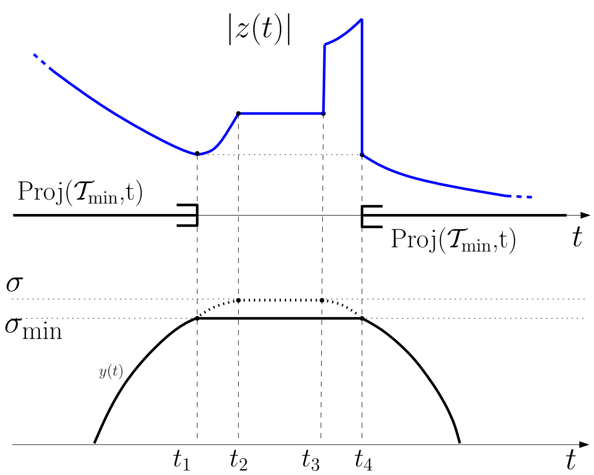

Fig. 3. A typical behavior of $|z(t)|$ when only a lower bound $0 < \sigma_{\min} \leq \sigma$ is known.

Figure 3 clarifies the result in Proposition 2 intuitively: a typical profile of $|z(t)|$ is shown when the observer uses a saturation value $0 < \sigma_{\min} < \sigma$. Note that the observer jump map (14) yields $|z(t_1)| = |z(t_4)|$.

Remark 2: Proposition 2 allows to use a certain pre-set value $\sigma_{\min}$ overcoming the problem of on-line detection of the saturation value, improving accuracy of the estimate. Furthermore, the results of Proposition 2 hold even if the measure of the signal $y(t)$ is affected by the quantization, or the measure of the jump times t_j's is imperfect, given that measures of $y(t)$ and t at quantization levels are available. □

Consider now the signal

$$y(t) = sat_\sigma(E_0 + E_1 \sin(\omega_1 t + \phi_1)), \qquad (15)$$

with known $\sigma > 0$, and unknown E_0, E_1, ω_1 and ϕ_1. Let $\chi = [\zeta, x, q]$ be the solution of the hybrid system (3) with flow map

$$F(\zeta,x,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\; F(\zeta,x,1) = \begin{bmatrix} x \\ -\theta_1\zeta + \theta_0 \\ 0 \end{bmatrix}, \qquad (16)$$

where $\theta_1 = \omega_1^2$, $\theta_0 = E_0\theta_1$, and the jump map $G(\chi)$ given by (5), such that $y(t) = \zeta(t,j)$ for all $t \in \mathcal{T}$. The flow set C and the jump set D are defined exactly as in Section II (in this case the hybrid trajectory $\chi(t,j)$ may hit only one of the boundaries $\zeta = \pm\sigma$).

Consider $\mathcal{T}_0$ and $\mathcal{T}_1$ given by (10), and define the estimation error as

$$\begin{aligned} z_x &= k_1\zeta + \xi_3 + \gamma_1 R(\xi_1,\xi_4)\Theta - x, \\ z_\Theta &= \gamma_2 R(\xi_1,\xi_4)^\top \zeta + \xi_2 - \gamma_1\Theta, \end{aligned} \qquad (17)$$

where k_1, γ_1, γ_2 are positive constants, $\xi_i \in \mathbb{R}$ for $i = \{1,3,4\}$, $\xi_2 \in \mathbb{R}^2$, $\Theta = [\theta_1, \theta_0]^\top \in \mathbb{R}^2$, and $R(\xi_1,\xi_4) = [\xi_1, \xi_4]^\top \in \mathbb{R}^{2\times 1}$. The observer flow map $F_o(\zeta,\xi,q)$ is selected[5] as

$$F_o(\zeta,\xi,1) = \begin{bmatrix} -k_1\xi_1 - \frac{\zeta}{\gamma_1} \\ -\gamma_2\left(y\dot{R}^\top + R^\top\left(k_1\zeta + \xi_3 + R\left(\xi_2 + \gamma_2 R^\top\zeta\right)\right)\right) \\ -k_1\xi_3 - k_1^2\zeta \\ -k_1\xi_4 - \gamma_1^{-1} \end{bmatrix},$$

$$F_o(\zeta,\xi,0) = 0 \qquad (18)$$

yielding

$$\begin{aligned} \dot{z}_x &= -k_1 z_x, \\ \dot{z}_\Theta &= -\gamma_2 R(\xi_1)^\top z_x - \gamma_2 R(\xi_1)^\top R(\xi_1) z_\Theta. \end{aligned}$$

[4] $\mathcal{T}$ is the hybrid time domain defined in Proposition 1.

[5] $R(\xi_1,\xi_4)$ is shortly referred to as R.

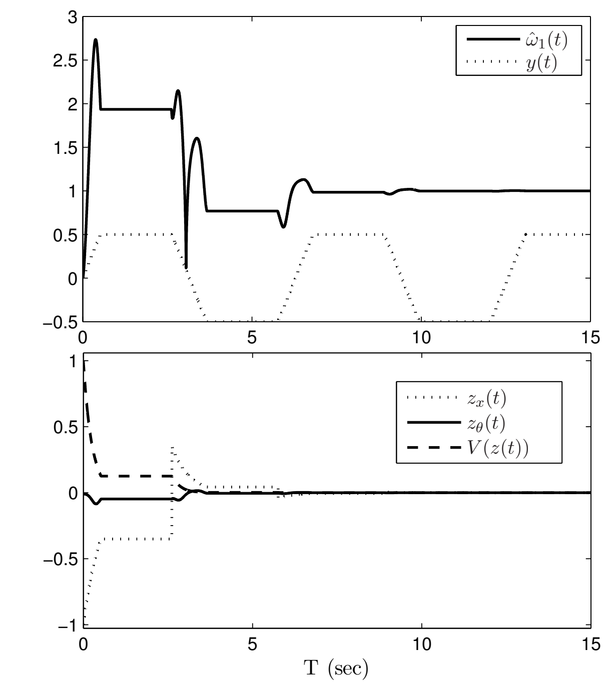

Fig. 4. Angular frequency estimation: 50% saturation, *i.e.* $\sigma = 0.5E_1$) (top). Evolution of the estimation errors $z_x(t)$, $z_\theta(t)$, and the Lyapunov function $V(z(t)) = |z(t)|^2$ (bottom).

The observer jump map $G_o(\zeta, \xi, q)$ is selected such that $|z^+| = |z|$, namely

$$G_o(\zeta,\xi,0)=\begin{bmatrix}-\xi_1\\ \xi_2+2\gamma_2\zeta R(\xi_1,\xi_4)^\top\\ -\xi_3-2k_1\zeta\\ -\xi_4\end{bmatrix}, G_o(\zeta,\xi,1)=\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\\ \xi_4\end{bmatrix}. \tag{19}$$

To conclude, assuming that $\sigma > |E_0| - |E_1|$ (which implies that the signal $y(t)$ is not permanently saturated) the claims in Propositions 1 and 2 hold, where F_H and G_H are defined using $F(\cdot)$ in (16) and the $F_o(\cdot)$ and $G_o(\cdot)$ in (18) and (19), respectively, and the estimates are evaluated as

$$\hat{\theta} = \frac{1}{\gamma_1}\left(\gamma_2\zeta R(\xi_1)^\top + \xi_2\right),$$
$$\hat{x} = k_1\zeta + \xi_3 + \gamma_1 R(\xi)\hat{\theta}.$$

III. Numerical Examples

The effectiveness of the proposed observers is now illustrated by means of some simulations.

Simulation results considering the hybrid observer of Proposition 1 and the signal $y(t) = sat_\sigma(E_1 \sin(\omega_1 t + \phi_1))$, with $E_1 = 1$, $\omega_1 = 1$, $\phi_1 = 0$, $\sigma = 0.5$, and observer gains $[k_1, \gamma_1, \gamma_2] = [2, 0.01, 0.3]$, are shown in Figure 4 together with the time histories of the Lyapunov function $V(z(t)) = |z(t)|^2$ and of the errors $z_x(t)$ and $z_\theta(t)$.

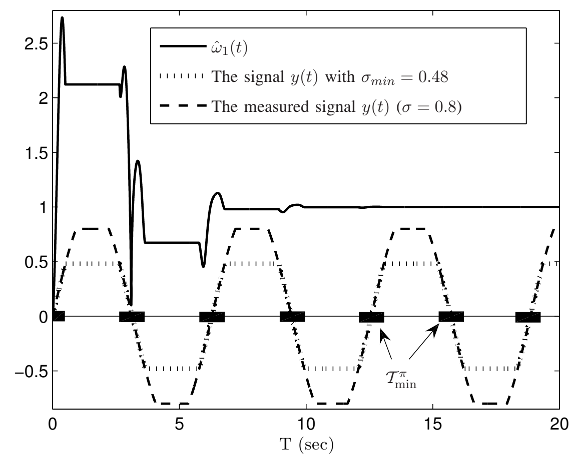

Fig. 5. Simulation with $\sigma_{\min} = 0.6\sigma$ and $\sigma = 0.8$. The estimate $\hat{\omega}_1(t)$ asymptotically converges to $\omega_1 = 1$ for $(t, j) \in \mathcal{T}_{\min}$, the projection of which on the time axis t is referred as $\mathcal{T}^\pi_{\min}$.

Figure 5 illustrates the effectiveness of the observer, with restriction $\mathcal{T}_{\min}$, when $\sigma_{min} = 0.6\sigma$ and $\sigma = 0.8$.

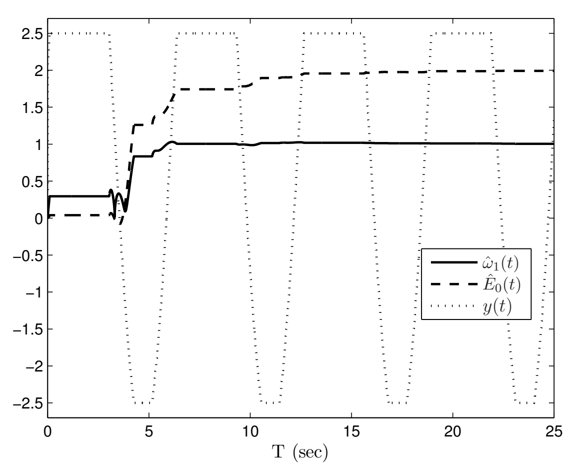

Fig. 6. Angular frequency and bias estimation.

Results in the case of a signal with constant bias with $E_0 = 2$, $E_1 = 5$, $\omega_1 = 1$, $\sigma = 0.5E_1$, are shown in Figure 6, where the observer parameters are

$[k_1, \gamma_1, \gamma_2] = [2, 1, 10]$.

Finally, the observer is tested with respect to measurement noise, *i.e.* the measured signal is $y(t) = sat_\sigma(5\sin t + d(t))$ with $\sigma = 0.8$ and $d(t) = 0.1\sin 10t$. The observer parameters are set as $[k_1, \gamma_1, \gamma_2] = [2, 0.03, 0.001]$ and simulation results are shown in Figure 7. The estimation error is sensible to the high frequency noise and, to limit the noise effect on the estimate, a small value of γ_2 has been selected (see [14] for details). Formal statements about robustness of the proposed results with respect to measurement noise requires an extensively use of the tools in [15] and is out of the scope of this paper.

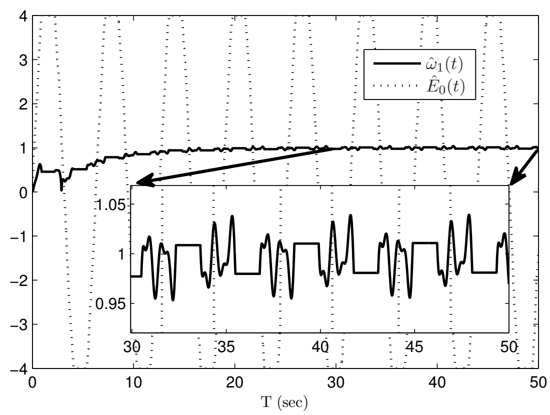

Fig. 7. Angular frequency estimation in the presence of measurement noise.

IV. CONCLUSIONS

We modeled a single frequency saturated signal via hybrid model with unknown parameters, related to the signal angular frequency and bias, estimated on-line by a hybrid observer. The approach we propose may be effective when the measured signal is cut-off due to sensor's limitations or distortions. Asymptotic properties and robustness with respect to the saturation levels are discussed and illustrated by means of simulations.

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