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H_∞-PD Controller for Suspension Systems with MR Dampers

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***Abstract*— In this paper, we consider the implementation of a static H_∞ output feedback controller to a quarter vehicle suspension system with a semi-active magnetorheological fluid (MRF) damper. Unlike most of the existing literature, all the states in the equation of motion are relative displacements and velocities between sprung, unsprung masses and road disturbance instead, in addition the input to the system is the acceleration of road disturbance. The measurements of the system are the relative displacement and velocity between sprung and unsprung masses only, which makes the practical implementation feasible. The controller of the system is composed of the H_∞-PD controller, which serves as a system controller to provide the desired damping force with the necessary robustness, and a modified version of the clip-optimal controller, which serves as a damper controller to track the the desired damping force. Satisfactory results are obtained through numerical simulations. To obtain the damper controller adopted in this work, an assumption regarding the dynamics of the MR damping force is verified through several numerical experiments. This dynamic equation indeed can also be served as an estimator of the magnetorheological damping force.**

keywords: **Magnetorheological fluid (MRF) damper, suspension system, semi-active control, H_∞-PD controller**

I. Introduction

For years, vibration attenuation of various dynamic systems has received broad attentions from both academic and industry. In the automobile industry the perceived comfort level and ride stability of a vehicle are two of the most important factors in a subjective evaluation of a vehicle. If a primary suspension is designed to optimize the handling and stability of the vehicle, the operator often perceives the ride to be rough and uncomfortable. On the other hand, if the suspension is designed for ride comfort alone, the vehicle may not be stable during maneuvers. As a result, the performance of primary suspensions is always defined by the compromise between ride and handling.

A semiactive suspension consists of a spring and a damper but, unlike a passive suspension, the value of the damper coefficient "c" can be controlled and updated. Various semi-active devices have been proposed to dissipate vibration energy in a structural or vehicle suspension system (see [1] and the references therein). The magneto-rheological (MR) dampers are new devices that use MR fluid to alter the damping coefficient. These fluids demonstrate dramatic changes in their rheological behaviors in response to a magnetic field.

To control the MR dampers, various control strategies have been proposed, just name a few, sliding mode control [2, 3] (needs a reference model), H_∞ control [4] (needs an inverse model of MR damper), clipped-optimal control [5] (on-off type). In this paper, an H_∞ output feedback controller, an extension of the work given by Gadewadikar [6], is proposed. The proposed controller which serves as a system controller keeps the robustness feature of a H_∞ controller and provides the desired damping force F_H, then it is integrated with a damper controller which generates the command voltage to change the viscosity of the MR damper, so that the MR damping force would be able to suppress the vehicle vibration semi-actively.

This paper is organized as follows. At first a quarter vehicle with a Bouc-Wen MR damper model is described in section II. Then the controller synthesis for the H_∞ output feedback controller and a modified clip-optimal damper controller are discussed in section III. Finally the effectiveness of the proposed controller and assumption are verified through numerical examples.

II. Quarter Vehicle Model with MR Damper

A. Quarter Vehicle Model

In this work we investigate only the vertical oscillating behavior of a vehicle. The response can be mathematically described by a relatively simple set of dynamic equations known as a quarter-car simulation.The reason favoring the quarter car is the fact that it covers the appropriate frequency range responsible for exciting vehicle vibrations and emphasizes those that excite modal resonances. The frequency response of the quarter car extends from approximately 0.5 to 20 Hz with some emphasis on roughness at the body bounce frequency and the axle resonance frequency.

The equations of motion of the quarter-car model

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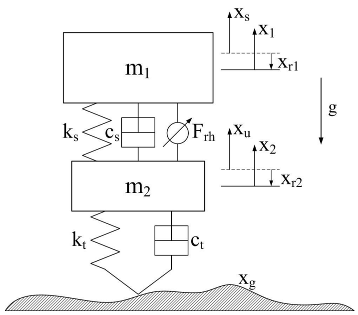

Fig. 1. Quarter Vehicle Suspension model

depicted in Fig. 1 can be written as

$$M\begin{bmatrix}\ddot{x}_s\\ \ddot{x}_u\end{bmatrix}+C\begin{bmatrix}\dot{x}_s\\ \dot{x}_u\end{bmatrix}+K\begin{bmatrix}x_s\\ x_u\end{bmatrix} = \begin{bmatrix}-m_1\\ -m_2\end{bmatrix}g+\begin{bmatrix}0\\ k_tx_g+c_t\dot{x}_g\end{bmatrix}+\begin{bmatrix}-1\\ 1\end{bmatrix}F_{rh}, \tag{1}$$

where the system matrices are defined as

$$M\triangleq\begin{bmatrix}m_1 & 0\\ 0 & m_2\end{bmatrix}, C\triangleq\begin{bmatrix}c_s & -c_s\\ -c_s & c_s+c_t\end{bmatrix}, K\triangleq\begin{bmatrix}k_s & -k_s\\ -k_s & k_s+k_t\end{bmatrix},$$

and the quantities

m_1, m_2 are the masses of vehicle body and axle,
x_s, x_u denote vertical displacements of m_1 and m_2,
x_g is the road disturbance,
k_s, c_s represent the stiffness and damping of the uncontrolled suspension,
k_t, c_t denote the stiffness, damping of the tyre.

To remove the gravitational force from the equations of motion, let's define the shifted state variables

$$x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}\triangleq\begin{bmatrix}x_s\\ x_u\end{bmatrix}-x_r,$$

where the the reference point, x_r, is the static equilibrium position

$$x_r=\begin{bmatrix}x_{r1}\\ x_{r2}\end{bmatrix}\triangleq K^{-1}\begin{bmatrix}-m_1\\ -m_2\end{bmatrix}g=\begin{bmatrix}-\frac{m_1+m_2}{k_t}-\frac{m_1}{k_s}\\ -\frac{m_1+m_2}{k_t}\end{bmatrix}g.$$

Then we have the equation of motion for x,

$$M\ddot{x}+C\dot{x}+Kx=\begin{bmatrix}-1\\ 1\end{bmatrix}F_{rh}.+\begin{bmatrix}0\\ k_tx_g+c_t\dot{x}_g\end{bmatrix}. \tag{2}$$

For convenience, we further define the state vector

$$x_P\triangleq\begin{bmatrix}x_{P1}\\ x_{P2}\end{bmatrix}\triangleq\begin{bmatrix}x_1-x_2\\ x_2-x_g\end{bmatrix}, \text{ or } x_P=T_Px-\begin{bmatrix}0\\ x_g\end{bmatrix},$$

where the transformation matrix is defined by $T_P\triangleq\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$. Note that the inverse matrix $T_P^{-1}=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$. Then the equations of motion (2) can be re-formulated as

$$M_P\ddot{x}_P+C_P\dot{x}_P+K_Px_P=\begin{bmatrix}-1\\ 0\end{bmatrix}F_{rh}-\begin{bmatrix}m_1\\ m_1+m_2\end{bmatrix}\ddot{x}_g, \tag{3}$$

where the transformed matrices are given by

$$M_P\triangleq T_P^{-\mathrm{T}}MT_P^{-1}, C_P\triangleq T_P^{-\mathrm{T}}CT_P^{-1}, K_P\triangleq T_P^{-\mathrm{T}}KT_P^{-1}.$$

B. Modified Bouc-Wen MR Damper Model

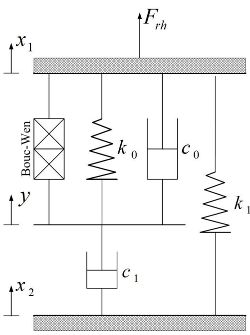

Fig. 2. Modified Bouc-Wen model

A modified Bouc-Wen model (see Fig. 2) for better predicting the response of the MR damper in the region of the yield point was proposed by Spencer [1] and is adopted in this work. The equations governing the force exerted by the MRF damper, F_{rh}, are reformulated as

$$\dot{x}_1-\dot{y}=\frac{1}{c_0+c_1}\left[-k_0(x_1-y)-\alpha z+c_1\dot{x}_{P1}\right], \tag{4}$$

$$\dot{z}=(\dot{x}_1-\dot{y})\left\{\delta-|z|^n\left[\beta+\gamma\mathrm{sgn}(\dot{x}_1-\dot{y})\mathrm{sgn}(z)\right]\right\}, \tag{5}$$

and

$$\begin{aligned}F_{rh}&=-c_1(\dot{x}_1-\dot{y})+c_1\dot{x}_{P1}+k_1(x_{P1}-\bar{x}_0),\\ &=\frac{c_1}{c_0+c_1}\left[k_0(x_1-y)+\alpha z\right]+\frac{c_0c_1}{c_0+c_1}\dot{x}_{P1}\\ &\quad+k_1(x_{P1}-\bar{x}_0),\end{aligned} \tag{6}$$

where

x_1-y and z are the internal relative displacement and hysteretic component of the MR damper, respectively,
δ, β, γ are positive constants, and
α is a scaling value for Bouc-Wen model,
k_0, k_1 are spring constants,
$\bar{x}_0$ corresponds to the initial displacement,

The voltage dependent parameters are modeled by

$$\alpha=\alpha_a+\alpha_bu,\ c_0=c_{0a}+c_{0b}u,\ c_1=c_{1a}+c_{1b}u,$$

where $\alpha_a, \alpha_b, c_{0a}, c_{0b}$ and c_{1a}, c_{1b} are positive constants. Furthermore, the command voltage is accounted for through the first-order filter

$$\dot{u} = -\eta(u - v), u(0) = 0, \tag{7}$$

where v is the command voltage sent to the current driver, and η is a positive number that reflects the lag time of the MR damper.

To reflect the situation of saturation of the magnetic field, the command input v is confined to be finite positive. As a result, u is also limited to be positive finite, that is,

$$0 \le v \le V_{\max} \text{ and } 0 \le u \le V_{\max},$$

where $V_{\max}$ is the maximum voltage to the current driver. It follows that all the related parameters α, c_0 and c_1 are all finite positive as well.

Remark 2.1: Equations (4) and (5) indicate that $(x_1 - y, z)$ and $(\dot{x}_{P1}, x_{P1} - \bar{x}_0)$ are the state vector and input of the MR damper model, respectively.

III. Controller Synthesis

The ride and handling characteristics of vehicles are governed by the stiffness and damping properties of the shock absorbers. However, by using magneto-rheological fluid (MRF) dampers, the suspension system stiffness and damping properties can be varied by the application of a magnetic field to the MRF damper.

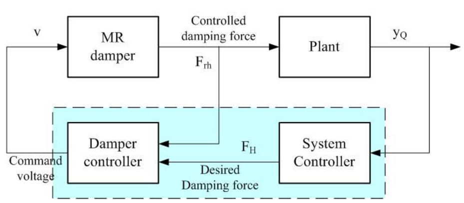

Fig. 3. Semi-active control systems for a plant integrated with an MR fluid damper

The schematic of a semiactive control system based on an MR fluid damper is illustrated in Fig. 3. The MR damper based semiactive control system consists of:

1) a system controller: generates the desired damping force according to the dynamic responses of the plant,
2) a damper controller: adjusts the command voltage to the current driver to track the desired damping force.

The damping force of the MR fluid damper should be monitored and/or predicted and fed to the damper controller to generate the command voltage according to the desired damping force generated by the system controller [7].

A. System Controller: H_∞ type PD Controller

At first, define the state variables for the quarter car model as

$$x_Q(t) \triangleq \begin{bmatrix} x_P(t) \\ \dot{x}_P(t) \end{bmatrix} = \begin{bmatrix} x_{P1} & x_{P2} & \dot{x}_{P1} & \dot{x}_{P2} \end{bmatrix}^{\mathrm{T}}, \tag{8}$$

equation (3) can be rewritten in the state-space form as

$$\dot{x}_Q = Ax_Q + BF_{rh} + B_w \ddot{x}_g(t), \tag{9}$$

where

$$A \triangleq \begin{bmatrix} 0 & I \\ -M_P^{-1}K_P & -M_P^{-1}C_P \end{bmatrix},$$
$$B \triangleq \begin{bmatrix} 0 \\ 0 \\ -\left(\frac{1}{m_1} + \frac{1}{m_2}\right) \\ \frac{1}{m_2} \end{bmatrix}, \; B_w \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}. \tag{10}$$

Next, consider the main performance criteria in vehicle suspension design that includes:

1) ride comfort - keep the transfer functions from road disturbance x_g to car body (sprung mass) acceleration $\ddot{x}_1$ small over the frequency range of $0 - 65$rad/sec.
2) road holding ability - require the transfer function from x_g to tyre deflection x_{Q2} should be small.
3) suspension deflection - keep the transfer function from x_g to suspension deflection x_{Q1} small enough to prevent excessive suspension bottoming.

For the time being, a perfect tracking is assumed, that is $F_{rh} = F_H$, where F_H is the desired input force. Accordingly it is naive to choose the performance variable [4]

$$z_H = C_1 x_Q + D_1 F_H, \tag{11}$$

where $C_1 \triangleq \begin{bmatrix} \frac{k_s}{m_1} & 0 & \frac{c_s}{m_1} & 0 \\ \alpha_H & 0 & 0 & 0 \\ 0 & \beta_H & 0 & 0 \end{bmatrix}$, $D_1 \triangleq \begin{bmatrix} \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}$, in which $\alpha_H > 0$ and $\beta_H > 0$ are scalar weightings. Assume that the output signals are the suspension deflection and the velocity of the sprung mass so that

$$y_Q = Cx_Q, \tag{12}$$

where $C \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Note that $CC^{\mathrm{T}} = I, CB_w = 0$. The goal is to determine the output feedback gain K in the feedback controller of the following form:

$$F_H = K_H y_Q = K_H C x_Q,$$

such that the closed-loop is stabilized and the H_∞-norm of the closed-loop transfer function from $\ddot{x}_g$ to

z_H, $\|T_{z_H x_g}\|_\infty$, is minimized, that is,

$$\frac{\int_0^\infty \|z_H(t)\|^2 dt}{\int_0^\infty \ddot{x}_g^2(t) dt} \leq \gamma^2. \tag{13}$$

The following theorem is a direct extension of [6]. For notational convenience, let us define

$$Q \triangleq C_1^{\mathrm{T}} C_1,\ R \triangleq D_1^{\mathrm{T}} D_1,\ S \triangleq C_1^{\mathrm{T}} D_1.$$

Theorem 3.1: Assume that (A, C_1) is detectable. Then the system defined by (9), (11) and (12) is output-feedback stabilizable with $\|T_{z_H x_g}\|_\infty < \gamma$, if and only if

1) (A,B) is stabilizable, and (A,C) is detectable and
2) there exist matrices K_H and L such that

$$K_H C + R^{-1}(B^{\mathrm{T}} P + S^{\mathrm{T}}) = R^{-1} L, \tag{14}$$

where $P^{\mathrm{T}} = P, P > 0$ is a solution to the Riccati equation

$$\begin{aligned} 0 = &A^{\mathrm{T}} P + PA + Q + \frac{1}{\gamma^2} P B_w B_w^{\mathrm{T}} P \\ &- (PB + S) R^{-1} (B^{\mathrm{T}} P + S^{\mathrm{T}}) + L^{\mathrm{T}} R^{-1} L. \end{aligned} \tag{15}$$

Remark 3.1: The difference between Theorem 3.1 and the one given in [6] is the non-zero "S" in (14).

Remark 3.2: The static gain can easily be obtained from equation (14),

$$\begin{aligned} K_H &= R^{-1}[L - (B^{\mathrm{T}} P + S^{\mathrm{T}})] C^{\mathrm{T}} (C C^{\mathrm{T}})^{-1}, \\ &= R^{-1}[L - (B^{\mathrm{T}} P + S^{\mathrm{T}})] C^{\mathrm{T}}, \end{aligned} \tag{16}$$

where the fact that $CC^{\mathrm{T}} = I$ has been implemented in this case. Substituting (16) into (14) leads to the relationship between L and P,

$$\begin{aligned} 0 &= [L - (B^{\mathrm{T}} P + S^{\mathrm{T}})][I - C^{\mathrm{T}}(CC^{\mathrm{T}})^{-1} C], \\ &= [L - (B^{\mathrm{T}} P + S^{\mathrm{T}})][I - C^{\mathrm{T}} C]. \end{aligned}$$

Hence, when the controller gain is pre-multiplied to the measurement vector we have

$$\begin{aligned} K_H y_Q = K_H C x_Q &= R^{-1}[L - (B^{\mathrm{T}} P + S^{\mathrm{T}})] C^{\mathrm{T}} C x_Q, \\ &= R^{-1}[L - (B^{\mathrm{T}} P + S^{\mathrm{T}})] x_Q, \end{aligned} \tag{17}$$

which takes the same form as a state feedback controller.

B. Damper Controller

To determine the damper controller, we define the following Lyapunov function of the closed-loop system

$$V \triangleq x_Q^{\mathrm{T}} P x_Q + \rho (F_{rh} - F_H)^2. \tag{18}$$

Note that the equation of motion (9) can be re-written as

$$\dot{x}_Q = A x_Q + B F_H + B(F_{rh} - F_H) + B_w \ddot{x}_g(t).$$

It follows that

$$\begin{aligned} \dot{V} &= \dot{x}_Q^{\mathrm{T}} P x_Q + x_Q^{\mathrm{T}} P \dot{x}_Q + 2\rho (F_{rh} - F_H)(\dot{F}_{rh} - \dot{F}_H), \\ &= x_Q^{\mathrm{T}} (\tilde{A}^{\mathrm{T}} P + P \tilde{A}) x_Q + 2 x_Q^{\mathrm{T}} P B_w \ddot{x}_g, \\ &\quad + 2\left[x_Q^{\mathrm{T}} P B + \rho (\dot{F}_{rh} - \dot{F}_H)\right](F_{rh} - F_H), \end{aligned}$$

where $\tilde{A} \triangleq A + BK_H C$. For the worst case of road disturbance $\ddot{x}_g^\star = \frac{1}{\gamma^2} B_w^{\mathrm{T}} P x_Q$, let the output feedback gain K and P satisfy (14) and (15), respectively. This renders

$$\begin{aligned} \dot{V}\big|_{\ddot{x}_g = \ddot{x}_g^\star} = &-z_H^{\mathrm{T}} z_H + \gamma^2 (\ddot{x}_g^\star)^{\mathrm{T}} \ddot{x}_g^\star \\ &+ 2\left[x_Q^{\mathrm{T}} P B + \rho (\dot{F}_{rh} - \dot{F}_H)\right](F_{rh} - F_H). \end{aligned}$$

Note that since $CB_w = 0$, we have

$$\dot{F}_H = K_H C \dot{x}_Q = K_H C A x_Q + K_H C B F_{rh}.$$

Assumption: As motivated by the clipped-optimal method [5], "assuming" that the command force F_{rh} satisfy

$$\dot{F}_{rh} = \dot{F}_H - \frac{1}{\rho} B^{\mathrm{T}} P x_Q + \beta_{rh} F_{rh} v. \tag{19}$$

Hence, we end up with

$$\dot{V}\big|_{\ddot{x}_g = \ddot{x}_g^\star} = -z_H^{\mathrm{T}} z_H + \gamma^2 (\ddot{x}_g^\star)^{\mathrm{T}} \ddot{x}_g^\star - \beta_{rh} F_{rh} (F_H - F_{rh}) v, \tag{20}$$

where $\beta_{rh} > 0$. In this work the assumption (19) will be verified through numerical examples at this moment.

Remark 3.3: Equation (19) can also be thought of as an estimator of magnetorheological force F_{rh}. Furthermore, owing to the identity (17) and the fact $S^{\mathrm{T}} x_Q = S^{\mathrm{T}} C^{\mathrm{T}} y_Q$, the second term of (19) can be replaced by

$$-B^{\mathrm{T}} P x_Q = (R K_H - S^{\mathrm{T}} C^{\mathrm{T}}) y_Q - L x_Q.$$

One of the simple feedback linearization techniques used to track the desired MR damping force, F_H, is the clipped-optimal method

$$v = V_{\max} H[F_{rh}(F_H - F_{rh})], \tag{21}$$

where $H(\cdot)$ is the Heaviside step function. Yet in this work we adopt a modified version of the above method which yields a smooth command voltage [8],

$$v = \mathrm{sat}_{[0, V_{\max}]}[\alpha_{FB} (F_H - F_{rh}) \mathrm{sgn}(F_{rh})], \tag{22}$$

where $\alpha_{FB} > 0$ is a free parameter, and the one-sided saturation function is defined by

$$\mathrm{sat}_{[0, V_{\max}]}(\cdot) \triangleq \max(0, \min(\cdot, V_{\max})).$$

For this case, the last term in (20) can be written in detail as

$$\begin{aligned} &-\beta_{rh} F_{rh} (F_H - F_{rh}) v \\ &= -\beta_{rh} |F_{rh} (F_H - F_{rh})| \mathrm{sat}_{[0, V_{\max}]}[\alpha_{FB} |F_H - F_{rh}|] \leq 0. \end{aligned}$$

Integrating (20) (and replacing $\ddot{x}_g^\star$ by $\ddot{x}_g$) renders

$$
\begin{aligned}
&V(T) - V(0) \\
&= \int_0^T [-z_H^{\mathrm{T}} z_H + \gamma^2 (\ddot{x}_g)^{\mathrm{T}} \ddot{x}_g - \beta_{rh} F_{rh} (F_H - F_{rh}) v] dt.
\end{aligned}
$$

Hence, selecting $x_Q(0) = 0, F_{rh}(0) = F_H(0)$ and the non-negativeness of $V(T)$, that is $V(T) \geq 0, \forall T$, guarantee that (13) is satisfied.

IV. Numerical Examples

Example 4.1: Consider a quarter car and an MR damper model with the parameters defined by Table I and II [3], respectively. Assume that the maximum input voltage of MR damper is $V_{\max} = 2.0$V.

TABLE I

QUARTER CAR MODEL PARAMETERS

Parameter	value
m_1	372 kg
m_2	45 kg
k_s	40 kN/m
k_t	190 kN/m
c_s	0 N s/m
c_t	0 N s/m

TABLE II

PARAMETERS FOR THE MR DAMPER (RD-1005-1) [3]

Coeff.		Coeff.	
α_a	12441 N/m	c_{0_a}	784 N · s/m
α_b	38430 N/m · V	c_{0_b}	1803 N · s/m · V
β	2059020 m^{-2}	c_{1_a}	14649 N · s/m
γ	136320 m^{-2}	c_{1_b}	34622 N · s/m · V
δ	58	n	2
η	190 s^{-1}	$\bar{x}_0$	0 m
k_0	3610 N/m		
k_1	840 N/m		

Then we implement the above H_∞ controller to the quarter car model with MR damper. The performance weighting parameters are chosen as $\alpha_H = 100, \beta_H = 50, \alpha_{FB} = 0.3, \beta_{FB} = 1$, for illustration purpose. It is easy to verify that the pair (A,B) is controllable, and both $(C_1,A),(C,A)$ are observable, since

$$
\begin{aligned}
&\operatorname{rank}\begin{bmatrix} \lambda I - A & B \end{bmatrix} = 4, \\
&\operatorname{rank}\begin{bmatrix} C \\ \lambda I - A \end{bmatrix} = \operatorname{rank}\begin{bmatrix} C_1 \\ \lambda I - A \end{bmatrix} = 4, \forall \lambda \in \mathscr{C}.
\end{aligned}
$$

After solving the coupled equations (14) and (15), we obtain the static output feedback gain

$$
K_H = \begin{bmatrix} 1.1168 \times 10^4 & 0.9444 \times 10^4 \end{bmatrix}.
$$

Figure 4 depicts the magnitude part of the Bode plot of the suspension system as MR dampers activated (assume $F_{rh} = F_H$) and inactivated ($F_{rh} = 0$). It is clear that at both of the two system resonant frequencies the magnitudes are attenuated.

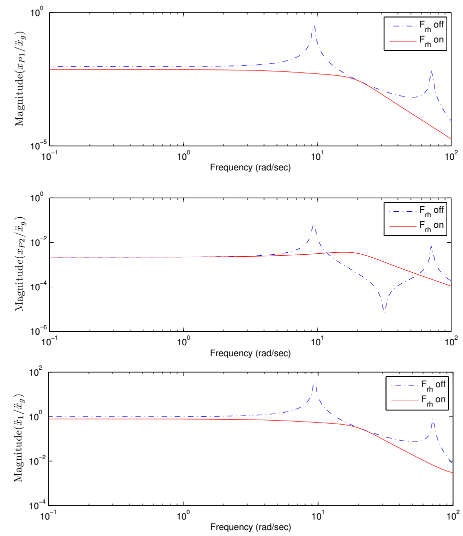

Fig. 4. Magnitude part of Bode plot for the system without and with using MR damper

Assume that the vehicle is subject to a grade C random road excitation. The system response of sprung and unsprung masses, supplied damping force (F_{rh}) and command voltage (v) are depicted in Figure 5 and 6, respectively. In addition, some typical values of the RMS of the acceleration of the sprung, the suspension deflection $(x_1 - x_2)$, and the tyre deflection $(x_2 - x_g)$ for the listed three different cases are given in Table III. These results indicate that for suspension deflection and tyre deflection the H_∞-PD controller provides a better performance compared with that of the passive suspension systems (input voltage $v = 0$V and $v = 2$V), while for the acceleration of sprung mass the H_∞ controller gives in-between performance.

TABLE III

RMS ANALYSIS FOR GRADE C ROAD EXCITATION TESTS

Damper	$\ddot{x}_1$(m/sec^2)	$x_1 - x_2$(m)	$x_2 - x_g$(m)
H_∞	0.9251	0.0016	0.0020
$v = 0$	0.5903	0.0037	0.0019
$v = 2$	1.3292	0.0018	0.0029

To verify the validity of the "assumed" dynamics of F_{rh} (19), a comparison is given in Fig. 7. As we can tell from the figure, the difference is insignificant. Actually,

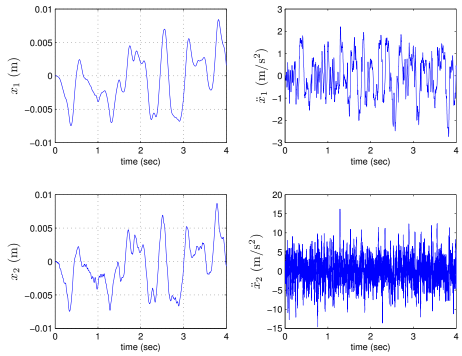

Fig. 5. Responses of sprung and unsprung masses under grade C random road excitation with augmented H_∞ controller

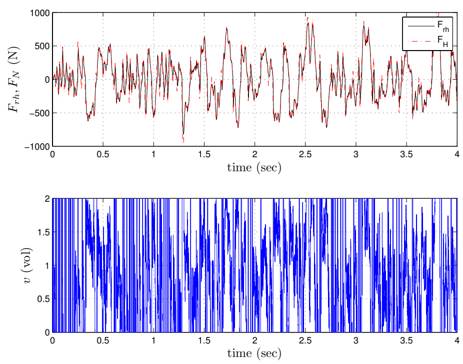

Fig. 6. Damping force F_{rh} and input voltage v under grade C random road excitation with augmented H_∞ controller

several examples conducted also show similar results.

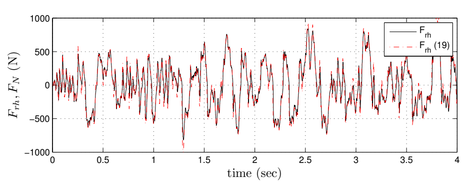

Fig. 7. Damping force F_{rh} (solid line) and the assumed damping force given by (19) (dot-dash line) ($\beta_{rh} = 0.5, \rho = 1$).

V. CONCLUSIONS AND FUTURE WORKS

The implementation of a static H_∞ output feedback controller to a quarter vehicle suspension system with magnetorheological fluid (MRF) damper is presented in this paper. Unlike most of the models used in existing literature, all the states are relative displacements and velocities between sprung mass, unsprung mass and road disturbance, the input is the acceleration from road disturbance instead. The H_∞ controller serves as a system controller to provide the desired damping force with the necessary robustness, while a modified version of the clip-optimal controller serves as a damper controller to track the the desired damping force. The proposed scheme is validated through numerical simulations under a grade C random road excitations, the results show that the H_∞-PD controller achieve a better performance than those with constant input voltages. In addition, the major assumption motivated by the clip-optimal control is verified through several examples. this alternatively implies the dynamics of the MR damper can be approximated by equation (19), or (19) can be treated as an estimator of the magnetorheological damping force.

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