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Stereo Matching for Calibrated Cameras without Correspondence

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***Abstract*— We study the stereo matching problem for reconstruction of the location of $3D$-points on an unknown surface patch from two calibrated identical cameras without using any a priori information about the pointwise correspondences. We assume that camera parameters and the pose between the cameras are known. Our approach follows earlier work for coplanar cameras where a gradient flow algorithm was proposed to match associated Gramians. Here we extend this method by allowing arbitrary poses for the cameras. We introduce an intrinsic Riemannian Newton algorithm that achieves local quadratic convergence rates. A closed form solution is presented, too. The efficiency of both algorithms is demonstrated by numerical experiments.**

***Index Terms*— Stereo Matching, Computer Vision, Correspondences, Newton's Algorithm, Lie Groups, Cholesky Decomposition.**

I. INTRODUCTION

The stereo matching problem is one of the challenging open issues in the area of computer vision [5]. Under the restriction that the $3D$ points are all lying on an unknown surface, the problem can be formulated as follows: Assume that two cameras observe a surface patch, as in Fig. 1. The two sets of image points $\{X_{1,i}\}$, and $\{X_{2,i}\}$, $X_{1,i}, X_{2,i} \in \mathbb{R}^3$ for $i = 1, \cdots, k$ are unordered, i.e. the pointwise correspondence between both sets is unknown. The only available information then is the Euclidian displacement (R, τ), $R \in SO(3)$ and $\tau \in \mathbb{R}^3$ between the cameras. From this stereo vision scenario three problems arise: (i) recover the geometry of the observed patch from the two images, (ii) establish a pointwise correspondence of both sets of image points, and (iii) find a homographic transformation from the points of one image to the points of the other. These three questions, which are closely related, have generated different approaches to the stereo matching problem. Brockett [2] considered the simplified matching problem for two finite point sets in $\mathbb{R}^3$ as an optimization on the rotation group $SO(3)$. He observed that the Gramians

$$N = \frac{1}{k}\sum_{i=1}^{k} X_{1,i}X_{1,i}^\top, \quad Q = \frac{1}{k}\sum_{i=1}^{k} X_{2,i}X_{2,i}^\top$$

are invariant up to permutations of the points $X_{1,i}$ and $X_{2,i}$, respectively, and proposed a solution by a gradient flow that achieves matching of the two Gramians.

Zhou and Ghosh [1] were interested in the recovery of the observed planar patch for coplanar cameras. Thus, they analyzed the case where only the simple translation $\tau = [h\, v\, 0]^\top$ between the cameras occurs and observed that the image points are related by a homography matrix $A \in \mathbb{R}^{3\times 3}$, whose parameters uniquely describe the unknown location of the surface patch. They expressed the problem as an optimization of a cost function $f : G_s \to \mathbb{R}$ on a Lie group G_s and developed a gradient flow algorithm, similar to Brockett's. In this way, a homographic transformation $A \in G_s$ is found without the necessity to explicitly compute the pointwise correspondences. This proposed method has only linear convergence.

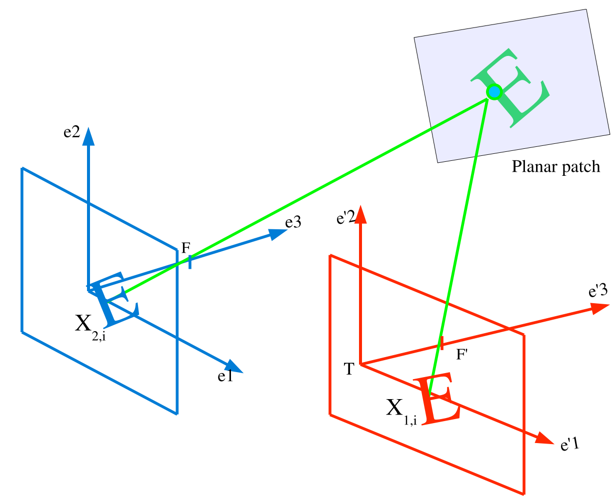

Fig. 1. Typical stereo vision scenario. Two coplanar cameras, left and right, observe a planar patch.

Li and Hartley [4] proposed a Newton-Schulz-like method to find the pointwise correspondences by matching associated Gramians. Two $k \times 2$ matrices X, Y are constructed for each pair of image point sets. It is assumed that the transformation between these matrices is given as $X = PYR$, where P is a permutation matrix and $R \in SO(2)$. This algorithm performs a Newton iteration to match the Gramians. No convergence analysis of the algorithm is given.

Based on the work by Ghosh *et al.*, we develop a Newton method to compute the planar surface parameters in the case of arbitrary camera poses. This leads to a smooth optimization problem posed on a non-compact homogeneous space M. The differentiable manifold M has been considered in [3] together with a Jacobi-type algorithm. However, this approach leads in general only to a linear convergent algorithm and thus is not fast enough for our purposes.

The Lie group G under consideration in the present work is the semi-direct product $G = \mathbb{R} \ltimes \mathbb{R}^n$. This Lie group G is actually a subgroup of G_s acting linearly on projective space $\mathbb{RP}^n$. A manifold version of Newton's method is formulated to minimize a smooth cost function $f : M \to \mathbb{R}$. This algorithm shows local quadratic convergence.

Later, based on the structure of the group elements of G and the properties of the Gramians N and Q, we elaborate a closed form solution for the same stereo matching problem.

The structure of this paper is as follows. In the next section, we present a more general problem formulation which includes the above problem as a low-dimensional case. This includes a characterization of the general homographies between image points, showing that they belong to a Lie group G and defining the homogeneous space M of tranformed Gramians. In section III, Newton and Cholesky algorithms are presented for computing the surface parameters and demonstrating local quadratic convergence. In section IV, we conclude with numerical experiments. These simulations show excellent convergence performance.

II. PROBLEM FORMULATION

In the sequel we assume that our cameras achieve projections from $\mathbb{R}^n$ and analyze the homographies between them. Let the first camera be equipped with the standard coordinate system of $\mathbb{R}^n$. Let the second camera be displaced by a Euclidean motion (R, τ), $R \in SO(n)$ and $\tau \in \mathbb{R}^n$ such that the coordinate transformation between both camera coordinate systems is given as

$$RX + \tau, \quad X \in \mathbb{R}^n \tag{1}$$

Let S denote an arbitrary hypersurface in $\mathbb{R}^n$ such that the last component of any element $P \in S$ is strictly positive. There exists then a unique number

$$\lambda : \mathbb{R}^n \to (0, \infty], \quad \lambda(X) \in (0, \infty] \tag{2}$$

such that $\lambda(X)X \in S$. For the case $\lambda = \infty$ the interpretation is that there is no finite value of $\lambda > 0$ for which $\lambda(X) \begin{bmatrix} X, 1 \end{bmatrix}^\top \in S$. Let $M \subset \mathbb{R}^n$ denote the open subset consisting of points where λ exists and is finite. We refer to λ as the the depth function of S. The associated homography for S then is the map

$$H(X) := \frac{RX + \frac{\tau}{\lambda(X)}}{e_n^\top \left(RX + \frac{\tau}{\lambda(X)}\right)}. \tag{3}$$

It can be shown that a homography is a semi-algebraic map, provided that the associated hypersurface is semi-algebraic. If the hypersurface is smooth, then the homography need not necessarily be smooth everywhere where it is defined.

Instead of starting with a hypersurface and then compute the homography, we can also reverse the process and start with an arbitrary, say smooth, analytic, or algebraic function $\lambda : M \to (0, \infty)$ and define a hypersurface as $S := \{\lambda(X)X | X \in M\}$. Then the associated homography is given as above.

Now let $X = \begin{bmatrix} x_1 & \cdots & x_{n-1} & 1 \end{bmatrix}^\top$ and consider affine hypersurfaces given as $S = \{p \in \mathbb{R}^n | p_n = \alpha_0 + \sum_{j=1}^{n-1} \alpha_j p_j\}$. Then $1/\lambda = a^\top X$, where $a = 1/\alpha_0 \cdot \begin{bmatrix} -\alpha_1 & \cdots & -\alpha_{n-1} & 1 \end{bmatrix}^\top$. If we donote the normalizing factor in the homography by $\kappa > 0$, we can write the homography as

$$H(X) = \kappa(R + \tau a^\top)X. \tag{4}$$

As $\tau = hR_\tau e_1$ holds for some rotation $R_\tau \in SO(n)$ and the scalar $h = \|\tau\| \geq 0$, we have that

$$H(X) = \kappa R_\tau (I + e_1 \widetilde{a}^\top) R_\tau^\top RX \tag{5}$$

for $\widetilde{a} := \|\tau\| R_\tau^\top Ra$. Thus, by transforming the image points

$$\begin{aligned} \widetilde{X}_{1,i} &:= \frac{\widehat{X}_{1,i}}{e_n^\top \widehat{X}_{1,i}}, \quad \widehat{X}_{1,i} := R_\tau^\top RX_{1,i}, \\ \widetilde{X}_{2,i} &:= \frac{\widehat{X}_{2,i}}{e_n^\top \widehat{X}_{2,i}}, \quad \widehat{X}_{2,i} := R_\tau^\top X_{2,i} \end{aligned} \tag{6}$$

we obtain the matching condition

$$A\widetilde{X}_{1,i} = \widetilde{X}_{2,\pi(i)} \tag{7}$$

with the linear operator $A := I + e_1 \widetilde{a}^\top$ and permutation $\pi : \{1, \ldots, k\} \to \{1, \ldots, k\}$. The set

$$G = \left\{ I_n + e_1 a^\top \in \mathbb{R}^{n \times n} \middle| 1 + e_1^\top a > 0, a \in \mathbb{R}^n \right\}. \tag{8}$$

forms a Lie group with Lie algebra

$$\mathfrak{g} := \left\{ e_1 b^\top \middle| b \in \mathbb{R}^n \right\} \tag{9}$$

and Lie bracket the matrix commutator. By exponentiating Lie algebra elements we obtain for any $g \in G$ the parameterization map

$$\nu : \mathbb{R}^n \to G, \quad \nu(b) := \exp(e_1 b^\top) = I_n + h(e_1^\top b) e_1 b^\top \tag{10}$$

with

$$h(b_1) = \begin{cases} \frac{e^{b_1} - 1}{b_1} & b_1 \neq 0 \\ 1 & b_1 = 0 \end{cases}. \tag{11}$$

Note that ν satisfies $\nu(0) = I_n$ and ν defines a global diffeomorphism onto the group G.

Lemma II.1 *Given an $(n \times n)$-matrix $N = N^\top > 0$ and let $M = \{ANA^\top | A \in G\}$. Then M is a smooth and connected n-dimensional manifold. The map*

$$\phi : G \to M, \quad \phi(A) := ANA^\top \tag{12}$$

is a global diffeomorphism. The tangent space of M at $X \in M$ is $T_X M = \{BX + XB^\top | B \in \mathfrak{g}\}$ (cf. [3]).

Correspondingly, we obtain a family of global parameterizations of the manifold M as

$$\mu_X : \mathbb{R}^n \to M, \quad \mu_X(b) := e^{e_1 b^\top} X (e^{e_1 b^\top})^\top. \tag{13}$$

Thus μ_X satisfies $\mu_X(0) = X$ and μ_X defines a global diffeomorphism onto the manifold M.

Following [1], the stereo matching problem without correspondences can be formulated as follows. From the normalized image points we form the two Gramians $N, Q \in \mathbb{R}^{n\times n}$

$$N = \tfrac{1}{k}\sum_{i=1}^{k} X_{1,i}X_{1,i}^\top, \quad Q = \tfrac{1}{k}\sum_{i=1}^{k} X_{2,i}X_{2,i}^\top. \qquad (14)$$

In the sequel we will always assume that N and Q are positive definite. This assumption corresponds to a generic situation in the stereo matching problem. Then, the stereo matching problem is equivalent to finding a transformation $A \in G$ and a permutation π on k elements, such that (7) holds for all $i = 1,\dots,k$. Of course, when the permutation matrix π is known then this amounts to solving the least squares problem of minimizing $\sum_{i=1}^{k} \|AX_{1,i} - X_{2,\pi(i)}\|^2$ over G. Often, however, such knowledge is not available and the question arises if one can find such an optimizing transformation A without knowing π.

III. STEREO MATCHING ALGORITHMS

The nice idea of [1] was to reformulate the exact task of solving the former equation via the weaker task of achieving the matching condition for the Gramians

$$Q = ANA^\top. \qquad (15)$$

Motivated by this we aim to solve the minimization of the least squares cost function

$$f : M \to \mathbb{R}, \quad f(X) = \|Q - X\|^2 \qquad (16)$$

where $\|Y\|^2 := \sum_{i,j=1}^{n} y_{ij}^2$.

Lemma III.1 *Let $N = N^\top$ be positive definite. The function $f(X) = \|Q-X\|^2$ has a unique critical point $X_c \in M$. The critical point X_c is characterized by the property that the first column coincides with that of Q.*

Proof: The derivative $\mathrm{D}f$ of $f : M \to \mathbb{R}$ evaluated at $X \in M$ acting on $(BX + XB^\top) \in T_X M$, with $B \in \mathfrak{g}$ is

$$\mathrm{D}f(X)\cdot(BX + XB^\top) = 4\operatorname{tr}\left(B^\top (X-Q)\, X\right). \qquad (17)$$

By the special form (9) of the matrix $B \in \mathfrak{g}$, (17) vanishes if and only if

$$e_1e_1^\top \cdot (X_c - Q)X_c = 0_n \qquad (18)$$

holds, i.e., by the positive definiteness of X_c, if and only if the first row of $X_c = X_c^\top$ and $Q = Q^\top$, respectively, are identical. On the other hand, $(X_c)_{ij} = N_{ij}$ holds for all $2 \le i,j \le n$, because the group action $G\times M \to M$ defined by $(A,N) \mapsto ANA^\top$ affects only the first row and the first column of N. Thus X_c is the unique critical point. ■

Note that in the noise free case there exists a group element $A \in G$ such that

$$Q - ANA^\top = 0_n.$$

Consequently, the unique global minimum X_c of the function f is characterized by $X_c = Q$ with critical value equal to zero.

A. NEWTON'S ALGORITHM

Now we will develop a Newton-type algorithm to minimize the composition of the cost function (16) with the global parameterization as in (13). The gradient and Hessian of this composed function can be explicitly computed as

$$\nabla(f\circ\mu_X)(0) = 4X(X-Q)e_1, \qquad (19)$$

$$\begin{aligned}\mathrm{H}_{f\circ\mu_X}(0) = {}& 4(X^2 + Xe_1e_1^\top X + e_1^\top (X-Q)e_1 X) \\ &+ \tfrac{1}{2}(X(X-Q)e_1e_1^\top + e_1e_1^\top (X-Q)X)).\end{aligned} \qquad (20)$$

Note that the Hessian at the unique critical point X_c simplifies to

$$\mathrm{H}_{f\circ\mu_{X_c}}(0) = 4\left(X_c^2 + X_ce_1e_1^\top X_c\right) \qquad (21)$$

and thus is positive definite. We now give a more precise description of those points where the Hessian is invertible. Due to the above simple form of the Hessian at a critical point we consider a modification of the Newton step as follows. For any $X \in M$ let

$$\widehat{\mathrm{H}}_{f\circ\mu_X}(0) = 4\left(X^2 + Xe_1e_1^\top X\right). \qquad (22)$$

The Newton-type algorithm we propose is defined by iterating a map

$$s : M \to M. \qquad (23)$$

Let $x^{\mathrm{opt}}(X)$ denote the solution of

$$\widehat{\mathrm{H}}_{f\circ\mu_X}(0)\, x = -\nabla(f\circ\mu_X)(0). \qquad (24)$$

Thus

$$x^{\mathrm{opt}}(X) = X^{-1}(I_n - \tfrac{1}{2}e_1e_1^\top)(Q-X)e_1 \qquad (25)$$

is well-defined for any $X \in M$. The algorithmic map s is given as

$$s(X) = \mu_X\left(x^{\mathrm{opt}}(X)\right). \qquad (26)$$

Theorem III.1 *Let a planar patch be given by*

$$z = \alpha_0 + \alpha_1 x_1 + \cdots + \alpha_{n-1}x_{n-1} \qquad (27)$$

being observed for two identical cameras, both with the same focal length f and the camera centers displaced by $\tau = e_1$. Let N and Q be the Gramians from the normalized image points of the planar patch as in (14). The algorithm defined by iterating

$$X_0 = N, \quad X_{t+1} = s(X_t) \qquad (28)$$

is locally quadratically convergent to the unique global minimum X_c of the cost function f (16). Furthermore, the sequence of matrices in the Lie group G

$$A_0 = I_n, \quad A_{t+1} = \nu(x^{\mathrm{opt}}(X_t))A_t \qquad (29)$$

converges locally to the optimal transformation

$$A = I_n + e_1a^\top \in G, \qquad (30)$$

with

$$a = -\tfrac{1}{\alpha_0}\begin{bmatrix}\alpha_1 & \cdots & \alpha_{n-1} & -1\end{bmatrix}^\top \qquad (31)$$

solving the stereo matching problem in $\mathbb{R}^n$.

Proof: It is seen by inspection that the algorithmic map s is smooth. It therefore suffices to check that the derivative of s at the unique critical point X_c vanishes. Let ξ denote an arbitrary tangent element of M at X_c. Then

$$\xi = e_1 x^\top X_c + X_c x e_1^\top \tag{32}$$

for a given, arbitrary element $x \in \mathbb{R}^n$. Note that the critical point X_c is characterized by the property $(Q - X_c)e_1 = 0$. Thus h(b)=1 for

$$b := e_1^\top (Q - X_c)(I_n - \frac{1}{2} e_1 e_1^\top) X_c^{-1} e_1. \tag{33}$$

The derivative of s at X_c therefore is

$$\mathrm{D}\, s(X_c)\xi = \xi - e_1 e_1^\top \xi - \xi e_1 e_1^\top + e_1^\top \xi e_1 e_1 e_1^\top. \tag{34}$$

By substituting $\xi = e_1 x^\top X_c + X_c x e_1^\top$ into (34) we see that

$$\mathrm{D}\, s(X_c)\xi = 0. \tag{35}$$

Now, let A_t be as in (29). From the definition of the algorithmic iteration, we get

$$X_{t+1} = s(X_t) = A_{t+1} N A_{t+1}^\top. \tag{36}$$

The matrices X_i are elements of the manifold M, the latter being diffeomorphic with the Lie group G (cf. Lemma II.1). As the sequence of matrices $(X_t | X_t \in M)$ is convergent to the critical point X_c, the sequence $(A_t | A_t \in G)$ is convergent to the optimal group element $A \in G$ such that $X_c = ANA^\top$ (cf. Lemma III.1). The matrix A is known to have the desired form. ■

B. CHOLESKY APPROACH FOR PLANAR SURFACES

An alternative approach of this problem can be developed using the Cholesky factorization of positive definite Gramians. Let

$$N = U_N U_N^\top, \quad Q = U_Q U_Q^\top \tag{37}$$

denote the unique Cholesky factorization of the Gramians N and Q, respectively, with upper triangular matrices U_N, U_Q with positive diagonal entries. Then, for group elements

$$A(x_1, \ldots, x_n) = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \tag{38}$$

we introduce another cost function

$$\begin{aligned} &\widetilde{f} \colon \mathbb{R}^n \to \mathbb{R}, \\ &\widetilde{f}(x_1, \ldots, x_n) := \|A(x_1, \ldots, x_n) U_N - U_Q\|^2 \end{aligned} \tag{39}$$

to be minimized. This function $\widetilde{f}$ is convex and its gradient and Hessian can be easily computed. In the special case $n = 3$, which is relevant for vision applications, we obtain them explicitly as

$$U_N = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & g \end{bmatrix}, \quad U_Q = \begin{bmatrix} r & s & t \\ 0 & u & v \\ 0 & 0 & w \end{bmatrix}. \tag{40}$$

$$\nabla \widetilde{f}(x, y, z) = 2 U_N (A(x, y, z) U_N - U_Q)^\top e_1 \tag{41}$$

$$\mathrm{H}_{\widetilde{f}(x,y,z)} = 2 U_N U_N^\top = 2N = 2 \begin{bmatrix} a^2+b^2+c^2 & bd+ce & cg \\ bd+ce & d^2+e^2 & eg \\ cg & eg & g^2 \end{bmatrix}. \tag{42}$$

It is clear that $\mathrm{H}_{\widetilde{f}(x,y,z)}$ is positive definite. A Newton iteration step for this problem then moves right into the minimum

$$\begin{aligned} \begin{bmatrix} x_{t+1} \\ y_{t+1} \\ z_{t+1} \end{bmatrix} &= \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} - \mathrm{H}^{-1}_{\widetilde{f}(x_t, y_t, z_t)} \nabla \widetilde{f}(x_t, y_t, z_t) \\ &= \begin{bmatrix} \frac{r}{a} \\ \frac{as-rb}{ad} \\ \frac{adt-cdr-aes+ber}{adg} \end{bmatrix} \end{aligned} \tag{43}$$

Thus, $A(x, y, z) \in G$ with

$$x = \tfrac{r}{a}, \; y = \tfrac{as-rb}{ad}, \; z = \tfrac{adt-cdr-aes+ber}{adg} \tag{44}$$

is the unique group element minimizing $\widetilde{f}$. Therefore, calculating Cholesky factors for N and Q and substituting into (44) represents an alternative way to solve the stereo matching problem in closed form. Observe that in the noise free case at the minimum

$$d = u, \quad e = v, \quad g = w, \tag{45}$$

$AU_N = U_Q$ and therefore the minimal value is equal to zero.

IV. NUMERICAL EXPERIMENTS

A. TWO CAMERAS OBSERVING A PLANAR PATCH

The objective of this experiment was to recover the plane parameters α, β, and γ from a set of observations or measurements on two images of the planar patch

$$z = \alpha + \beta x + \gamma y \tag{46}$$

without knowing point-to-point correspondences between the images. For the simulation, 2000 $3D-$points on the plane defined by

$$z = 21.6478 + 0.414214x, \tag{47}$$

i.e.,

$$\alpha = 21.6478, \; \beta = 0.414214, \; \gamma = 0 \tag{48}$$

were generated such that they formed the letter "E". The points were uniformly randomly distributed on the letter. All points were projected onto the left and onto the right image, firstly, assuming there is no noise in the detected features and secondly, assuming presence of noise.

The two cameras were assumed to have focal length $f = 1$. The second camera was assumed to have a displacement of $\tau = \begin{bmatrix} 10.0 & 4.3 & -6.7 \end{bmatrix}^\top$ from the first camera and a rotation of $R = R_z(\frac{\pi}{6}) R_x(\frac{\pi}{72}) R_y(\frac{\pi}{12})$

$$R = \begin{bmatrix} 0.8309 & -0.4995 & -0.2452 \\ 0.4927 & 0.8652 & -0.0929 \\ 0.2586 & -0.0436 & 0.9650 \end{bmatrix}. \tag{49}$$

1) Noise free simulation: Images for both cameras were calculated by projecting perspectively each of the points from the original set. The resulting images are shown in Fig. 2. The calculated Gramians N and Q from these image points were

$$\begin{aligned} N &= \begin{bmatrix} 4.0394 & -2.2089 & -1.8830 \\ -2.2089 & 1.6256 & 0.9535 \\ -1.8830 & .0953 & 1.0 \end{bmatrix}, \\ Q &= \begin{bmatrix} 12.5868 & -3.944 & -3.3511 \\ -3.944 & 1.6256 & 0.9535 \\ -3.3511 & 0.9535 & 1.0 \end{bmatrix}. \end{aligned} \tag{50}$$

2) Simulation with noisy data: To simulate noise in the measurement random points of a Gaussian distribution with zero mean and standard deviation 0.025 were added to the projections of both images. The resulting image points are shown in Fig. 3. The corresponding Gramians N and Q were

$$N = \begin{bmatrix} 4.9758 & -2.2248 & -3.3721 \\ -4.0234 & 1.6603 & 0.9563 \\ -3.3721 & 0.9563 & 1.0 \end{bmatrix}, \qquad Q = \begin{bmatrix} 12.8834 & -4.0234 & -3.3721 \\ -4.0234 & 1.6603 & 0.9563 \\ -3.3721 & 0.9563 & 1.0 \end{bmatrix}. \tag{51}$$

3) Newton's algorithm without noise: Newton's method, *cf.* Section (III), needed 12 iterations to find the solution. This is illustrated in Fig. 4.

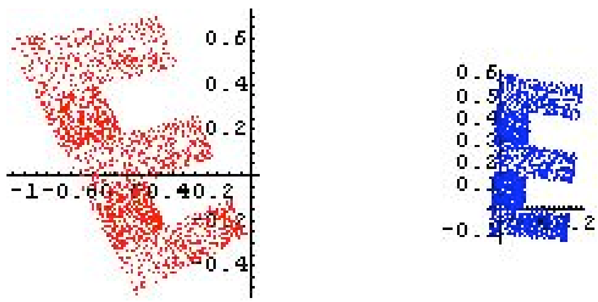

Fig. 2. Projected noise free image points for the left and right camera, respectively.

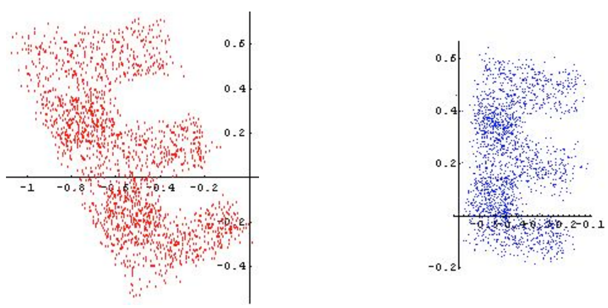

Fig. 3. Projected noisy image points for the left and right camera, respectively.

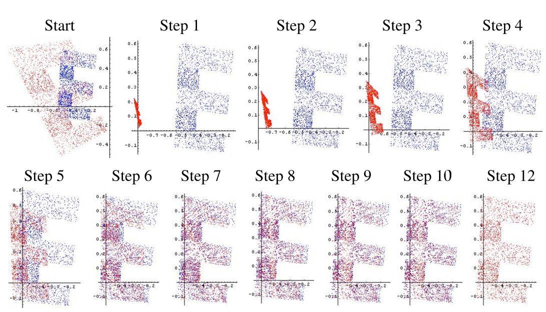

Fig. 4. Development of the Newton algorithm in the noise free case. Up left are the projected image points when the algorithm starts. The next figures show how the matrix A obtained after each iteration maps the red points to the blue ones until they perfectly match.

4) Newton's algorithm with noise: The resulting image points and their evolution along the algorithm's iterations are shown in Fig. 5. The recovered parameters in both cases,

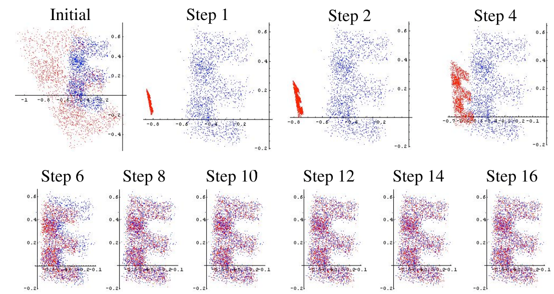

Fig. 5. Up left are the projected images simulating noise in the measurements. The blue points correspond to the left image and red to the right image.

with and without noise are summarized in tables I and II. The algorithm needed 16 iterations in the noisy case to achieve the solution.

5) Cholesky algorithm without noise: The Cholesky approach of Section (III-B) was also applied to the noiseless data. After Cholesky factorization of the Gramians, we used (44) to obtain the matrix parameters, and from that the plane parameters. The point-to-point correspondence achieved by the resulting matrix is shown in Fig. 6, the corresponding reconstructions in Fig. 7. The corresponding parameters are shown in Table I and Table II.

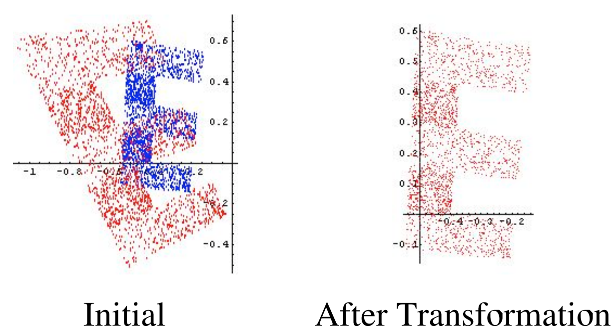

Fig. 6. Left are the projected image points of the original "E" on the planar patch. Right, the image points of the right camera are mapped to the points of the left camera by the matrix A, obtained with by Cholesky decomposition (Section III-B) of the Gramians Q and N using noiseless data.

6) Cholesky algorithm with noise: Using (44) the matrix A and the corresponding plane parameters were calculated. In Fig. 8 the point-to-point correspondence achieved by the resulting matrix is presented. In Fig. 9 the reconstructions on the planar patch are illustrated.

V. CONCLUSION

In this paper we have presented two algorithms to solve the stereo matching problem without correspondence. Firstly, an iterative Newton-like algorithm and secondly, a method based on Cholesky factorization leading to a closed form

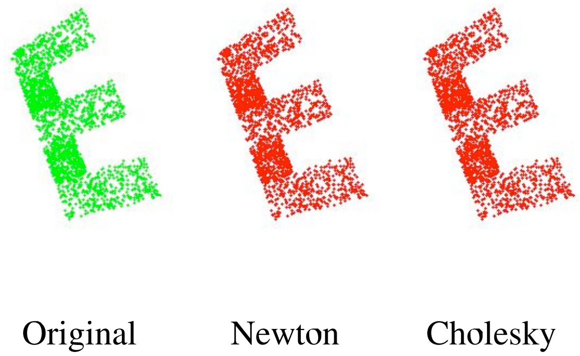

Fig. 7. Reconstruction of space points with noise free data. Left is the original data set, in the middle the reconstruction using the solution by Newton's method and in the right the reconstruction with the solution by the Cholesky method. The reconstructions were calculated using the image points and obtained plane parameters, respectively.

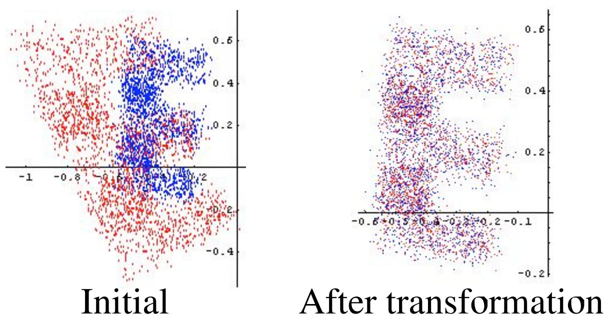

Fig. 8. Left are the projected image points of the left (blue) and right (red) camera and on the right the pointwise correspondence achieved by the obtained matrix with the Cholesky method using noisy data.

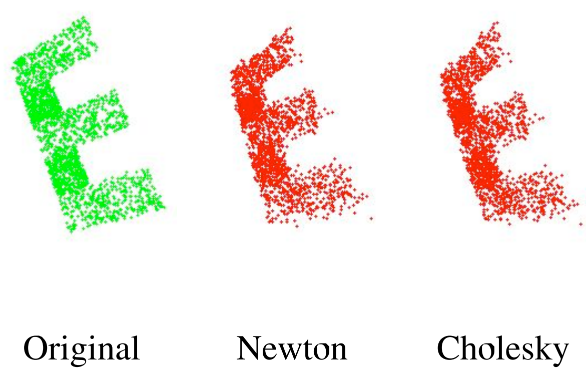

Fig. 9. Reconstructions of space points of the Newton and the Cholesky algorithms performing on noisy data. On the left the original "E", in the middle, the reconstruction from the Newton solution and in the right the reconstruction from the Cholesky solution.

solution. The results of the Newton iteration were excellent with noiseless data as one would expect. But even in the presence of noise it showed to be robust. This algorithm achieved a locally quadratically fast convergence.

The computation of the Gramians N and Q must be done for both algorithms, Newton and Cholesky. However,

Method	Data	a_1	a_2	a_3
	True value	0.01913	0	-0.046194
Newton	Noiseless	0.01913	-7.65×10^{-15}	-0.0462
	Noisy	0.02266	0.00258	-0.0448
Cholesky	Noiseless	0.01913	3.153×10^{-15}	-0.046194
	Noisy	0.02242	0.002221	-0.04488

TABLE I

TRUE VALUES AND ESTIMATED ONES FROM THE SIMULATIONS FOR MATRIX A.

Method	Data	α	β	γ
	True value	21.6478	0.414214	0
Newton	Noiseless	21.6478	0.414214	1.66×10^{-13}
	Noisy	22.326	0.5058	0.0576
Cholesky	Noiseless	21.6478	0.414214	-6.82×10^{-14}
	Noisy	22.28	0.4995	0.0495

TABLE II

TRUE VALUES AND ESTIMATED PARAMETERS FOR THE PLANE EQUATION $z = \alpha + \beta x + \gamma y$.

Method	Data	Final cost
Newton	Noiseless	5.687×10^{-23}
	Noisy	1403.32
Cholesky	Noiseless	6.349×10^{-23}
	Noisy	46059.11

TABLE III

FINAL COST ACCOMPLISHED IN EACH EXAMPLE WITH THE COST FUNCTION $f = \|Q - ANA^\top\|^2$ CALCULATED WITH THE MATRIX A OBTAINED IN EACH CASE.

to achieve the closed form solution exploiting Cholesky factorization is of very low computational cost because the two matrices to be factorized are simply 3×3. In our experiments this method showed a good performance as well, even with noisy data.

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