Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Reliable Regulation in Decentralized Control Systems Subject to Polynomial Exogenous Signals

Arturo Locatelli, and Nicola Schiavoni

Abstract—This paper addresses the design of a reliable control system for a linear, asymptotically stable plant. Specifically, the considered problem consists of finding a regulator which, besides guaranteeing closed loop asymptotic stability and zero error regulation when all the instrumentation is in operation, also ensures that these properties are preserved, at their maximum possible extent, when some sensors and/or actuators faults occur, that is, some of the feedback loops open. Thus, a single regulator has to be found, able to contemporarily solve a certain number of classical regulator problems.

Referring to a fully decentralized control structure, the paper presents a constructive necessary and sufficient condition for the problem to admit a solution when the exogenous signals are polynomial in time.

I. Introduction

The problem of designing control systems tolerant with respect to faults has received some interest in the past. See, for instance, the recent book [1].

Within this general context, the Reliable Regulator Problem (RRP) considered in this paper makes reference to a linear environment and consists of finding a regulator which, besides guaranteeing closed loop asymptotic stability and zero error regulation when all the instrumentation is in operation, also ensures that these properties are preserved, at their maximum possible extent, when some sensors and/or actuators faults occur, that is, some of the feedback loops open. Thus, the problem basically amounts to synthesizing a single regulator capable of contemporarily solving a certain number of standard regulation problems [2], [3]. In this sense, it follows the stream of research which led to [4]-[14], among many other contributions. All the mentioned papers made reference to only constant exogenous signals and decentralized regulators, apart from [12], where also some periodic exogenous signals and centralized regulators are considered. Further, the recent paper [15] dealt with polynomial exogenous signals and centralized regulators, and showed that a solution to RRP there exists if and only if all the single regulator problems composing the overall RRP separately admit a solution.

Here, reference is made to polynomial exogenous signals, but the case where the regulator is fully decentralized is considered. Under this new constraint, the separate solvability of the single regulator problems composing the overall RRP is not sufficient anymore for a solution to RRP to exist.

As in [15], a necessary and sufficient condition for the solvability of RRP is given, whose proof is constructive and proposes a decentralized regulator of the least order, which has a small gain and supplies the control system with a generalized form of the unconditional stability property [7].

The layout of the paper is the following. Next section formally introduces RRP and Sections III, IV and V are devoted to the presentation of the necessary, sufficient, and constructive necessary and sufficient conditions for its solvability, respectively. Then, Sections VI contains a couple of illustrative examples and Section VII some concluding remarks. The proofs of the main results are sketched in the Appendix.

II. The Reliable Regulator Problem

This section presents the problem considered in the paper, which is stated with reference to the control system represented in Fig. 1.

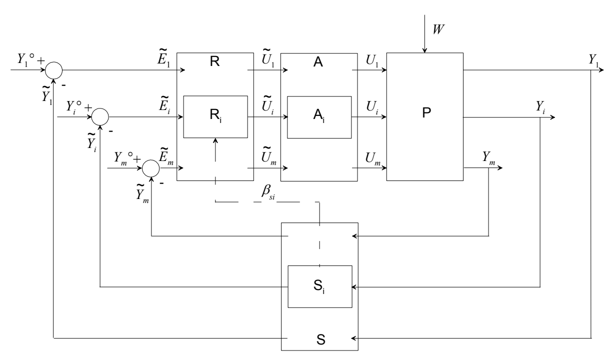

Fig. 1. The control system.

Loosely speaking, the problem consists of designing a decentralized stabilizing controller R for the plant P, also ensuring zero error regulation, in the presence of exogenous signals polynomial in time, both when the multivariable actuator A and sensor S are in operation and when faults occur at some local actuator A_i and/or sensor S_i. In order to face plant parameters variations, it is known that the controller must incorporate an appropriate internal generator of the modes of the exogenous signals, which make it not

Manuscript received February 21, 2008, received in revised form September 2, 2008. This work was developed within the National Research Project Identification and Adaptive Control of Industrial Systems funded by the Italian Ministry of University and Research.

A. Locatelli is with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Piazza L. da Vinci 32, 20133 Milano, Italy (e-mail: arturo.locatelli@polimi.it).

N. Schiavoni is with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Piazza L. da Vinci 32, 20133 Milano, Italy (corresponding author; phone: +39-02-2399-3554; fax: +39-02-2399-3412; e-mail: nicola.schiavoni@polimi.it).

978-1-4244-3124-3/08/$25.00 ©2008 IEEE

asymptotically stable.

The plant P under control is asymptotically stable, square and described by

$$Y(s) = G(s)U(s) + H(s)W(s) \ , \tag{1}$$

where s is the complex variable, U, W and Y are the Laplace transforms of the m-dimensional, p- dimensional and m-dimensional control, disturbance and output vectors, respectively, and $G(s) := \{G_{ij}(s)\}$ and $H(s)$ are two rational transfer function matrices of appropriate sizes, which are assumed strictly proper and proper, respectively.

The set point and the disturbance are signals polynomial in the time t; their Laplace transforms Y^o and W are strictly proper and take on the form

$$Y^o(s) = \frac{1}{\gamma(s)}\hat{y}^o(s) \ , \tag{2.a}$$

$$W(s) = \frac{1}{\gamma(s)}\hat{w}(s) \ , \tag{2.b}$$

where

$$\gamma(s) = s^r \ , \tag{3}$$

with $r \geq 1$ integer, whereas $\hat{y}^o$ and $\hat{w}$ are unknown polynomial vectors of degree less than r.

The assumption that the denominator γ is common to the vectors Y^o and W does not imply any loss of generality since cancellations can occasionally occur between γ and the single elements of $\hat{y}^o$ and $\hat{w}$; however, it is assumed that no root of γ is completely hidden by $\hat{y}^o$ and $\hat{w}$, so that a signal going to infinity as goes t^{r-1} is actually applied to the plant.

As a whole, the multivariable sensor S and actuator A are described by

$$\widetilde{Y}(s) = \text{diag}\{\beta_{s1}, \beta_{s2}, \ldots, \beta_{sm}\}Y(s) \ , \tag{4.a}$$

$$U(s) = \text{diag}\{\beta_{a1}, \beta_{a2}, \ldots, \beta_{am}\}\widetilde{U}(s) \ , \tag{4.b}$$

respectively, where, for $i \in M := \{1, 2, \ldots, m\}$, $\beta_{si} = 1$ ($\beta_{ai} = 1$) if the i-th local sensor S_i (actuator A_i) is in operation and $\beta_{si} = 0$ ($\beta_{ai} = 0$) in the opposite case.

On this regard, let assume that all the local sensors and actuators are subject to fault and any combination of such simple faults may contemporarily occur.

The regulator R to be designed makes its output, with Laplace transform $\widetilde{U}$, to depend on the sensed error, with Laplace transform

$$\widetilde{E}(s) := Y^o(s) - \widetilde{Y}(s) \ . \tag{5}$$

More specifically, it is fully decentralized, i.e., it is constituted by a set of m local regulators R_i's, each one of which determines the i-th component $\widetilde{U}_i$ of $\widetilde{U}$ as a function of the i-th component $\widetilde{E}_i$ of $\widetilde{E}$ only.

The actual control system error is

$$e(t) := y^o(t) - y(t) \ , \tag{6}$$

with Laplace transform $E(s) := Y^o(s) - Y(s)$.

Observe that $E_i = \widetilde{E}_i$ only if $\beta_{si} = 1$ and $\widetilde{U}_i = U_i$ only if $\beta_{ai} = 1$, E_i being the i-th component of E and U_i the i-th component of U.

The regulator R is basically linear, that is, the relationship it establishes between $\widetilde{E}$ and $\widetilde{U}$ is linear for each condition of the actuators A_i's and sensors S_i's. However, R_i is assumed to be aware of the status of the local sensor, that is, it knows the value taken on by β_{si} and modifies accordingly the way $\widetilde{U}_i$ depends on $\widetilde{E}_i$. Note that this assumption does not jeopardize the fully decentralized nature of the regulator.

Specifically, in the absence of faults, that is, if $\beta_{si}\beta_{ai} = 1$, $i \in M$, it turns out that

$$\widetilde{U}(s) = R(s)\widetilde{E}(s) \ , \tag{7}$$

where $R(s) := \text{diag}\{R_1(s), R_2(s), \ldots, R_m(s)\}$, because of the decentralization requirement.

If $\beta_{si} = 0$, R_i sets the effect of $\widetilde{E}_i$ on $\widetilde{U}_i$ to zero, so that also $U_i = 0$, which is equivalent to setting $R_i(s) = 0$. In fact, in this case $\widetilde{E}_i \neq E_i$ does not carry any piece of information useful for control. As a consequence, letting a nonzero signal $\widetilde{E}_i$ to actually influence the output $\widetilde{U}_i$ of the regulator (which must include appropriate reduplications of the generator of the exogenous signals [2], [3]) would render impossible the zeroing of the error components different from the i-th one.

On the contrary, $\beta_{ai} = 0$ renders impossible the zeroing of the i-th error variable, but does not prevent the other errors from going to zero. Then, there is no need to detect the status of the actuator and take specific actions. However, even in this case $R_i(s)$ can be considered as zeroed.

Now, let

$$f := \{i \mid i \in M, \beta_{si}\beta_{ai} = 0\} \subseteq M \tag{8}$$

specify any given pattern of faults. Note that f is the empty set if no fault occurs.

Then, for any $f \subseteq M$, denote by $R_f(s)$ the transfer function matrix obtained from $R(s)$ after zeroing the entries on the diagonal with indices in f. Then, (7) is substituted by

$$\widetilde{U}(s) = R_f(s)\widetilde{E}(s) \ . \tag{9}$$

Thus, if it is desired to design $R(s)$ such as to guarantee asymptotic stability and zero error regulation of all the error variables if no fault occurs and to preserve this property to the maximum possible extent under any pattern of faults (8), the following problem has to be solved.

Reliable regulator problem (RRP)

Find a diagonal proper transfer function matrix $R(s)$ such that, for all $f \subseteq M$:

(i) The closed loop systems (1), (4), (5), (9) are asymptotically stable;

(ii) The control system error (6) is such that the regulation constraints $\lim_{t\to\infty} e_i(t) = 0 \quad , \quad i \in M - f$, hold true in systems (1)-(5), (9) for all perturbations of $G(s)$ and $H(s)$, which preserve the asymptotic stability of the closed loop systems (1), (4), (5), (9).

□

A couple of remarks are now in order. First, what is required concerning stability is the maximum one can call for. In fact, asymptotic stability of (1), (4), (5), (7) can never be achieved in failed conditions, since the system resulting from the cascade connection of S, R and A with input Y and output U will contain not asymptotically stable parts which are unreachable and/or unobservable. Second, the situation where $f = M$ from one hand renders necessary the asymptotic stability of the plant, and from another hand is plainly trivial, since no error has to be zeroed. Hence, in the sequel the only nontrivial patterns of faults, that is, the elements of the set $F := \{f \mid f \subset M\}$ will be considered.

III. Necessary Conditions for the Reliable Regulator Problem

The RRP essentially consists of solving simultaneously a set of classical regulator problems in a linear, time-invariant framework. Hence, necessary solvability conditions can immediately be derived from [2], [3], where necessary and sufficient solvability conditions are given for a single standard regulator problem. For this reason the forthcoming lemma, which presents them, is given without proof.

Lemma 1

If the RRP admits a solution, then, all the principal minors of $G(0)$ are different from zero.

□

This lemma can be restated in a form which is more suitable for the following developments.

For any $m \times m$ matrix Λ, let $\Lambda(\varphi)$ denote the matrix obtained from Λ after deleting its rows and columns with indices in the set $\varphi \subset M$; conventionally, if φ is the empty set no row and column is deleted. Analogously, for any z , let adopt the notation $\Lambda(z;\varphi)$ for a matrix function $\Lambda(z)$.

Lemma 1 states that, if RRP is solvable, then

$$\det(G(0;f)) \neq 0 \quad , \quad \forall f \in F \quad . \tag{10}$$

Observe that $G(0)$ is the plant dc gain from U to Y and is well defined since $G(s)$ is asymptotically stable by assumption. The matrix $G(0;f)$ represents the dc gain, as seen from the regulator if the pattern of faults is f . Then, condition (10) simply means that the plant, deprived of the inputs and outputs with indices in f , possesses a nonsingular dc gain, and in particular $\det(G(0)) \neq 0$.

As shown in [15], the condition of Lemma 1 is also sufficient for RRP to admit a solution when a centralized regulator is sought, that is, when $R(s)$ is allowed to be a full matrix. In the present context, where a fully decentralization requirement has to be encompassed, this is not so anymore. In fact, the following Lemma 2 holds, where $G_D(s) := \mathrm{diag}\{G_{ii}(s)\}$.

Lemma 2

If the RRP admits a solution, then

$$\det(G(0;f))\det(G_D(0;f)) > 0 \quad , \quad \forall f \in F \quad . \tag{11}$$

□

Of course, condition (11) is more demanding than (10).

IV. Sufficient Conditions for the Reliable Regulator Problem

As a first step towards the statement of a sufficient solvability condition, a third lemma is now introduced, which is a slight variation of Lemma 2 in [15]. It guarantees the solvability of RRP and supplies an explicit expression of $R(s)$ under the assumption that a suitable matrix, to be called V , can be found. Later in the paper (Theorem 1) an explicit formula for such a matrix V will be supplied, so getting a condition of a constructive nature.

Lemma 3

If there exists an $m \times m$ real diagonal matrix V , such that, for all $f \in F$, the eigenvalues of the matrices $G(0;f)V(f)$ all have positive real part, then RRP admits a solution.

Furthermore, there exists $\bar{\varepsilon} > 0$ such that, for all $\varepsilon \in (0, \bar{\varepsilon})$, the transfer function matrix

$$R(s) = V \frac{1}{\gamma(s)} \sum_{i=1}^{r} s^{r-i} \varepsilon^{i(i+1)/2} \tag{12}$$

solves RRP.

□

The regulator of Lemma 3 is of the least possible order, as it only contains an m-fold reduplication of the system generating the exogenous signals, which is anyhow necessary in order to steer the control system errors to zero if all the instrumentation is in operation [2], [3], and supplies the control system with a generalized form of the unconditional stability property, as defined in [7], in the sense that any reduction of ε , with respect to a given value belonging to the interval $(0, \bar{\varepsilon})$, determines a new regulator (12) which still solves the RRP.

Further, trivial, but cumbersome, computations show that the regulator (12) can be rewritten as

$$R(s) = V \frac{1}{\gamma(s)} \varepsilon \prod_{i=2}^{r} \left(s + \varepsilon^i + o\left(\varepsilon^i\right)\right),$$

where $o(\vartheta)$ is such that $\lim_{\vartheta \to 0} o(\vartheta)/\vartheta = 0$, which means that the regulator introduces in the loop $m(r-1)$ real negative transmission zeros, m of which have order of magnitude ε^i, $i = 2,3,\ldots,r$.

The second step towards sufficiency concerns the existence and actual determination of a matrix V with the properties required in Lemma 3.

Some particular cases have already been dealt with in [4], where it has been shown that if $G(0)$ is triangular (besides being nonsingular) or else diagonally dominant, than it can simply be set $V := \mathrm{diag}\{G_{ii}(0)/|G_{ii}(0)|\}$. Of course, in the former case only the signs of the entries of V are relevant, so that each one of them can be scaled by any arbitrary positive factor.

The following lemma shows that an appropriate V can always be found under no condition additional to (11).

Lemma 4

If condition (11) holds, then there exists $\bar{\zeta} > 0$ such that, for all $f \in F$ and for any $\zeta \in (0, \bar{\zeta})$, the eigenvalues of the matrices $G(0;f)T(\zeta;f)$, where $T(\zeta) := G_D(0)Z(\zeta)$, $Z(\zeta) := \mathrm{diag}\{\zeta, \zeta^2, \ldots, \zeta^m\}$, all have positive real part.

□

It is clear that, for any $\zeta \in (0, \bar{\zeta})$, the matrix $V := G_D(0)Z(\zeta)$ has the properties required in Lemma 3, so that condition (11) is also sufficient.

V. Necessary and Sufficient Conditions for the Reliable Regulator Problem

By combining the results presented in the preceding sections, and in particular those stated in Lemmas 2-4, the following constructive necessary and sufficient condition for RRP to admit a solution, encompassing the fully decentralization constraint considered in this paper, can immediately be proven.

Theorem 1

The RRP admits a solution if and only if

$$\det(G(0;f))\det(G_D(0;f)) > 0 \quad , \quad \forall f \in F . \tag{13}$$

Furthermore, if (13) is satisfied, then there exist $\bar{\zeta} > 0$ and a real function $\bar{\varepsilon}(\cdot) > 0$ such that, for any $\zeta \in (0, \bar{\zeta})$ and any $\varepsilon \in (0, \bar{\varepsilon}(\zeta))$, the diagonal transfer function matrix

$$R(s) = G_D(0)Z(\zeta)\frac{1}{\gamma(s)}\sum_{i=1}^{r} s^{r-i}\varepsilon^{i(i+1)/2} \tag{14}$$

solves RRP.

□

This theorem shows that the solvability of RRP does not depend on r, and turns out to depend only on the plant dc gain from U to Y, which can easily be determined experimentally, since the plant is asymptotically stable.

As for the regulator, it can always be chosen in the class of the least order and small gain ones, according to (14), where the values of ε has to be finely tuned by trial and error so that condition (i) of the statement of RRP is satisfied, after ζ has been chosen so that, for all $f \in F$, the eigenvalues of the matrices $G(0;f)T(\zeta;f)$, all have positive real part.

On this respect, see [16].

To the best of the authors' knowledge, RRP has been tackled till now for step exogenous signals and under the decentralization constraint almost exclusively (with the exception of [12], [15]).

In this framework, the roots of the above necessary and sufficient condition for the solvability of RRP are in Theorem 1 of [4], where a condition is given, essentially equivalent to the one here, but stated as sufficient only and not explicit in terms of the problem data, i.e., in the form of Lemma 3. The same sufficient condition also appears as Theorem 6 in [7], where it is erroneously conjectured (Remark 2) that it is not also a necessary one.

Further, Theorem 1 can also be viewed as a generalization of Theorem 11.2 of [5], where however reference is made to decentralized PI regulators only. The condition of Theorem 1 constitutes also a generalization of Theorem 3.1 of [8], where only 2 control channels ($m = 2$), though MIMO and not necessarily square, are considered. The papers [9]-[11], [13], [14] deal with variations and extensions of the problem in [8], but do not give totally general, explicit and simple solutions to the problem of finding the regulator transfer function matrix.

VI. Illustrative Examples

Example 1 According to [17] and [14], the relationships between the flow rates of dopamine and sodium nitroprusside (control variables) and main arterial pressure and cardiac output (controlled variables) in a patient subject to anesthesia are described by

$$G(s) = \begin{bmatrix} \dfrac{-6}{1+0.67s} & \dfrac{3}{1+2s} \\ \dfrac{12}{1+0.67s} & \dfrac{5}{1+2s} \end{bmatrix},$$

which satisfies condition (13). For this system, the reference signals are taken as unitary ramps, so that $\gamma(s) = s^2$.

Setting $T(\zeta;f)$ as in Lemma 4, it follows that, for all $f \in F$, the eigenvalues of the matrices $G(0;f)T(\zeta;f)$ all have positive real part for any positive ζ. The choice $\zeta = 0.5$ supplies $\bar{\varepsilon} \cong 0.36$. Then, the decentralized regulator (14) has been applied with $\varepsilon = 0.18$.

Figure 2 shows the transients of the error variables if no fault occurs ((a), (b)) and a single loop is open ((c), (d)). In (a) and (c) $y_1^o(t)=t$ and $y_2^o(t)=0$, whereas in (b) and (d) $y_1^o(t)=0$ and $y_2^o(t)=t$.

Apparently, RRP is solved, even though the transients are sluggish and underdamped.

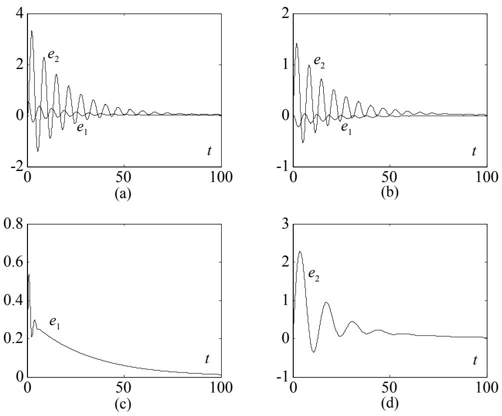

Fig. 2. Error responses for Example 1: (a), (b) system without faults; (c) system with a fault in loop 2; (d) system with a fault in loop 1.

Example 2 The head box of a paper machine, already considered in [18] and [4], and characterized by two control inputs (stock flow and air flow) and two controlled outputs (total pressure and stock level), is described by

$$G(s)=\begin{bmatrix} \dfrac{2.2s+0.004}{s^2+0.26s+0.0004} & \dfrac{1.3s}{s^2+0.26s+0.0004} \\ \dfrac{0.1s+0.004}{s^2+0.26s+0.0004} & \dfrac{-0.013}{s^2+0.26s+0.0004} \end{bmatrix},$$

which satisfies condition (13). The reference signals are again taken as unitary ramps.

Since $G(0)$ is triangular, in accordance with the remark before Lemma 4, one can set $V=\mathrm{diag}\{0.01,-0.01\}$. Then, $\bar{\varepsilon}\cong 0.033$ and the decentralized regulator (14) has been applied with $\varepsilon=0.025$.

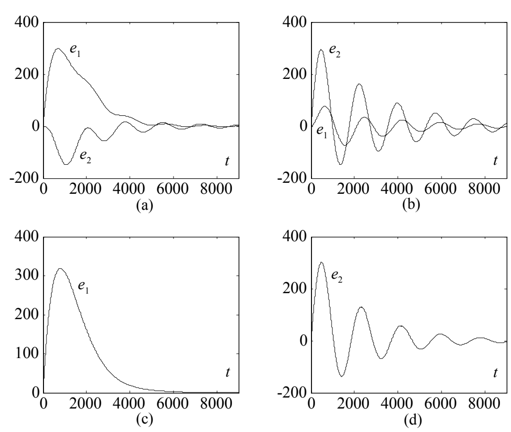

Fig. 3. Error responses for Example 2: (a), (b) system without faults; (c) system with a fault in loop 2; (d) system with a fault in loop 1.

Figure 2 shows the transients of the error variables when no fault occurs ((a), (b)) and a single loop is open ((c), (d)). In (a) and (c) $y_1^o(t)=t$ and $y_2^o(t)=0$, whereas in (b) and (d) $y_1^o(t)=0$ and $y_2^o(t)=t$.

Again, RRP is solved, even though the transients are unsatisfactory. This is not only due to the regulator very simple structure, but is also a consequence of the fact that one of the plant poles ($s=-0.0015$ and $s=-0.26$) is very close to the origin of the complex plane.

VII. Concluding Remarks

This paper presented some new results on the problem of designing control systems reliable with respect to faults in the instrumentation. The case of exogenous signals polynomial in time has been dealt with, and a necessary and sufficient condition for solving the problem by means of a fully decentralized regulator has been given.

The here extension of the result of [15], relative to a centralized framework, to encompass the decentralization constraint is conceptually nontrivial. Indeed, in the latter case, the solvability of the single classical regulator problems composing an RRP turns out to be no more sufficient for its solution.

The main result is of a constructive nature, but the suggested regulator has a least order, so that it may produce sluggish and underdamped transients in the control system. Hence, the determination of design criteria leading to more efficient, though more complex, regulators still remain a problem deserving attention.

Other significant extensions concern both the presence of exogenous signals with Laplace transforms having multiple nonzero poles on the imaginary axis, and the case where the zero-non-zero structure of the regulator transfer function matrix must comply with constraints more general than the diagonal one dealt with in this paper.

Appendix

Proof of Lemma 2 (sketch)

Without loss of generality ([2], [3]), the regulator takes on the form $R(s):=\mathrm{diag}\{n_i(s)/(s^r d_i(s))\}$, where the degree of $n_i(s)$ is less than or equal to the degree of $s^r d_i(s)$, $d_i(s)$ is monic and $n_i(0)\neq 0$, $i\in M$.

For any $f\in F$, the (rational) characteristic equation of the control system is $\det(I+G(s;f)R(s;f))=0$.

Let κ_f be the cardinality of f, $m_f:=m-\kappa_f$, $\Psi_f:=M-f:=\{i_{f1},i_{f2},\ldots,i_{fm_f}\}$, $G_{ij}(s):=\nu_{ij}(s)/\delta_{ij}(s)$, where the degree of $\nu_{ij}(s)$ is less than the degree of $\delta_{ij}(s)$, $\delta_{ij}(s)$ is monic and Hurwitz, $i\in M$, $j\in M$.

Then, the (polynomial) characteristic equation becomes

$\det(P(s;f))=0$, where $P(s;f):=\{p_{hj}(s;f)\}$,

$p_{hh}(s;f):=(s^r d_{i_{fh}}(s)+G_{i_{fh}i_{fh}}(s)n_{i_{fh}}(s))\Delta(s;f)$,

$p_{hj}(s;f):=G_{i_{fh}i_{fj}}(s)n_{i_{fj}}(s)\Delta(s;f)$, $h\neq j$,

and $\Delta(s;f)$ is the monic least common multiple of the $\delta_{ij}(s)$'s, $i\in\Psi_f$, $j\in\Psi_f$.

Asymptotic stability implies that the coefficients of the terms with highest and lowest degree of $\det(P(s;f))$ have the same sign. The former originates from $\prod_{i\in\Psi_f}\delta_{ii}(s)s^r d_i(s)$ and is equal to 1, since the $\delta_{ii}(s)$'s and $d_i(s)$'s are monic. Then, the latter is $\det(G(0;f))\prod_{i\in\Psi_f}n_i(0)\Delta^{m_f}(0;f)$ and must be positive too, which, in turn, implies $\det(G(0;f))\prod_{i\in\Psi_f}n_i(0)>0$, since $\Delta^{m_f}(0;f)>0$, because $\Delta(s;f)$ is monic and Hurwitz.

Considering in particular $\bar{f}_i:=M-\{i\}$, $i\in M$, the above condition is equivalent to $g_{ii}(0)n_i(0)>0$, so that, for any $f\in F$, $\det(G(0;f))\det(G_D(0;f))>0$, because $\prod_{i\in\Psi_f}G_{ii}(0)=\det(G_D(0;f))$.

□

Proof of Lemma 4 (sketch)

For any $f\in F$, the characteristic equation of $G(0;f)T(\zeta;f)$ is $\sum_{i=0}^{m_f}(-1)^i(\eta_{fi}+o(\zeta))\zeta^{\mu_i}s^{m_f-i}=0$, where $\eta_{f0}:=1$ and η_{fi}, $i\in M_f:=\{1,2,...,m_f\}$, is the determinant of the matrix constituted by the first i rows and columns of $G(0;f)\,G_D(0;f)$, whereas $\mu_0:=0$, $\mu_i:=\sum_{h=1}^{i}i_{fh}$, under the assumption that the indices i_{fh} satisfy $i_{f1}<i_{f2}<...<i_{fm_f}$ (here m_f and the i_{fh}'s are as defined in the proof of Lemma 2).

For $\zeta=0$, the characteristic equation possesses exactly m_f roots at the origin. Then, by continuity, the lemma is proven if it is shown that the multivalued algebraic function $s(\cdot;f)$, representing the roots as functions of ζ, has all the branches $s_h(\cdot;f)$ moving towards the right half plane for small positive ζ.

By applying standard arguments [19], [20], it turns out that

$s_h(\zeta;f)=(\xi_{fh}+o(\zeta))\zeta^{i_{fh}}$, $h\in M_f$,

where $\xi_{fh}=\eta_{fh}/\eta_{fh-1}$, $h\in M_f$.

Now, since $\eta_{fh}=\det(G(0;\chi_{fh})\,G_D(0;\chi_{fh}))$, where $\chi_{fh}:=f\cup\{i_{fh+1},i_{fh+2},...,i_{fm_f}\}$, it turns out that $\xi_{fh}>0$, in view of condition (11). Then, there exists $\bar{\zeta}_f>0$ such that, for all $\zeta\in(0,\bar{\zeta}_f)$, the eigenvalues of $G(0;f)\,T(\zeta;f)$ all have positive real part.

Hence, the proof of the lemma follows by letting $\bar{\zeta}:=\min_{f\in F}\bar{\zeta}_f$.

□

REFERENCES

[1] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, Springer-Verlag, Berlin, 2003.

[2] E. J. Davison, "The robust control of a servomechanism problem for linear time-invariant multivariable systems", *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 25-34, 1976.

[3] B. A. Francis, and W. M. Wonham, "The internal model principle of control theory", *Automatica*, vol. 12, pp.457-465, 1976.

[4] A. Locatelli, R. Scattolini, and N. Schiavoni, "On the design of reliable robust decentralized regulators for linear systems", *Large Scale Systems*, vol. 10, pp. 95-113, 1986.

[5] J. Lunze, *Robust Multivariable Feedback Control*, Prentice-Hall, Hemel Hempstead, 1989.

[6] R. D. Braatz, M. Morari, and S. Skogestad, "Robust reliable control", in *Proc. American Contr. Conf.*, 1994, pp. 3384-3388.

[7] P. J. Campo, and M. Morari, "Achievable closed-loop properties of systems under decentralized control: conditions involving the steady-state gain", *IEEE Trans. Automat. Contr.*, vol. AC-39, pp. 932-943, 1994.

[8] A. N. Gündeş, and M. G. Kabuli, "Reliable stabilization with integral action in decentralized control systems", *Automatica*, vol. 32, pp. 1021-1025, 1996.

[9] A. N. Gündeş, "Reliable decentralized integral-action controller design for multi-channel systems", in *Proc. Conf. Decision Contr.*, 1999, pp. 965-966.

[10] A. N. Gündeş, and M. G. Kabuli, "Two-channel decentralized controller design with integral action", in *Proc. American Contr. Conf.*, 1999, pp. 3831-3832.

[11] A. N. Gündeş, and M. G. Kabuli, "Reliable decentralized integral-action controller design", *IEEE Trans. Automat. Contr.*, vol. 46, pp. 296-301, 2001.

[12] A. Locatelli, and N. Schiavoni, "Reliable regulation in the presence of bounded exogenous signals", in *Proc. American Contr. Conf.*, 2001, pp. 1015-1020.

[13] V. Kariwala, J. F. Forbes, and E. S. Meadows, "Integrity of systems under decentralized integral control", *Automatica*, vol. 41, pp. 1575-1581, 2005.

[14] A. N. Mete, A. N. Gündeş, and A. N. Palazoglu, "Reliable decentralized PID stabilization of MIMO systems", in *Proc. American Contr. Conf.*, 2006, pp. 5306-5311.

[15] A. Locatelli, and N. Schiavoni, "Reliable Regulation in Centralized Control Systems Subject to Polynomial Exogenous Signals", in *Proc. Mediterranean Conf. Contr. Autom.*, 2008, pp. 1228-1233.

[16] E. J. Davison, "Multivariable tuning regulators: the feed-forward and robust control of a general servomechanism problem", *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 35-47, 1976.

[17] B. W. Bequette, *Process Control - Modeling, Design and Simulation*, Prentice Hall, 2003.

[18] V. Peterka, and K. J. Åström, "Control of multivariable systems with unknown but constant parameters", in *Proc. IFAC Symp. Identification Systems Parameters Estimation*, 1973, pp. 535-544.

[19] G. A. Bliss, *Algebraic Functions*, American Mathematical Society, New York, 1933.

[20] I. Postletwaite, and A. G. J. MacFarlane, *A Complex Variable Approach to the Analysis of Linear Multivariable Systems*, Springer-Verlag, Berlin, 1979.