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Controlling the longitudinal dynamics of a vehicle using sensor based haptic feedback

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Abstract— This paper considers a sensor based haptic feedback law for control of the longitudinal dynamics of a vehicle. The environment impedance is implemented in parallel as force feedback to the vehicle throttle as well as haptic feedback to the accelerator pedal. A non-linear gain is used in the feedback design to introduce a singularity in the energy dissipation relationship collocated with an observed obstacle. A consequence of this is that the vehicle would theoretically have to dissipate infinite energy to impact an obstacle. Using this approach we prove that the vehicle cannot impact an obstacle given no actuator saturation. The proposed control architecture has the potential to enhance vehicle safety while adding to the driving experience.

I. INTRODUCTION

In the last decade the automotive industry has invested heavily in making the driving experience more enjoyable and safe [12]. Driver assist systems for obstacle avoidance, fault diagnosis and maintenance are being deployed by various vendors [3]. Most of the existing warning systems depend on human vision or hearing to provide the user with a warning or alarm for each impending collision or the presence of a fault. Such systems are often annoying and distracting, leading to a less enjoyable driving experience and quite possibly compromising safety. Haptic feedback provides a natural and unobtrusive feedback mechanism that does not overload the human sensory process. Force Feedback Pedals [3] are an effective mechanism to provide the driver with a feeling for the environment and potential dangers in a manner that integrates naturally with the act of driving the vehicle.

Bilateral force feedback and teleoperation systems have a rich history dating back to early sixties [7]. In the mid eighties, Hogan [6] (see also parallel work by Asada and Slotine [1]) introduced a rigorous theoretical framework for modeling and control design of bilateral force feedback systems for robotic manipulators using impedance control. Since then, a number of practical teleoperational schemes have been proposed [7]. The concept of using impedance control as basis for obstacle avoidance was partially addressed by Hogan [6]. Hogan's approach was inspired by early work of Khatib [8], using an artificial potential field. In the mid nineties, Hennessey et. al [5] proposed virtual bumpers for two dimensional collision avoidance, suggesting a spring-damper system attached to a vehicle. Alec Gorjestani et. al [4] applied virtual bumpers to implement a truck collision

avoidance system. Recently, researchers at Delft university are investigating the issues involved in bilateral force feedback, teleoperation and collision avoidance [9], [11].

In this paper, we propose an integrated force feedback and obstacle avoidance control for longitudinal vehicle dynamics. We use the bond graph modelling technique to present the framework for virtual energy coupling between driver input, vehicle dynamics and the environment. The environment impedance is implemented as a virtual subsystem based on sensor feedback. The high gain feedback required to provide obstacle avoidance is obtained by incorporating a singularity (located at the obstacle) in the model of energy transfer to and from the virtual environment. This ensures that the system must dissipate infinite energy through the virtual environmental impedance prior to contact with an obstacle. The main result of the paper, Theorem 2.1, proves that the vehicle cannot impact the obstacle, regardless of the input provided by the driver. A simulation is provided to demonstrate the performance of the proposed design. The authors believe that the proposed control architecture has the potential to enhance vehicle safety while adding to the driving experience.

II. PROBLEM FORMULATION

Figure 1 shows a bond graph model for the longitudinal dynamics of a vehicle, coupled to a static spring force model for the accelerator pedal, and sensor feedback from the environment. The model is divided into three subsystems; namely the *user subsystem* (§II-A), the *vehicle subsystem* (§II-B) and the *virtual environment* subsystem (§II-C), that are discussed in the following subsections.

A. User Subsystem

The upper block of Figure 1 shows the user subsystem that models the response of the force feedback accelerator pedal. The subsystem consists of two sources S_f and MS_f, and an impedance Z_{user} as shown in Figure 1. The independent source S_f represents the driver of the vehicle and corresponds to a displacement θ degrees on the accelerator pedal. The displacement θ is the throttle setting for the motor in the vehicle dynamics described in §II-B. The dual signal is the haptic force E_γ felt by the driver through the accelerator pedal. The signal $\theta(t)$ is modelled as an exogenous input to the system, although it is clear that in a real world system the driver will provide additional feedback by reacting to the force feedback sensed through the accelerator pedal. The modulated source MS_f represents an actuator attached to the mounting point of the spring system on the accelerator

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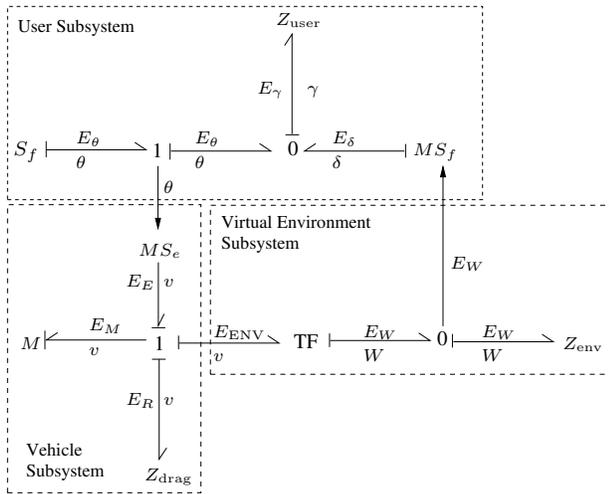
Fig. 1. Bond graph model of the complete system

pedal. The source MS_f actuates the base attachment point of the spring, displacing this point in order to compress or release the spring depending on the signal E_W received from the sensor system. The angle of displacement caused by the source MS_f is denoted by δ and the reflected force is denoted E_δ. The impedance Z_{user} is a simple spring impedance in series between the driver and an actuated mounting point. The displacement of the spring is denoted by an angle γ while the associated force is denoted E_γ. The constitutive equation for a spring is given by

$$E_\gamma = \frac{C}{L}\gamma + E_\gamma^0 \qquad (1)$$

where C is the spring constant, L is the moment arm, γ is the spring displacement in degrees and E_γ^0 is the force due to pre-compression of the spring. The standard rules for a 0-junction of a bond graph yield $E_\theta = E_\delta = E_\gamma$ and $\gamma = \theta + \delta$. Thus, the reflected force experienced by the driver is given by

$$E_\theta = \frac{C}{L}(\theta + \delta) + E_\gamma^0. \qquad (2)$$

B. Vehicle Subsystem

The vehicle dynamics are represented in the vehicle subsystem, the lower left block of Figure 1. The subsystem receives an input signal θ from the user subsystem that regulates the throttle of the engine, represented by the modulated effort source MS_e supplying the force E_E. The vehicle dynamics are represented by the 1-junction in the vehicle subsystem. The velocity v is equated for all bonds attached to the 1-junction, and the effort across the junction is given by

$$E_E = E_M + E_R + E_{Env} \qquad (3)$$

The mass of the vehicle is represented as M and leads to the inertial force $E_M = M\dot{v}$; the leftmost bond. The resistive forces (rolling resistance, air resistance, etc) are modelled as $E_R = Rv + Dv^2$ for R, D positive constants; the lower bond. The rightmost bond with variables (E_{Env}, v)

couples the vehicle subsystem to the virtual environmental subsystem. This coupling is virtual in nature and must be implemented in the feedback design (see §III). The bond (E_{Env}, v) has a flow-out causality, thus the flow, or vehicle velocity v, is imposed on the environment, while E_{Env} is the force reflected back on the vehicle by the environment. According to this convention, the environment is modelled as an impedance.

From (4) and substituting for the inertial admittance and the resistive and environmental impedances, the vehicle dynamics are given by

$$M\dot{v} + Rv + Dv^2 = E_E - E_{Env} \qquad (4)$$

where E_E is specified by the throttle signal θ obtained from the user subsystem and E_{Env} must be implemented in feedback.

C. Virtual Environment subsystem

The final subsystem, the lower right dashed block in Figure 1, represents the interaction of the vehicle with the environment. The term "virtual" is used for the subsystem because the interaction is not physical in nature and no actual energy exchange takes place between the two subsystems. The causality of the bond (E_{Env}, v) infers that the environmental subsystem is an impedance Z_{Env}, that is that it accepts flow and reflects effort. We propose a simple proportional impedance

$$E_W = \eta W \qquad (5)$$

where (E_W, W) are the coupled force velocity variables associated with the environment system and η is a constant representing the virtual viscosity of the environment.

The bond signals (E_W, W) are coupled to the bond signals for the vehicle (E_{Env}, v) by a non linear transfer function TF (see Figure 1). The construction of the transfer function is of fundamental importance to the collision avoidance technique proposed in this work. We propose to transform the effort variable as

$$E_W = E_{Env}\left(\frac{d}{c_0}\right), \qquad (6)$$

where $c_0 > 0$ is a proportionality constant and d is the distance from vehicle to the closest obstacle in the environment. To ensure the power flowing into and out of the transfer function is equal, one has

$$W = c_0\left(\frac{v}{d}\right) \qquad (7)$$

The velocity in sensor space, or the virtual velocity, W is inversely related to the distance d between the vehicle and the closest obstacle. When the closest obstacle is sufficiently distant from the vehicle, the transfer function acts like a normal linear transfer function. As the distance to an obstacle decreases, the term W will increase hyperbolically, with a singularity at $d = 0$. If the vehicle has non-zero actual velocity close to collision the virtual velocity will be close to infinite. Since the impedance (5) generates a force that impedes the motion of the vehicle then this will reflect an almost infinite force opposing the motion of the vehicle in

such situations. Recalling Equations 5, 6 and 7, the virtual force applied to the vehicle is given by

$$E_{\text{Env}} = \frac{\eta c_0^2}{d^2} v. \tag{8}$$

Substituting this equation into Eq. 4 provides an expression for the closed loop dynamics of the system

$$M\dot{v} = -\left(R + \frac{\eta c_0^2}{d^2}\right)v - Dv^2 + E_E$$

where E_E is an exogenous input. As $d \to 0$ the positive value $\eta c_0^2/d^2$ should dominate all other terms to obtain the approximate system

$$M\dot{v} \approx -\frac{\eta c_0^2}{d^2} v$$

with dynamics $v(t) \to 0$. Theorem 2.1 provides a rigorous formulation of this intuition.

Scenario: Consider a scenario where a vehicle at time t is at a position $x(t)$ and is moving towards an obstacle at a position $x_0 > x(0)$. The velocity of the vehicle is $v = \dot{x}$. At any given time the distance between the vehicle and the obstacle is defined as $d(t) = x(t) - x_0$. This implies that $\dot{d} = v = \dot{x}$. Note that $d(0) < 0$.

Theorem 2.1: Let $(x(t), v(t))$ represent the position and velocity of a vehicle subject to dynamics induced by the bond graph as shown in Figure 1, for piecewise continuous and bounded $\theta(t)$ in the scenario described above. Let $(x(0), v(0))$ be chosen such that $d(0) < 0$ and let the maximal power supplied by the engine be finite. Then either $d(t) < 0$ for all time or there exists T_0 such that[1] $d(T_0^-) = 0, v(T_0^-) = 0$ and $d(t) < 0$ for all $0 \le t < T_0$.

Proof: Clearly, being the solution of a second order system with piecewise continuous input, $(x(t), v(t))$ will be continuous on any time interval where $d(t) \ne 0$. If $d(t) < 0$ does not hold for all time this implies that there is a smallest time $T_0 > 0$ for which $d(T_0^-) = 0$ and $d(t) < 0$ for all $0 \le t < T_0$ (note that $d(0) < 0$).

The proof proceeds as a proof by contradiction. Assume that $v(T_0^-) = 0$ does not hold. We will first show that then $v(t)$ necessarily has unbounded variation.

We know from the definition of a limit that there must exist $\epsilon > 0$ such that for all $\delta > 0$ there exists a time t with $t \in (T_0 - \delta, T_0)$ and $|v(t)| > \epsilon$. We want to show that similarly $|v(t)|$ must be small for some time t in any arbitrarily small time interval before T_0. Again, this is shown by contradiction. Assume that there exists $\delta > 0$ such that for all times t with $t \in (T_0 - \delta, T_0)$ we have $|v(t)| \ge \epsilon/2$. From the continuity of $v(t)$ we know that there are only two cases possible corresponding to $v(t)$ negative or positive. In case 1, $v(t) \le -\epsilon/2$ for all $t \in (T_0 - \delta, T_0)$. Integrating $\dot{d}(t) = v(t)$ from $T_0 - \delta$ to T_0^- one obtains

$$d(T_0^-) - d(T_0 - \delta) = \int_{T_0-\delta}^{T_0^-} v(t)\mathrm{d}t \le \delta \cdot \left(-\frac{\epsilon}{2}\right) < 0$$

[1]For a signal $\sigma(t)$ defined on $t \in [0, T)$ then

$$\sigma(T^-) = \lim_{t < T, t \to T} \sigma(t).$$

and hence $0 = d(T_0^-) < d(T_0 - \delta)$, a contradiction to $d(t) < 0$ for all $0 \le t < T_0$. In case 2, $v(t) \ge \epsilon/2$ for all $t \in (T_0 - \delta, T_0)$. For this case we look at the energy balance of the system. From the law of conservation of energy, we can write

$$E_0 + E_{\text{Supplied}} = E_{\text{Stored}} + E_{\text{Dis,int}} + E_{\text{Dis,env}} \tag{9}$$

where $E_0 = \frac{1}{2}Mv(0)^2$ is the initial kinetic energy of the vehicle, E_{Supplied} is the energy supplied to the vehicle through the engine, $E_{\text{Stored}} = \frac{1}{2}M(v(t)^2 - v(0)^2)$ is the additional kinetic energy stored at time t, $E_{\text{Dis,int}}$ is the internal dissipation due to rolling resistance, air drag, etc. and $E_{\text{Dis,env}}$ is the virtual energy dissipation due to the virtual environmental impedance. From Equation 9 it follows that

$$E_{\text{Dis,env}} \le E_0 + E_{\text{Supplied}}$$

We are interested in the energy dissipation in the time interval $(T_0 - \delta, T_0)$. Using Equations 5, 6 and 7 one obtains

$$\eta c_0^2 \int_{T_0-\delta}^{T_0} \left(\frac{v(t)}{d(t)}\right)^2 \mathrm{d}t \le E_0 + B \cdot \delta$$

where $B > 0$ is a bound on the maximal power supplied by the engine. Since $v(t) \ge \epsilon/2$ on $(T_0 - \delta, T_0)$ we get

$$\eta c_0^2 \cdot \frac{\epsilon}{2} \int_{T_0-\delta}^{T_0} \frac{v(t)}{d(t)^2} \mathrm{d}t \le E_0 + B \cdot \delta$$

Using $\dot{d}(t) = v(t)$ and taking limits, therefore

$$-\eta c_0^2 \cdot \frac{\epsilon}{2} \left[\frac{1}{d(T_0^-)} - \frac{1}{d(T_0 - \delta)}\right] \le E_0 + B \cdot \delta$$

which implies

$$d(T_0^-) \le \frac{\eta c_0^2 \epsilon}{\eta c_0^2 \epsilon [d(T_0 - \delta)]^{-1} - 2(E_0 + B \cdot \delta)}$$

The right hand side is strictly negative ($d(T_0 - \delta) < 0$), implying $d(T_0^-) < 0$, a contradiction to $d(T_0^-) = 0$. We have now shown that for all $\delta > 0$ there exists a time t with $t \in (T_0 - \delta, T_0)$ and $|v(t)| < \epsilon/2$. Picking the right values of δ, we can hence construct two time sequences $(t_n)_{n \in \mathbb{N}}$ and $(t_n')_{n \in \mathbb{N}}$ such that $0 < T_0 - t_n' < T_0 - t_n < 1/n$, $|v(t_n)| < \epsilon/2$ and $|v(t_n')| > \epsilon$ for all $n \in \mathbb{N}$. In particular, this shows that $v(t)$ has unbounded variation as claimed.

The remainder of the proof now focuses on the kinetic energy

$$T(t) = \frac{1}{2}Mv(t)^2$$

From Equations 4 and 12 we get

$$\dot{T}(t) = \alpha v\theta - \eta v^2 \frac{c_0^2}{d^2} - Rv^2 - Cv^3$$
$$\le \alpha v\theta - Cv^3$$

and hence

$$|\dot{T}(t)| \le \alpha |v(t)| \cdot |\theta(t)| + C|v(t)|^3$$

This implies

$$0 < \frac{3M\epsilon^2}{8} < \int_{t_n}^{t_n'} \dot{T}(t)\mathrm{d}t < \frac{1}{n}(\alpha v_{\max}\theta_{\max} + Cv_{\max}^3)$$
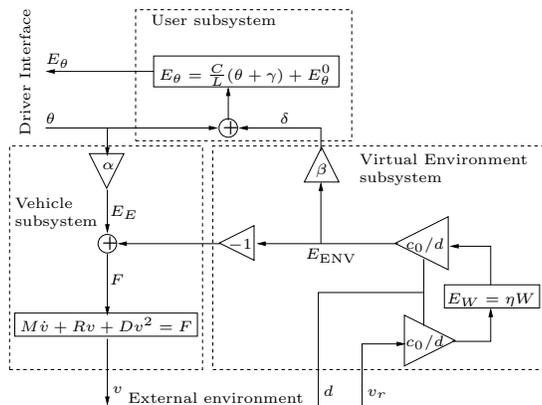
Fig. 2. Block diagram of the proposed system showing the signal flow and feedback loops.

for all $n \in \mathbb{N}$, where $v_{\max}$ is an upper bound for the continuous signal $|v(t)|$ on the compact time interval $[t_n, t'_n]$ and $\theta_{\max}$ is an upper bound for the bounded signal $\theta(t)$. This is a contradiction since the right hand side goes to zero for n to infinity. We hence conclude that $v(T_0^-) = 0$. ∎

The case $(d(T_0^-), v(T_0^-)) = (0, 0)$ corresponds to the the vehicle coming to a complete halt just touching the obstacle. This point is a singularity in the bond graph model shown in Figure 1 and the theory of bond graphs cannot be used to predict the future behavior of the system beyond such a singularity. For this reason the theorem can only consider the evolution of the system up to such point. One would hope that a sensible driver would take the opportunity to reverse away from the obstacle.

III. FEEDBACK DESIGN

In this section the structure and implementation of the control system is discussed. In the implementation of the feedback loops it is best to represent the system in block diagram form as shown in Figure 2. Once again the system can be thought of as three coupled subsystems; the user subsystem (§III-A), the vehicle subsystem (§III-B) and the virtual environment subsystem (§III-C).

A. Haptic feedback for driver

The driver interface is represented by the exogenous input θ, representing the accelerator displacement, and a system output E_θ, representing the force experienced by the driver. The force E_θ is the output of the spring system, a static affine relationship represented by the uppermost block in Figure 2. The spring system has input $(\theta + \delta)$, where δ represents the state of the actuator that adjusts the mounting point of the spring. The value of δ is derived from the value of E_W multiplied by a gain β.

The main challenge in designing the user interface feedback is to map the virtual effort E_W to a meaningful haptic feedback force E_θ that provides the driver with a 'feel' for the environment. A detailed ergonomic study of the haptic interface is beyond the scope of the present study, however, it is instructive to make some of the initial calculations. Studies

in ergonomics suggests that the normal operating force E_θ on the accelerator pedal should be between 20 N to 60 N as it will not induce any fatigue to drivers of average build [10]. Recalling Eq. 2 one sets $E_\gamma^0 = 20$. Let $\theta_{\max}$ represent the maximum displacement of the accelerator pedal. Then,

$$\frac{C}{L} = \frac{40}{\theta_{\max}} \tag{10}$$

ensures that the force encountered at maximum depression (for $\delta = 0$) is 60N. Only the ratio C/L is used in the calculations. Normally the moment arm would be designed with reference to the foot size of an average driver and the value of C would be chosen according to Eq. 10.

The haptic feedback to the driver is an additional force derived from the virtual environmental force E_W. This force can be allowed to be much larger than normal operating conditions for the accelerator as it is intended to provide the driver with a warning of impending collision. Ergonomic studies [10] suggest that a maximum return force $E_\theta^{\max}$ of 300N is acceptable. Such a force is insufficient to injure the driver and is perceived by all drivers as an almost infinite resistance. Let $E_W^{\max}$ denote the largest environmental force that is likely to be experienced in normal operation of the vehicle. Substituting the maximum value $E_\theta^{\max} = 300N$ into Eq. 2 and using Eq. 10 one obtains

$$\beta E_W^{\max} = 6\theta_{\max} \tag{11}$$

where $E_W^{\max}$ is the maximum environmental force that will be experienced. The value of $E_W^{\max}$ can be estimated by estimating the maximum braking force that the vehicle is capable of applying. This is directly linked to the maximum environmental force that the vehicle is capable of modelling.

B. Feedback to Engine Throttle and Braking System

In the approach taken in this paper, the engine of the vehicle is controlled through the throttle setting. The throttle is considered to be a two-sided input to the vehicle engine, that is, it can be used to generate both acceleration and deceleration forces to the vehicle. In modern hybrid cars, with regenerative braking systems, the implementation of braking through a single throttle setting may be relatively straightforward. In a more traditional vehicle, with separate drive train and braking systems, the coordination of the actuators would be a challenging design task in itself. Although these issues are important for a practical implementation of the proposed algorithm it is beyond the scope of the present paper and we will focus on developing the underlying concepts and demonstrating their theoretical viability.

Without loss of generality (suitable scaling in the choice of constants α and c_0), the throttle setting of the car is taken equal to the force input E_E experienced by the car. The throttle setting is the sum of the user input signal θ, multiplied by a gain α, minus a feedback term E_{Env} generated by the virtual environment interaction. In the implementation of the proposed closed-loop system, the environmental force E_{Env} exists only as a signal in the control algorithm, generated by reference to sensor input. However, if the environmental

force were a real force, then additional throttle would be required to overcome its effect. Thus, the equivalent system response is obtained by subtracting the corresponding additional throttle, which would have been required to overcome environment impedance, from the actual throttle setting of the system. Recalling Eq. 8 one obtains

$$F = \alpha\theta - \frac{\eta c_0^2}{d^2}v. \tag{12}$$

The negative gain in the feedback loop ensures that the environment acts against the motion of the vehicle as a resistive impedance. The transfer function gain c_0 controls the aggression in the braking mechanism of the controller. The higher the value of c_0, the earlier the braking mechanism comes in action, implying gradual and softer deceleration. Lower values of c_0 imply late braking and thus higher deceleration rates. The value of c_0 can be tuned for different scenarios and driver expectations.

C. Sensor systems and modelling the environment

The final subsystem of the implementation is the virtual environment subsystem on the lower right of Figure 2. This subsystem is fully implemented in software. The measured signals required are the range to an obstacle d and the relative velocity of the host vehicle with respect to the obstacle v_r. The range measurement relies on an external sensor system. Many high end vehicles are already equipped with microwave radar systems, used for adaptive cruise control [13], that provide range measurements to an obstacle. For a stationary obstacle, the velocity measurement can be obtained from the vehicle odometre. In the case of a moving obstacle it is necessary to derive relative velocity information from the range sensor. If the range is obtained using a radar system then the doppler shift of the received signal can be used to obtain accurate relative velocity measurements, especially at higher speeds where direct differentiation of the signal becomes noisy. At low speeds, where the doppler shift is unreliable, a low pass filtered differentiation of the range signal can be effectively used. Beyond establishing that suitable technology exists it is beyond the scope of the present paper to delve more deeply into the exact nature of the sensing systems used in a real world system.

The range measurement is required in the implementation of the two non-linear gain blocks, that together model the transfer function in the bond graph Figure 1. The lower block models the forward transfer function on velocity, while the upper block models the inverse transfer function on force, equal to the forward transfer of velocity. Note that as the range to the closest obstacle becomes large, the effective gain in these blocks is negligible and the system defaults to the normal open loop control of the vehicle. It is only when d becomes sufficiently small that the nonlinear gain becomes significant and the vehicle and driver begin to feel the environmental impedance as a reflected force.

IV. RESULTS

The proposed feedback system was simulated using a Matlab/Simulink environment. For each of the tests the variables of interest are the velocity of the vehicle, the distance of the vehicle from the obstacle, the feedback to the engine throttle and the haptic feedback to the driver. The term 'host vehicle' means the vehicle with an onboard virtual impedance controller. The coefficients of rolling resistance and air resistance are taken as 0.01N.m^{-1} and 0.7N.m^{-1} respectively [2]. The mass of the vehicle is taken as 1800Kg. Following the discussion in Section III-A, the value of the spring constant C for an accelerator pedal of length (from pivot to pedal pad) $L = 0.18\text{m}$ and maximum driver depression angle $\theta_{\max}$ of 20 degrees is 0.36Nm.Deg^{-1}. For E_γ^0 of 20N, the spring needs to be pre-compressed by $\theta_0 = 10\text{Deg}$.

From initial simulation studies involving a vehicle approaching a stationary object, it was observed that for a vehicle travelling at an average speed of 16.6m.s^{-1} (60Km.h^{-1}) and reasonably close (30m) to an obstacle, the maximum force E_W generated by the environment on the vehicle is around 10KN. To show the full range of response of the system we choose $E_W^{\max} = 10\text{KN}$ and compute $\beta = 0.03$ from Equation 11.

The value of c_0 is based on the maximum deceleration achievable by a vehicle. Consider the case when the maximum declaration rate of the vehicle is $-a_{\max}$. The minimum stopping distance $D_{\min}$ of the vehicle is given by

$$D_{\min} = \frac{v_0^2}{2a_{\max}} \tag{13}$$

where v_0 is the velocity of the vehicle. When brakes are applied, the component of total engine force due to the driver (the throttle mechanism) is negligible [11]. This means that the $\alpha\theta$ factor becomes zero. For the maximum deceleration one has $E_E = -ma_{\max}$, where m is the vehicle mass. Recalling Eq. 12 and substituting for $d = D_{\min}$ and $v = v_0$, one may solve for c_0:

$$c_0 = \sqrt{\frac{mv_0^3}{4\eta a_{\max}}}. \tag{14}$$

Equation 14 can be used to tune the constant c_0 for different scenarios. Thus, if the closed-loop system must be able to stop the car from a distance of 30m with velocity 16.6m.s^{-1} then these values are the $D_{\min}$ and v_0 values in the calculation of c_0. For the simulations, the maximal deceleration is taken as -7.35m.s^2, which is $-0.75g$, where g is the gravitational constant.

The simulation was carried out to test the capability of the model in an emergency situation, i.e. how the controller (and thus the host vehicle) reacts when the driver is ignoring the haptic feedback. The driver is assumed to be giving a constant input to the accelerator pedal. The initial velocity of the vehicle is 15 m/s (54 Km/h) and the distance of a stationary obstacle is 300 metres from the vehicle. The results of the simulation run are shown in Figure 3.

The controller acts to decelerate the vehicle to almost negligible velocity, despite the drivers failure to brake, thus avoiding a collision. While the vehicle is sufficiently distant from the obstacle, the transfer function (TF) behaves almost
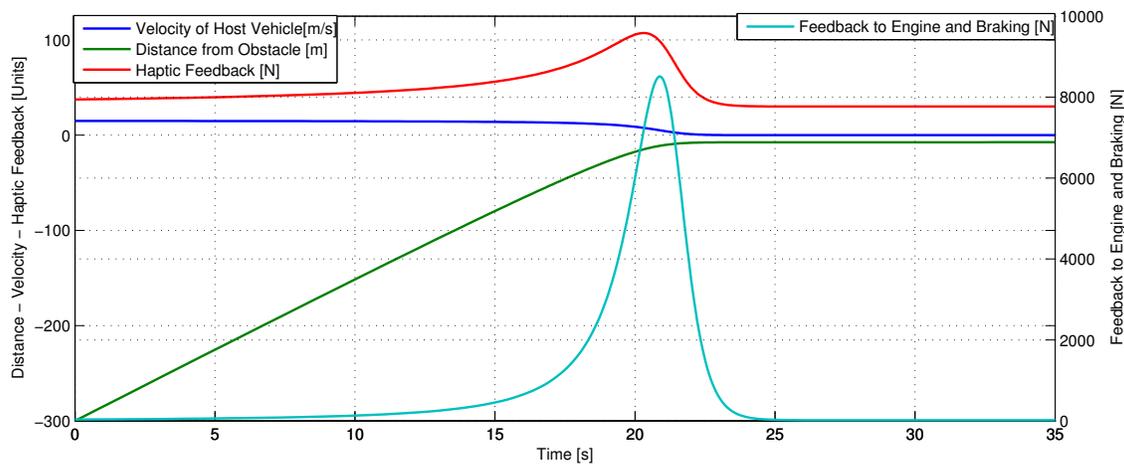
Fig. 3.   Critical stop.

linearly, i.e. the distance and velocity of the vehicle decrease linearly with respect to time. As the vehicle comes in close proximity of the obstacle, the non-linearity of TF comes into play. This results in rapid deceleration of the vehicle at around $t = 20$s with the velocity decreasing to negligible by $t = 23$s. After deceleration the feedback to the engine throttle decreases once again as the result of reduced velocity and distance, but never reaches zero. This is due to the fact that the driver is still trying to 'push' the vehicle but the controller is balancing the force of the engine throttle induced by the driver against the virtual environment resistance. The haptic feedback provided to the driver is initially around 30N, corresponding to normal driving conditions, jumping to over 100N during to the deceleration manoeuver, and then dropping again to around 20N once the vehicle is nearly stationary.

In addition to providing the driver with significant haptic feedback during the critical stop manoeuvre, the behaviour of the system in the limiting situation is of interest. In the limiting case only slow movement of the vehicle is possible for forces on the accelerator in the normal regime of operation. This property is advantageous in parking situations where the driver may learn to play against the haptic feedback of the system to obtain more precise control of the vehicle at slow speeds in a cluttered environment.

V. CONCLUSIONS

In this paper we have addressed the problem of real-time obstacle avoidance for road vehicles. An impedance control technique was utilised to implement our framework for the case of longitudinal vehicle dynamics. This is achieved by introducing a non-linear transfer function that incorporates a singularity at the obstacle. Due to the singular nature of the transfer function the energy dissipated by the virtual environmental subsystem prior to collision is infinite and the associated reflected 'virtual' force is used to brake the vehicle as well as provide haptic feedback to the driver.

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