Design and Diagnosis of the Basic Feedback Loop

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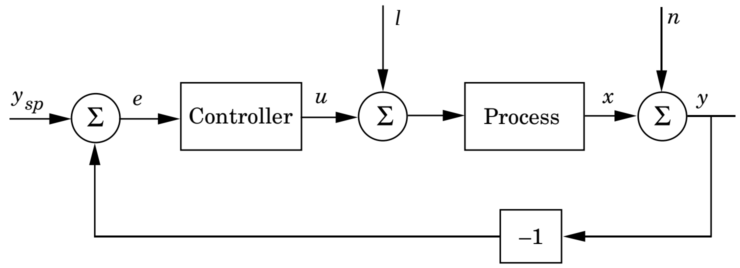
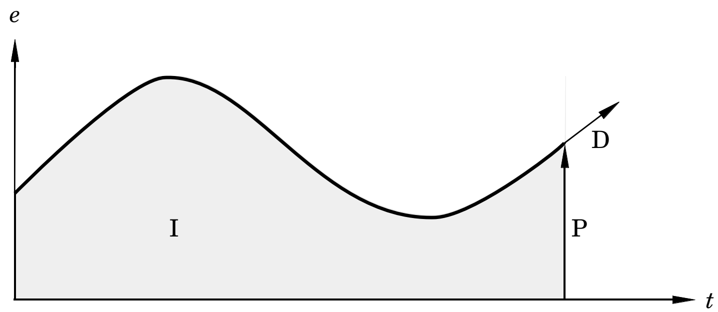
The Basic Feedback Loop



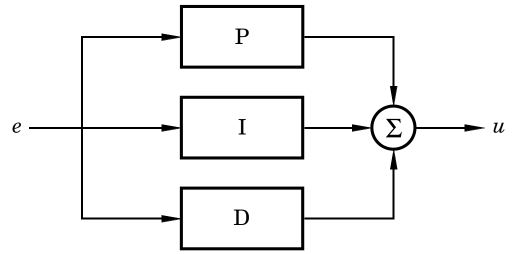
The PID Controller

The textbook version:



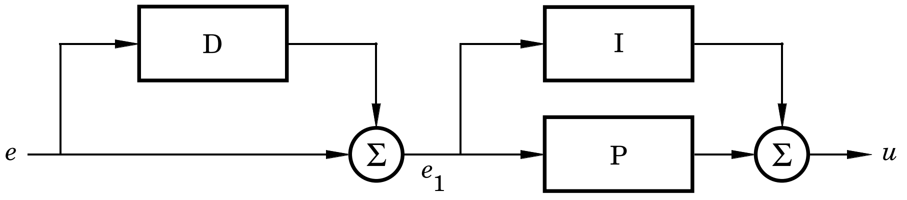
$$u = K\left(e + \frac{1}{T_i}\int e(t)dt + T_d\frac{de}{dt}\right)$$

Parallel form



$$u = K\left(e + \frac{1}{T_i}\int e(t)dt + T_d\frac{de}{dt}\right)$$

Series form



$$e_1 = e + T_d' \frac{de}{dt}$$

$$u = K' \left(e_1 + \frac{1}{T_i'} \int e_1(t) dt \right)$$

Relations between parallel and series form

Series form → parallel form:

$$K = K' \frac{T_i' + T_d'}{T_i'}$$

$$T_i = T_i' + T_d'$$

$$T_d = \frac{T_i' T_d'}{T_i' + T_d'}$$

Parallel form → series form (Requirement: $T_i > 4T_d$):

$$K' = \frac{K}{2} \left(1 + \sqrt{1 - \frac{4T_d}{T_i}} \right)$$

$$T_i' = \frac{T_i}{2} \left(1 + \sqrt{1 - \frac{4T_d}{T_i}} \right)$$

$$T_d' = \frac{T_i}{2} \left(1 - \sqrt{1 - \frac{4T_d}{T_i}} \right)$$

Setpoint handling

$$u = K \left(br - y + \frac{1}{T_i} \int e(t) dt - T_d \frac{dy}{dt} \right)$$

One additional parameter: b

Also: Filters, ramping modules, feed-forward, ...

Noise handling

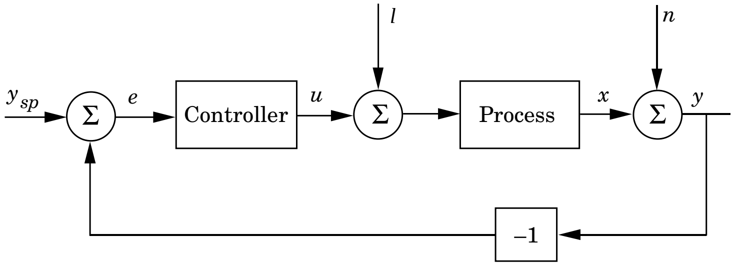
$$U = -K \left(1 + \frac{1}{sT_i} + \frac{sT_d}{1 + sT_d/N} \right) Y$$

or better

$$U = -K \left(1 + \frac{1}{sT_i} + sT_d \right) \frac{1}{(1 + sT_d/N)^2} Y$$

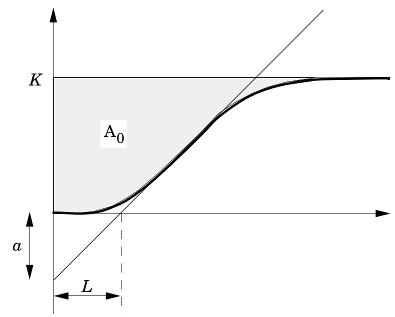
One additional parameter: N

Specifications



- Load disturbance rejection
- Setpoint following
- Measurement noise amplification
- Robustness with respect to process variations

Ziegler-Nichols' step response method



Design criterion: Decay ratio 0.25

Two parameters: a and L

Ziegler-Nichols' step response method

Controller	K	T_i	T_d
P	$1/a$		
PI	$0.9/a$	$3L$	
PID	$1.2/a$	$2L$	$0.5L$

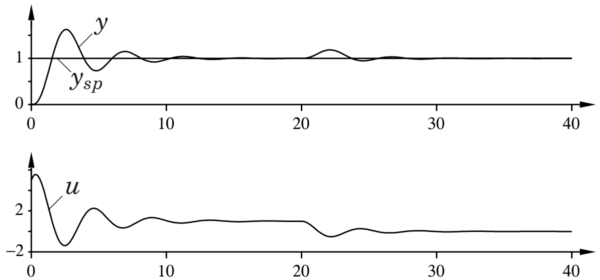
Example: ZN step response method

Process:

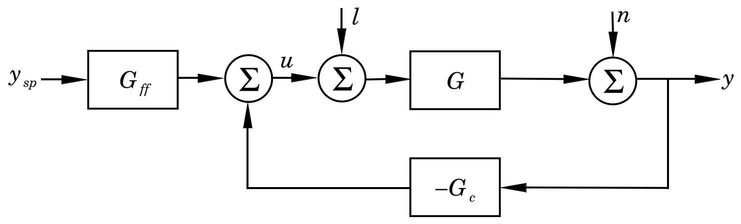
$$G(s) = \frac{1}{(s+1)^3}$$

Controller:

$$K = 5.50$$
$$T_i = 1.61$$
$$T_d = 0.403$$



Optimization methods

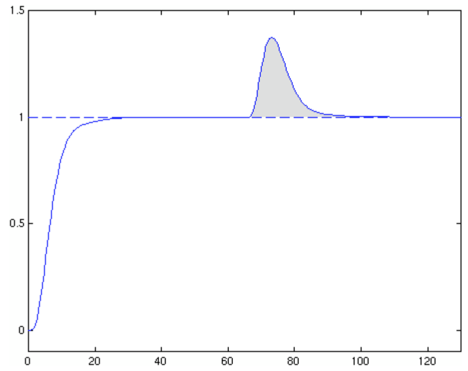


Specifications on:

- Load disturbance rejection
- Setpoint following
- Measurement noise amplification
- Robustness with respect to process variations

Tuning parameter

Load disturbances

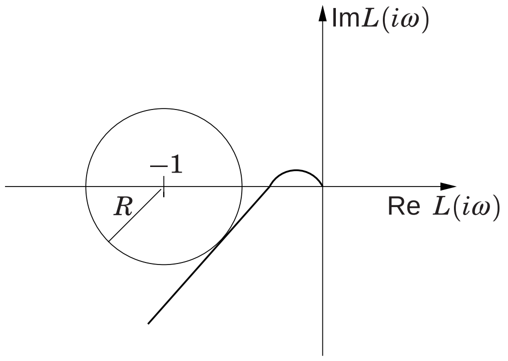


Minimize

$$IE = \int e(\tau)d\tau = \frac{1}{k_i} = \frac{T_i}{K}$$

at load disturbances

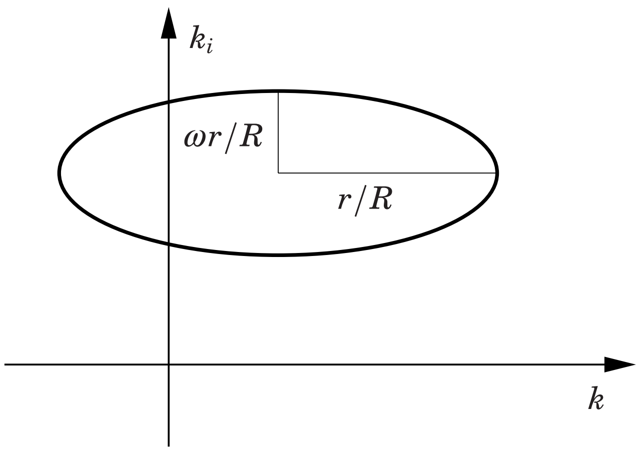
Robustness



$$R = 1/M_s$$

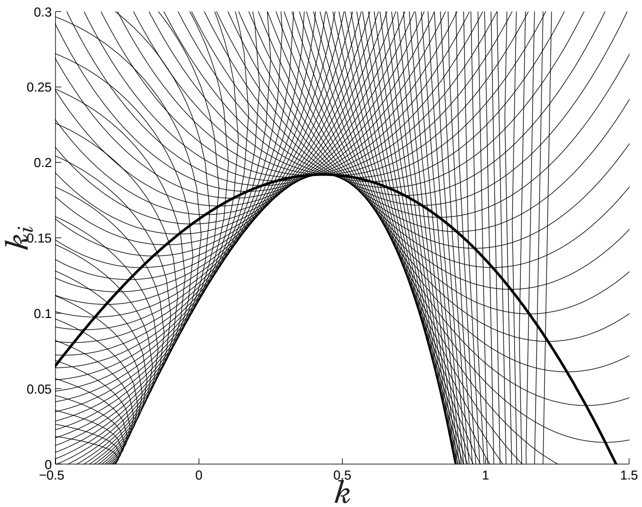
M_s is a useful tuning parameter. Range [1.2,2].

Robustness



$$r(\omega) = |G_p(i\omega)|$$

Robustness



MIGO Design

M-constrained Integral Gain Optimization

Maximize k_i (i.e., minimize IE) such that $L(i\omega)$ is outside the M_s-circle.

M_s tuning parameter.

$M_s = 1.4$ good default value.

Requires accurate process model.

AMIGO Design

Approximate M-constrained Integral Gain Optimization

Approximate the MIGO design by

- Using simple process models
- Based on step responses or frequency responses
- Three parameters needed
- Fitting controller parameters to a large test batch
- Using simple tuning rules like Ziegler-Nichols

AMIGO Design

Process model:

$$G_p(s) = \frac{K_p}{1 + sT} e^{-sL}$$

Processes with integration:

$$G_p(s) = \frac{K_v}{s} e^{-sL} \qquad K_v = \frac{K_p}{T}$$

Relative time delay: $\tau = \dfrac{L}{L + T}$

Dynamic gain: $a = K_p \dfrac{L}{T} = K_v L$

AMIGO – PI – Test batch

$$P_1(s) = \frac{e^{-s}}{1+sT}, \qquad T = 0.01, 0.05, 0.1, 0.3, 0.5, 1, 2, 3, 5, 10, 20, 100$$

$$P_2(s) = \frac{e^{-s}}{(1+sT)^2}, \qquad T = 0.01, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, 1.3, 1.5, 2, 4, 6, 8, 10, 20, 100$$

$$P_3(s) = \frac{1}{(s+1)(1+sT)}, \qquad T = 0.01,\ 0.02,\ 0.05,\ 0.1,\ 0.2,\ 0.5$$

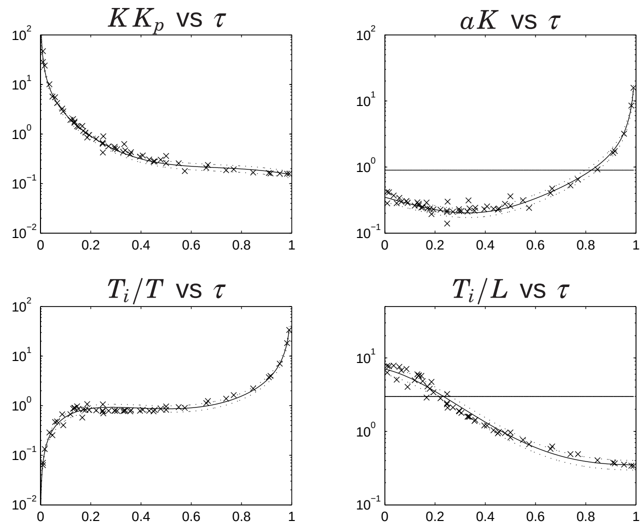
$$P_4(s) = \frac{1}{(s+1)^n}, \qquad n = 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8$$

$$P_5(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)}, \qquad \alpha = 0.1, 0.2, 0.5, 0.7$$

$$P_6(s) = \frac{1-\alpha s}{(s+1)^3}, \qquad \alpha = 0.1,\ 0.2,\ 0.5,\ 1,\ 2$$

$$P_7(s) = \frac{1}{s^2+2\zeta s+1}, \qquad \zeta = 0.5,\ 0.7,\ 0.9$$

AMIGO – PI Design



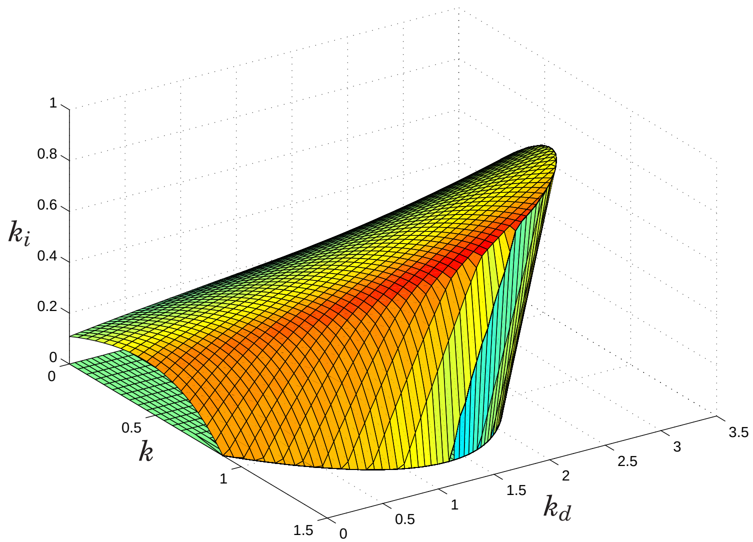
AMIGO – PI Design

$$K = \frac{1}{K_p}\left(0.15 + 0.35\frac{T}{L} - \left(\frac{T}{L+T}\right)^2\right)$$

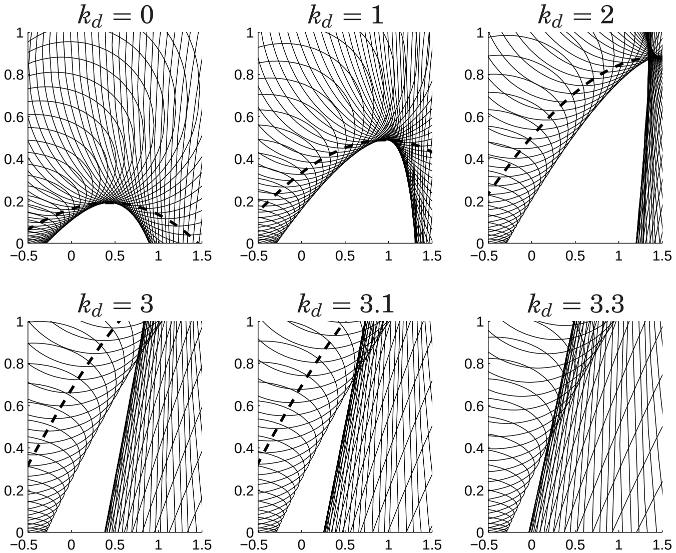
$$T_i = 0.35L + \frac{6.7LT^2}{T^2 + 2LT + 10L^2}$$

3 parameters needed!

MIGO – PID Design

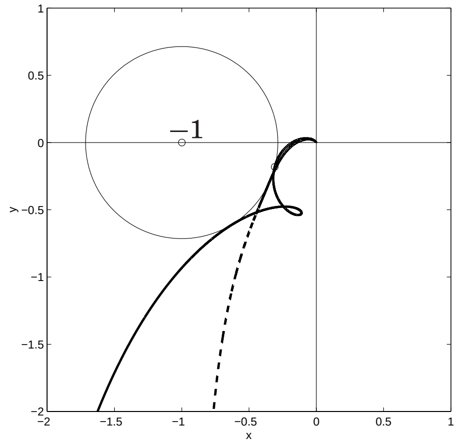


MIGO – PID Design



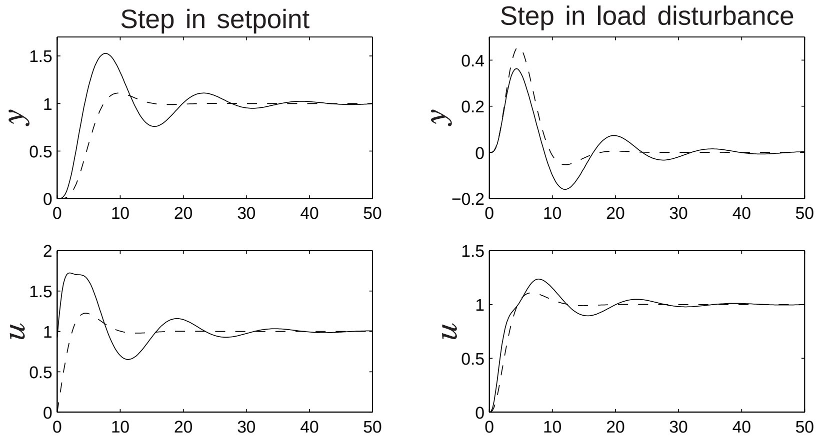
MIGO – PID Design

Problems with PID control. Additional constraints required.



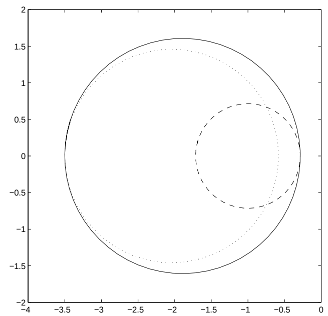
MIGO – PID Design

Problems with PID control. Additional constraints required.



MIGO – PID Design

Use M circle instead of M_s circle



$M_s = \max_\omega |S(i\omega)| = \max_\omega \left| \frac{1}{1+PC(i\omega)} \right|$ (Dashed)

$M_p = \max_\omega |T(i\omega)| = \max_\omega \left| \frac{PC(i\omega)}{1+PC(i\omega)} \right|$ (Dotted)

MIGO – PID Design

Suggested additional constraints:

- $T_i = \alpha T_d$
- $L(i\omega)$ has negative curvature and monotone phase
- $\partial k_i / \partial k = 0$ (Used in the following)

AMIGO – PID – Test batch

$$P_1(s) = \frac{e^{-s}}{1+sT}, \qquad T = 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, 1.3, 1.5, 2, 4, 6, 8, 10, 20, 50, 100, 200, 500, 1000$$

$$P_2(s) = \frac{e^{-s}}{(1+sT)^2}, \qquad T = 0.01, 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, 1.3, 1.5, 2, 4, 6, 8, 10, 20, 50, 100, 200, 500$$

$$P_3(s) = \frac{1}{(s+1)(1+sT)^2}, \qquad T = 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 2, 5, 10$$

$$P_4(s) = \frac{1}{(s+1)^n}, \qquad n = 3, \quad 4, \quad 5, \quad 6, \quad 7, \quad 8$$

$$P_5(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)}, \qquad \alpha = 0.1, 0.2, 0.3, 0, 4, 0.5, 0.6, 0.7, 0.8, 0.9$$

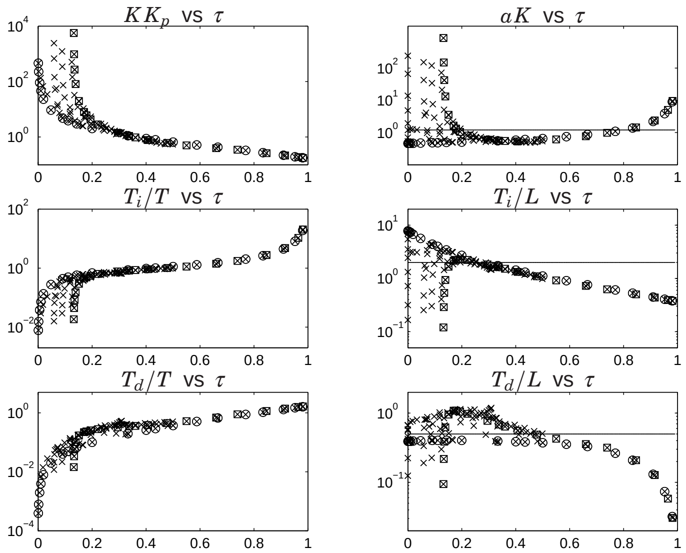
$$P_6(s) = \frac{1}{s(1+sT_1)} e^{-sL_1}, \qquad L_1 = 0.01, 0.02, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9, 1.0, \quad T_1 + L_1 = 1$$

$$P_7(s) = \frac{T}{(1+sT)(1+sT_1)} e^{-sL_1}, \qquad T_1 + L_1 = 1, \qquad T = 1, 2, 5, 10 \qquad L_1 = 0.01, 0.02, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9, 1.0$$

$$P_8(s) = \frac{1-\alpha s}{(s+1)^3}, \qquad \alpha = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1$$

$$P_9(s) = \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}, \qquad T = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$$

MIGO – PID Design



AMIGO – PID Design

Why large spread of controller parameters for small τ?

Processes with transfer functions

$$P(s) = \frac{K_v}{s(1+sT_1)} \quad \text{and} \quad P(s) = \frac{K_p}{(1+sT_1)(1+sT_2)}$$

can be controlled with arbitrarily high gains in the PID controller.

These processes have $\tau < 0.13$, with equality when $T_1 = T_2$.

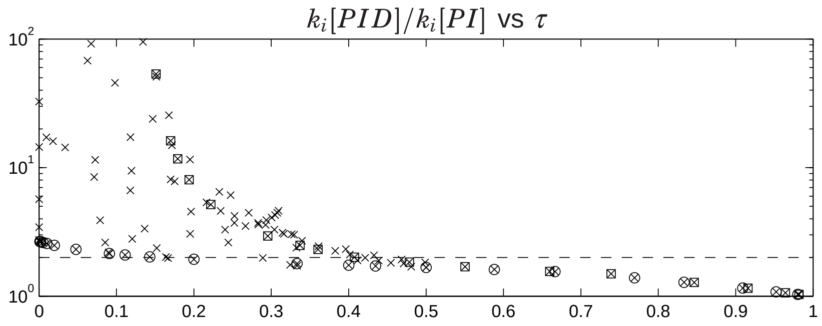
Consequence

Modeling with the structure

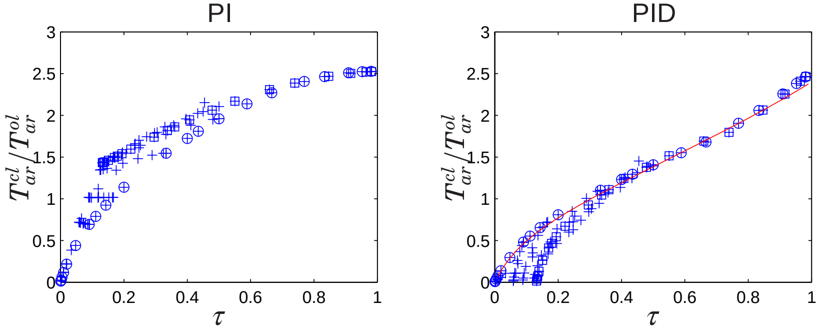
$$P(s) = \frac{K_p}{1 + sT} e^{-sL}$$

imposes fundamental limitations that may not be present in the true process!

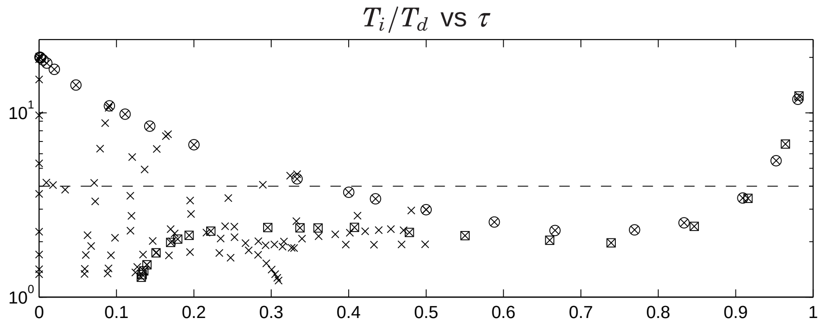
PI or PID?



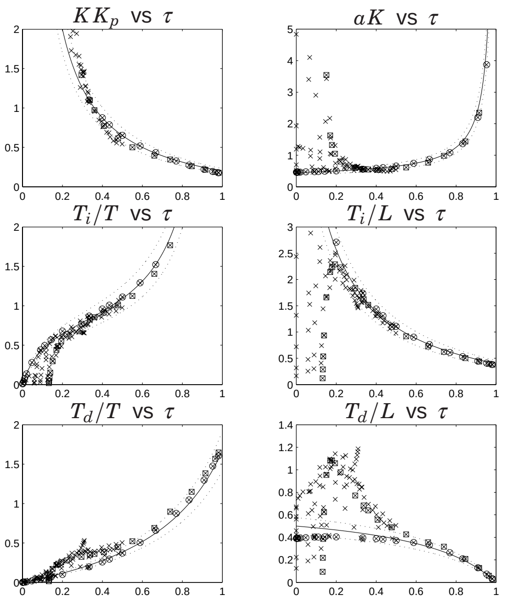
Average Residence Times



Ratio T_i/T_d



AMIGO – PID Design



AMIGO – PID Design

$$K = \frac{1}{K_p}\left(0.2 + 0.45\frac{T}{L}\right)$$

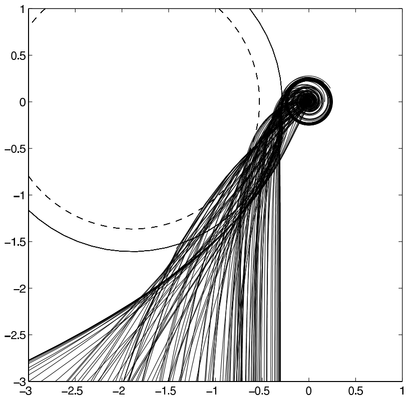
$$T_i = \frac{0.4L + 0.8T}{L + 0.1T}L$$

$$T_d = \frac{0.5LT}{0.3L + T}$$

Efficient for $\tau > 0.2$.

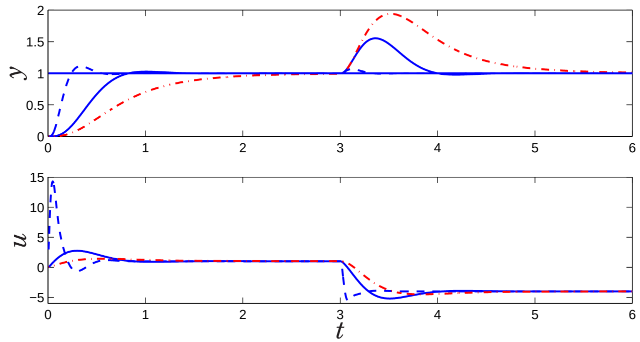
Conservative for $\tau < 0.2$.

AMIGO – PID Design



Example – Lag-dominant process

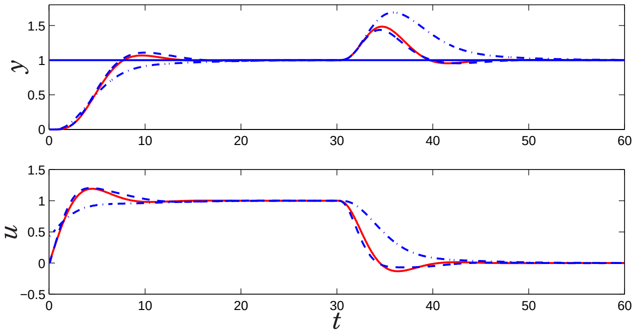
$$P(s) = \frac{1}{(1+s)(1+0.1s)(1+0.01s)(1+0.001s)}$$



Solid = AMIGO-PID, Dashed = MIGO-PID, Dashed-dotted = MIGO-PI

Example – Balanced lag and delay

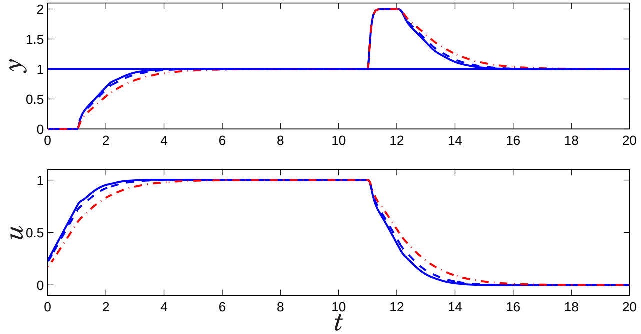
$$P(s) = \frac{1}{(s+1)^4}$$



Solid = AMIGO-PID, Dashed = MIGO-PID, Dashed-dotted = MIGO-PI

Example – Delay dominant

$$P(s) = \frac{1}{(1+0.05s)^2} e^{-s}$$



Solid = AMIGO-PID, Dashed = MIGO-PID, Dashed-dotted = MIGO-PI

AMIGO – Summary

PI

$$K = \frac{1}{K_p}\left(0.15 + 0.35\frac{T}{L} - \left(\frac{T}{L+T}\right)^2\right)$$

$$T_i = 0.35L + \frac{6.7LT^2}{T^2 + 2LT + 10L^2}$$

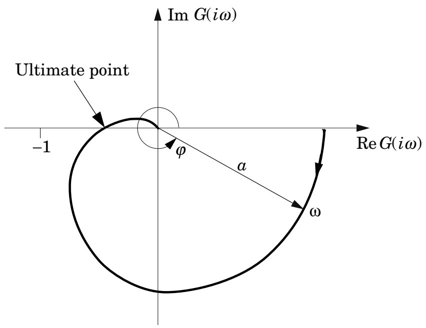
PID

$$K = \frac{1}{K_p}\left(0.2 + 0.45\frac{T}{L}\right)$$

$$T_i = \frac{0.4L + 0.8T}{L + 0.1T}L$$

$$T_d = \frac{0.5LT}{0.3L + T}$$

Ziegler-Nichols' frequency response method



Design criterion: Decay ratio 0.25

Two parameters: K_{180} and T_{180}

AMIGO – Frequency response

Process model:

$$K_p = |G_p(0)|$$

$$K_{180} = |G_p(i\omega_{180})|$$

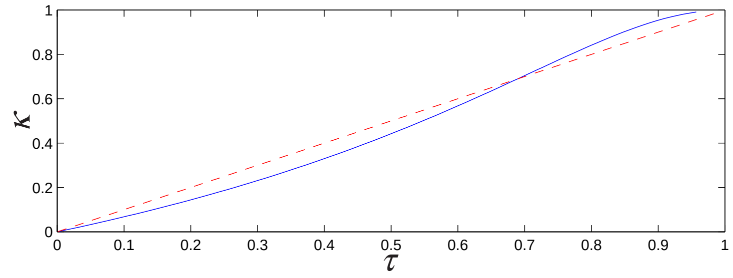
$$T_{180} = \frac{2\pi}{\omega_{180}}$$

Integrating processes: K_{180} and T_{180}

Gain ratio: $\kappa = \dfrac{K_{180}}{K_p}$

Relation between κ and τ

For FOTD process:



$$\tau = \frac{\pi - \arctan\sqrt{1/\kappa^2 - 1}}{\pi - \arctan\sqrt{1/\kappa^2-1} + \sqrt{1/\kappa^2 - 1}}$$

AMIGO – Frequency response

PI

$$KK_{180} = 0.15$$

$$\frac{T_i}{T_{180}} = \frac{0.8}{1 + 3.7\kappa}$$

PID

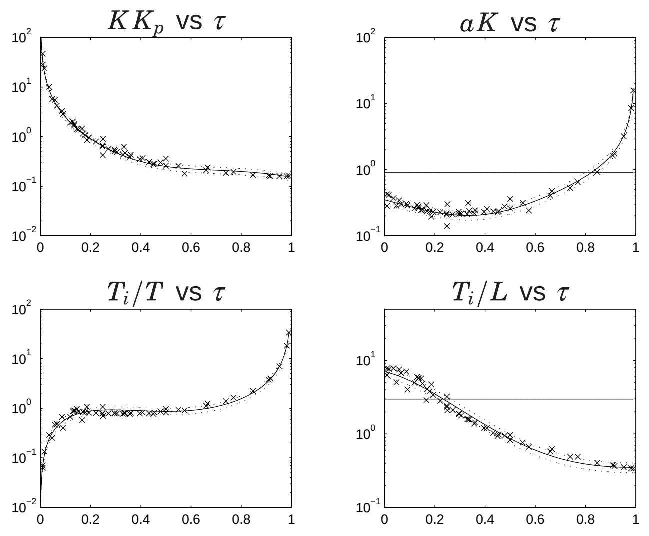
$$K = (0.3 - 0.1\kappa^4)/K_{180}$$

$$T_i = \frac{0.6}{1 + 2\kappa} T_{180}$$

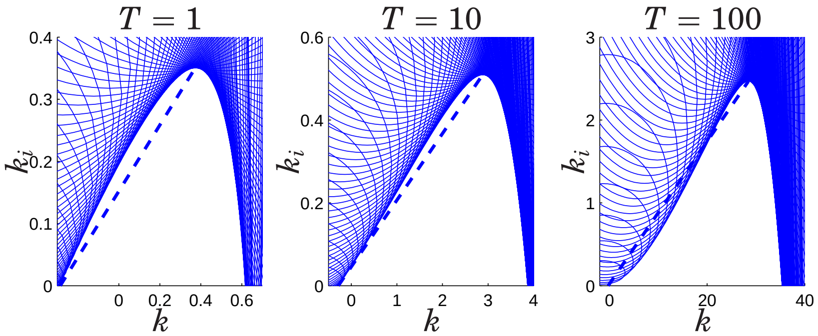
$$T_d = \frac{0.15(1-\kappa)}{1 - 0.95\kappa} T_{180}$$

Efficient for processes with $\kappa > 0.2$

Is the gain too high?



AMIGO – Detuning PI



AMIGO – Detuning PI

$$k_i = \begin{cases} k_i^0 \dfrac{\alpha + KK_p}{\alpha + K^0 K_p} & \text{for } KK_p \geq \dfrac{k_i^0 K_p (L+T)}{\beta(\alpha + K^0 K_p)} - \alpha \\ \beta \dfrac{(\alpha + KK_p)^2}{K_p(L+T)} & \text{for } KK_p < \dfrac{k_i^0 K_p (L+T)}{\beta(\alpha + K^0 K_p)} - \alpha \end{cases}$$

$$\alpha = \frac{M_s - 1}{M_s}$$

$$\beta = \begin{cases} M_s \left(M_s + \sqrt{M_s^2 - 1} \right) / 2 & \text{for design based on } M_s \\ M(M-1) & \text{for design based on } M \end{cases}$$

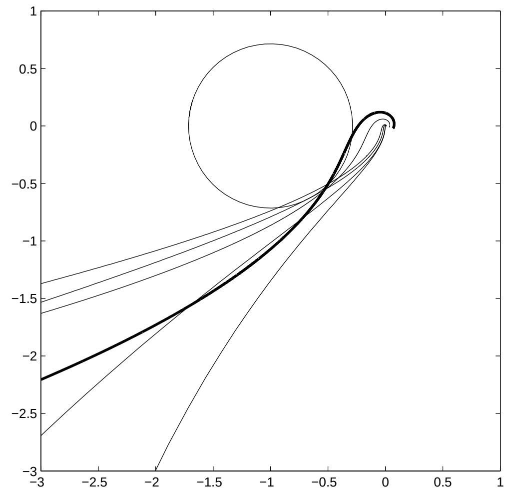
AMIGO – Detuning PI, Example

$$P(s) = \frac{1}{1 + 1000s} e^{-s}$$

AMIGO Design:

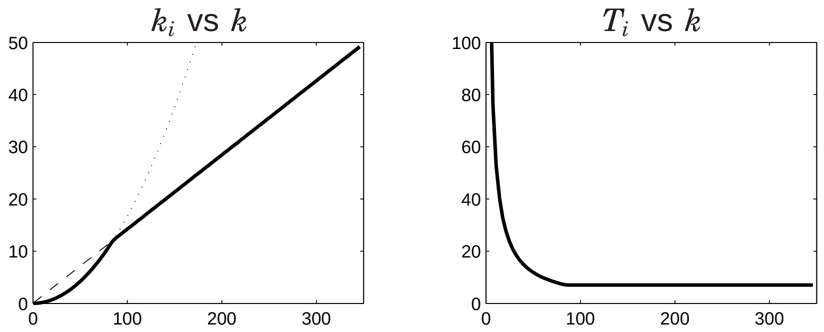
$$K = 349$$
$$T_i = 7.04$$

AMIGO – Detuning PI, Example

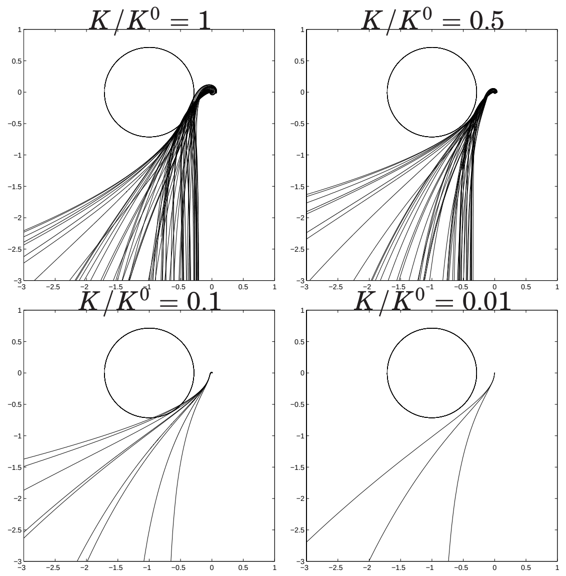


Gain reduction factors: 1, 0.5, 0.1, 0.05, 0.01, and 0.005

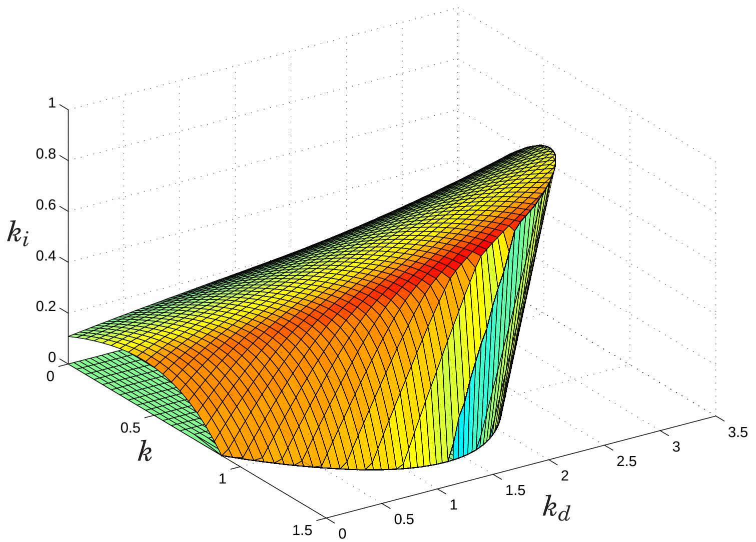
AMIGO – Detuning PI, Example



AMIGO – Detuning PI, Test Batch



AMIGO – Detuning PID

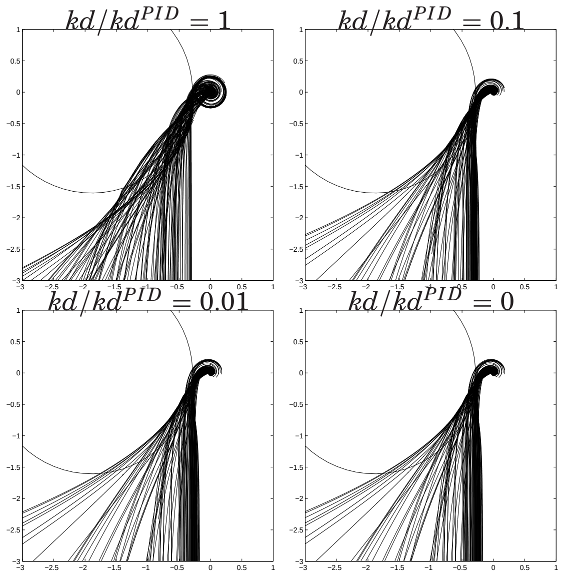


AMIGO – Detuning PID

$$K = K^{PI} + \frac{k_d}{k_d^{PID}}(K^{PID} - K^{PI})$$

$$k_i = k_i^{PI} + \frac{k_d}{k_d^{PID}}(k_i^{PID} - k_i^{PI})$$

AMIGO – Detuning PID



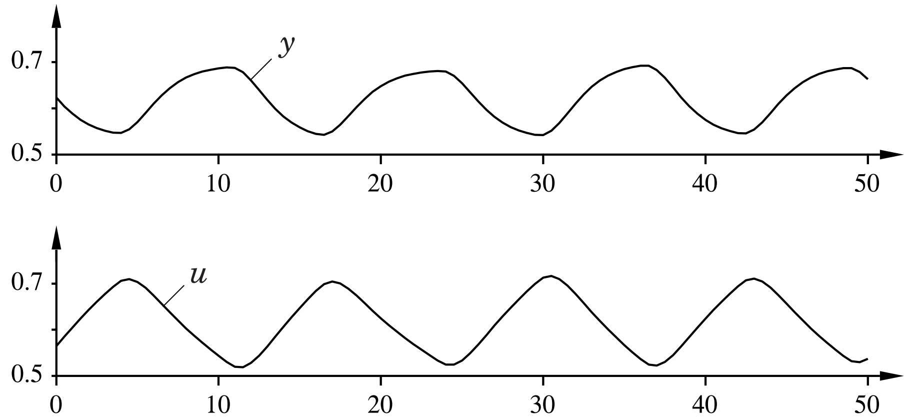
Before you start tuning ...

investigate the process!

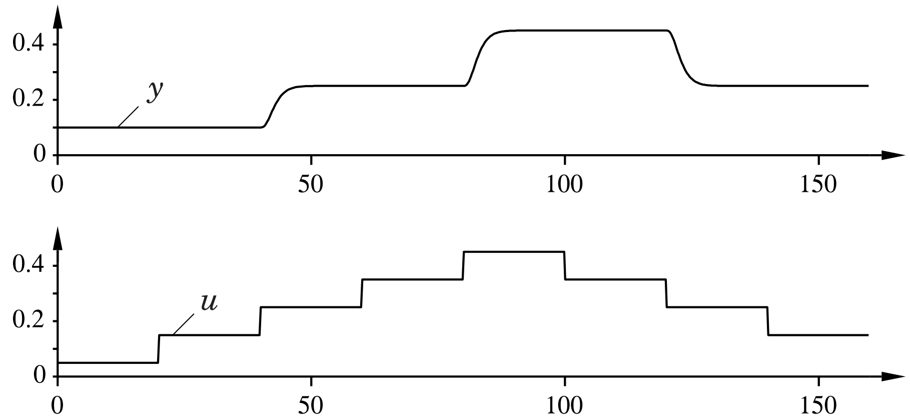
Process = Everything outside the PID algorithm!

- Are there any scaling factors?
- Are there any filters?
- Avoid dead-times
- Sensors and actuators OK?
- Friction or hysteresis?
- Other nonlinearities?
- Controller series or parallel?
- Is dynamics really the limiting factor?

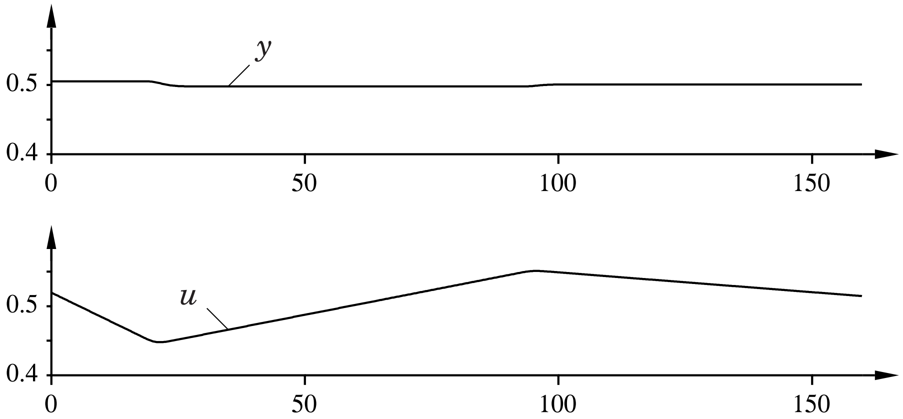
Control with friction (Stick-slip motion)



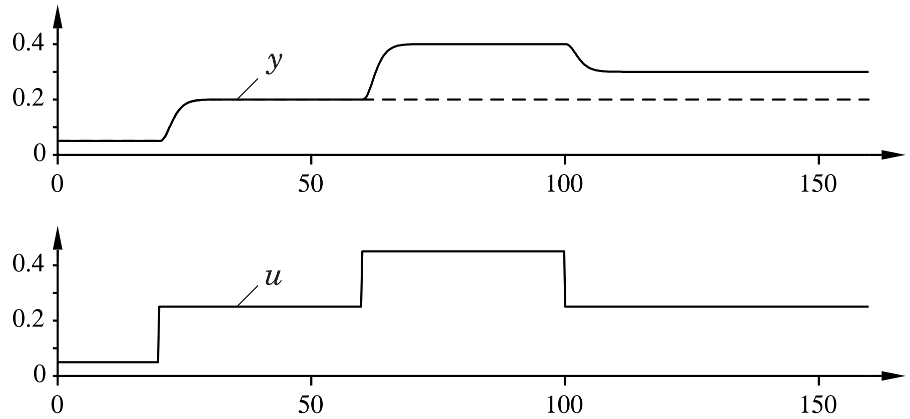
Diagnosis of friction



Control with hysteresis



Diagnosis of hysteresis



Automatic performance monitoring

- We have improved process control.
- We have lost human performance monitoring.
- We need automatic performance monitoring.

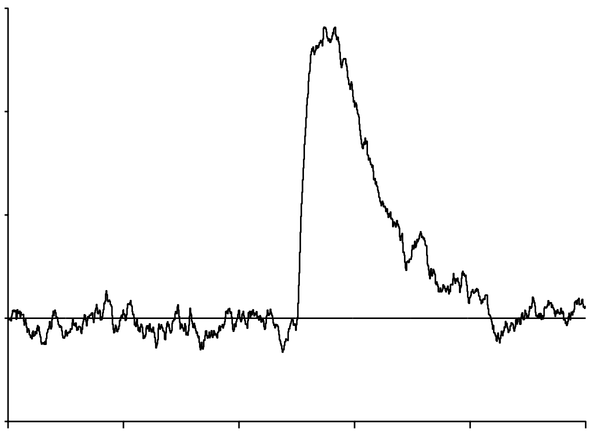
Reasons for poor control loop performance

- Equipment problems
  - Stiction in valves
  - Sensor faults
- Poor controller tuning
  - Never tuned?
  - Nonlinear plant
  - Time-varying plant
- Oscillating load disturbances

Two monitoring tools:

- Detection of oscillating control loops
- Detection of sluggish control loops

Oscillation detection



Oscillation detection

Determine

$$IAE = \int_{t_{i-1}}^{t_i} |e(t)|dt$$

between zero crossings of the control error.

Good control: IAE small

Load disturbances: IAE large

Oscillation detection

The loop is oscillating if the *rate* of load disturbances becomes high.

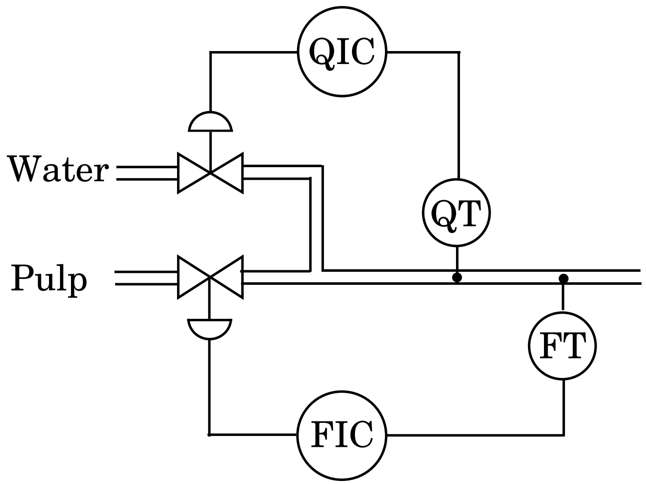
The loop is oscillating if more than n_{lim} load disturbances are detected during a supervision time T_{sup}.

3 parameters: IAE_{lim}, n_{lim}, T_{sup}.

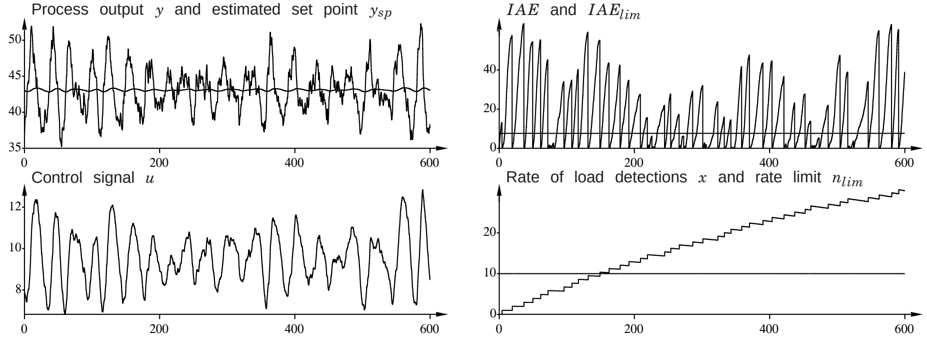
Suggestion:

$$IAE_{lim} = T_i/\pi$$
$$n_{lim} = 10$$
$$T_{sup} = 50T_i$$

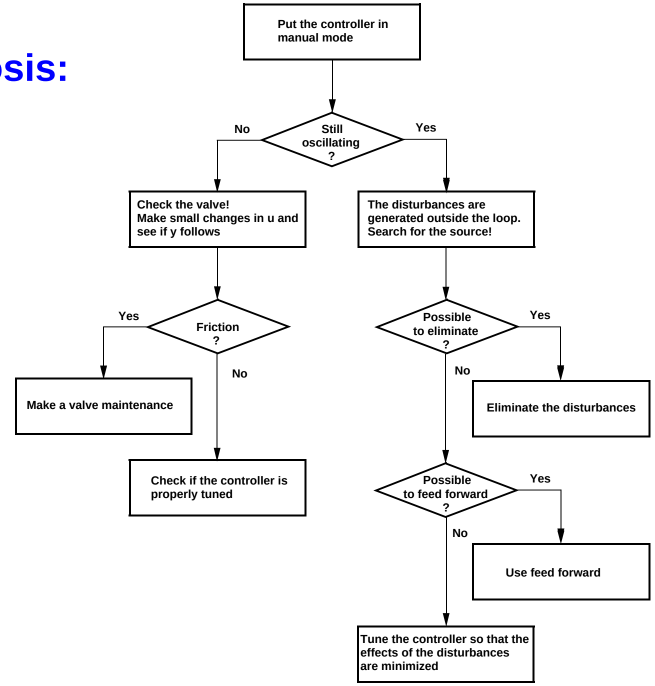
Example – Pulp concentration control



Example



Diagnosis:



Frövi Paper Mill

Oscillation detection procedure used in Honeywell TDC3000.

91% of the loops in the carton board mill are supervised.

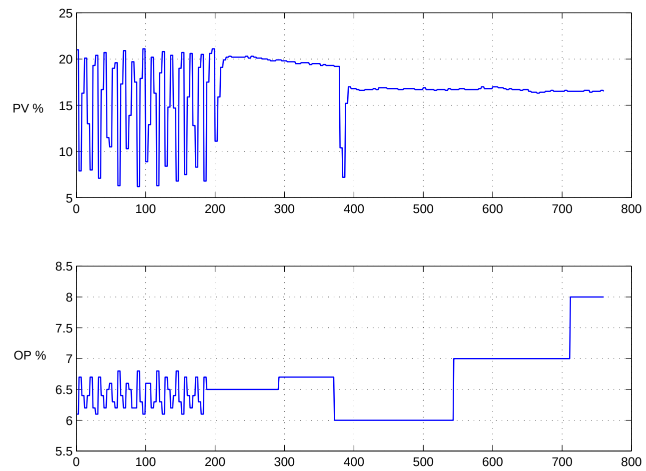
Each loop has an *Oscillation index* that is increased every time a detection is made.

The Oscillation index is reset to zero at maintenance or tuning.

Top-ten list presents the worst loops.

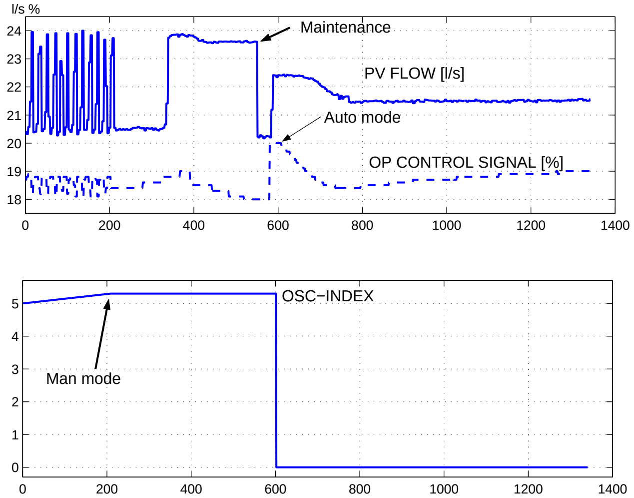
Frövi Paper Mill

Pressure control loop



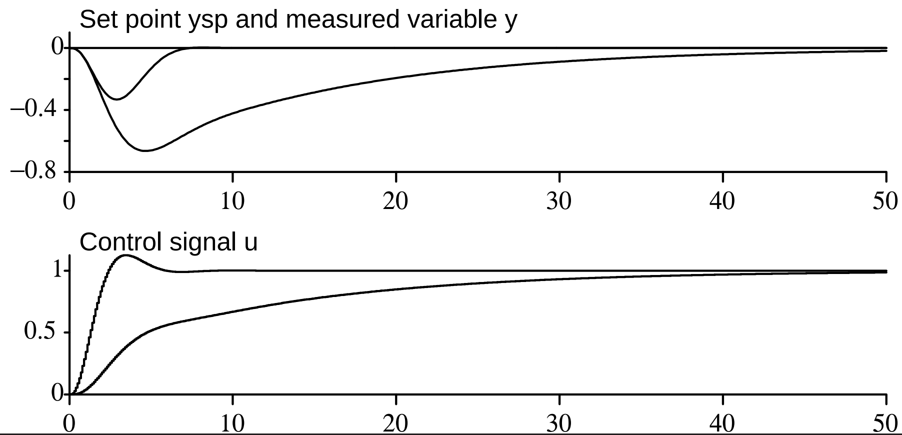
Frövi Paper Mill

Flow control loop



Detection of sluggish control loops

Good and bad control of load disturbances:



Idle Index

$$I_i = \frac{t_{\text{pos}} - t_{\text{neg}}}{t_{\text{pos}} + t_{\text{neg}}} \qquad I_i \in [-1, 1]$$

I_i large $\Rightarrow$ Sluggish control

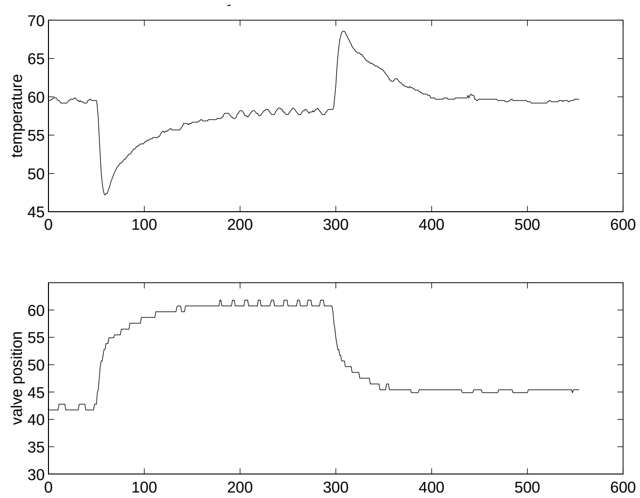
Previous example: $I_i = 0.82$ and $I_i = -0.68$

Recursive version

Filtering important

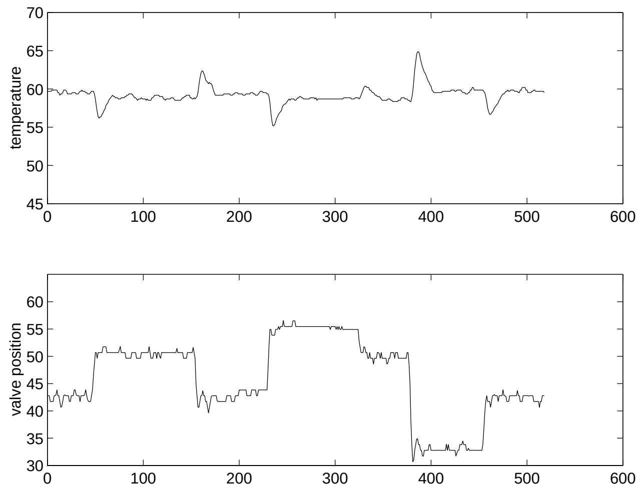
Automatic

Control of a heat exchanger



$$K = 0.01 \qquad T_i = 30s \qquad I_i = 0.8$$

Control of a heat exchanger



$$K = 0.025 \qquad T_i = 8s \qquad I_i = 0.3$$

Implemented where?

The DCS system or an external computer? What is needed?

- Measurement signal
- Measurement signal range
- Setpoint
- Control signal
- Control signal range
- Controller parameters
- Control mode: Man/Auto/Tracking
- Filters etc.
- Sampling interval

This information is normally available in the DCS system only.

Design and Diagnosis

of the Basic Feedback Loop

Design

- 4 considerations
- 3 parameters needed
- No universal tuning rule

Diagnosis

- Before you start
- The loop will change
- Control the control

References

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[3] T. Hägglund and K. J. Åström. Revisiting the Ziegler-Nichols tuning rules for PI control – Part II the frequency response method. *Asian Journal of Control*, submitted, 2004.

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