A Probabilistic Approach to Process Identification and Control

A Case Study in Pulp Bleaching Focused on the Stationary Probability Density Function

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Outline

- Stochastic Processes
- Probabilistic Framework
- Perspective for Identification and Control
- Bleaching Reactor
- Identification
- Controller Design
- Conclusions

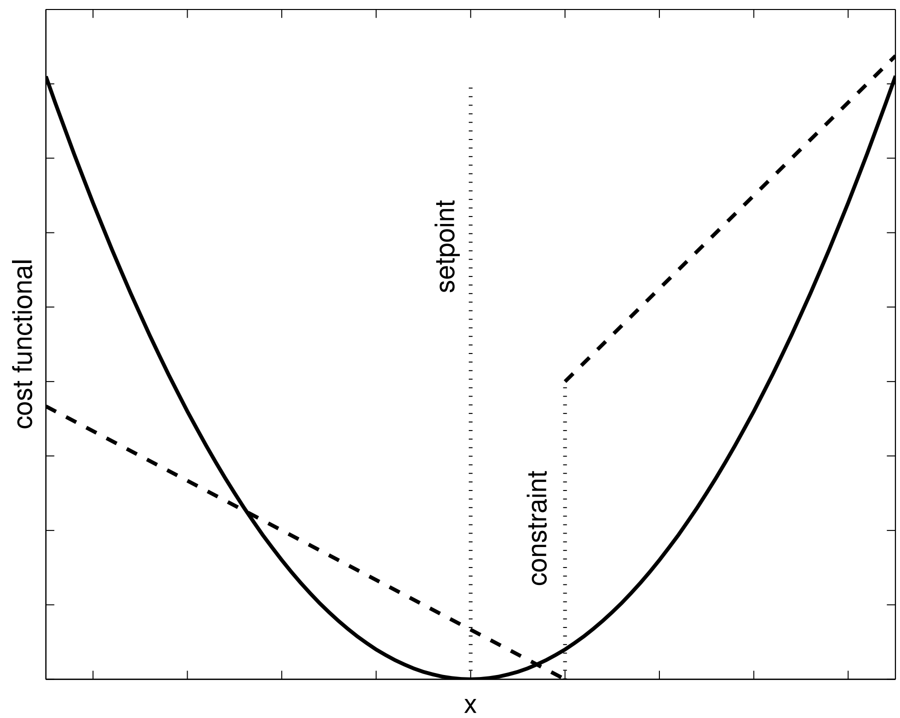
Stochastic Processes

Practical realities of the processing and manufacturing industries:

- Nonlinear processes,
- (Non-Gaussian) random disturbances – prevents true equilibrium,
- Quality specifications

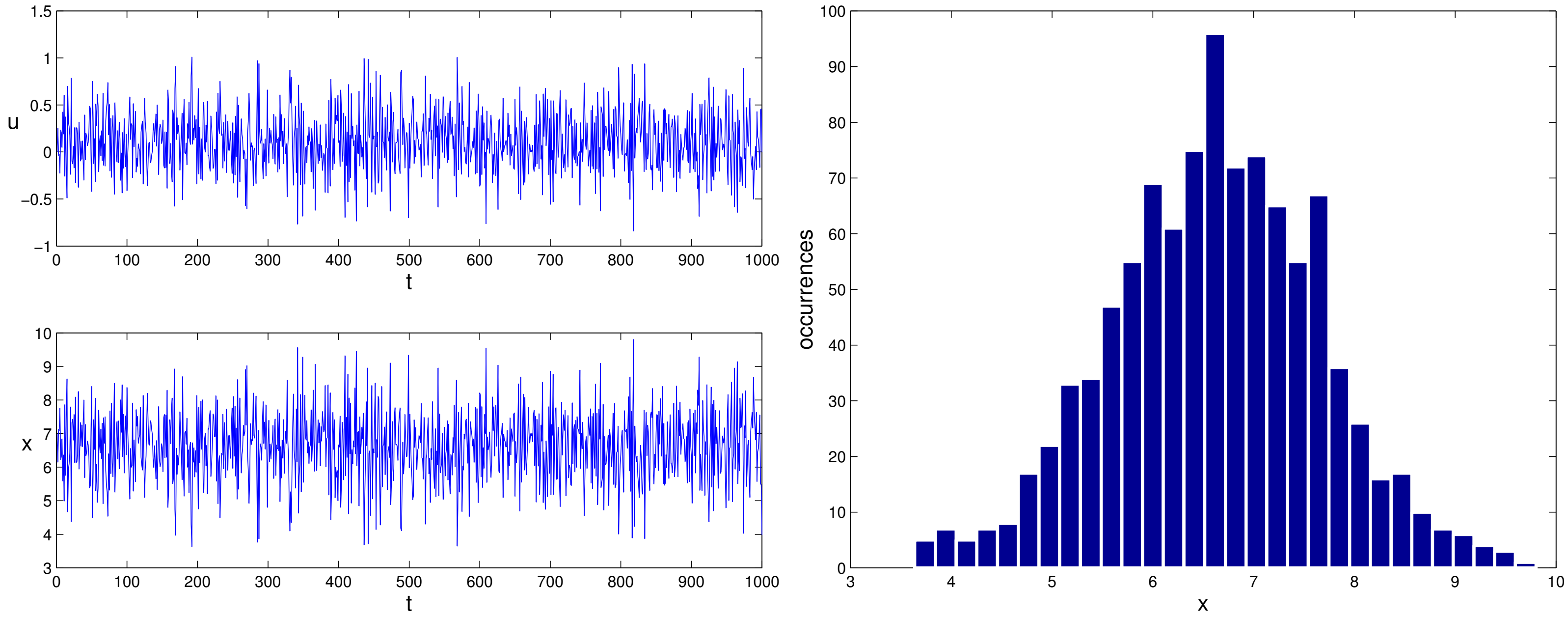
Quantifying Performance

– quadratic loss functional about setpoint; nonquadratic loss functional for quality specification



Process Time Series, Dispersion

– time series and histogram for process data



Mathematical Framework

$$\mathbf{x}_{t+1} = \mathbf{f}\left(\mathbf{x}_t, \mathbf{u}_t\right) + \mathbf{w}_t, \quad \mathbf{u}_t = \mathbf{k}\left(\mathbf{x}_t\right)$$

$$\mathbf{w}_t \sim p_{\mathbf{w}}\left(\mathbf{w}_t\right)$$

$$p\left(\mathbf{x}_{t+1}\right) = \int_{\mathcal{D}_{\mathbf{x}}} p_{\mathbf{w}}\left(\mathbf{x}_{t+1} - \mathbf{f}\left(\mathbf{x}_t, \mathbf{k}\left(\mathbf{x}_t\right)\right)\right) p\left(\mathbf{x}_t\right) d\mathbf{x}_t \quad (*)$$

A.H. Jazwinski (1970), H. Tong (1990)

Model Identification

$$\hat{\mathbf{x}}_{t+1} = \hat{\mathbf{f}}_{\mathbf{a}}\left(\mathbf{x}_t, \mathbf{u}_t\right) \quad \longrightarrow \quad \mathbf{e}_t = \hat{\mathbf{x}}_t - \mathbf{x}_t$$

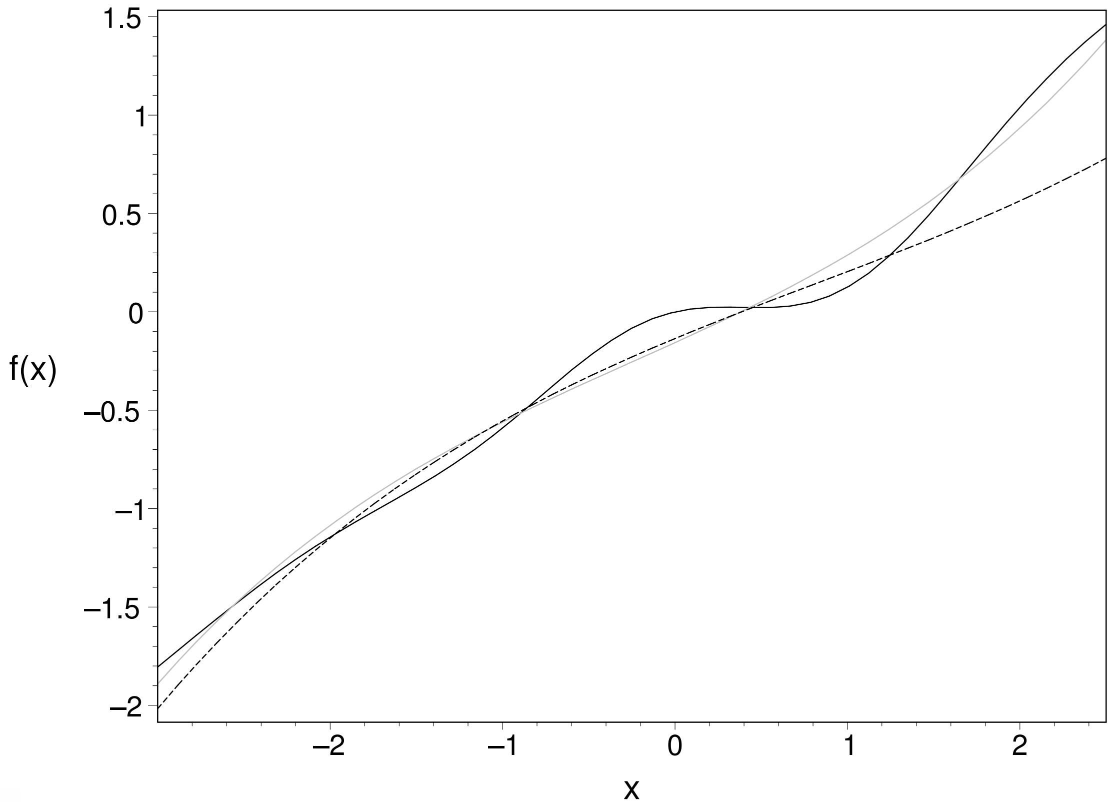
$$\longrightarrow \quad J = \frac{1}{N}\sum_{t=1}^{N} \mathbf{e}_{t+1}^T \mathbf{e}_{t+1}$$

PEM – L. Ljung (1999)

$$\int_{\mathcal{D}} \left(\hat{\mathbf{f}}_{\mathbf{a}}\left(\mathbf{x}, \mathbf{u}\right) - \mathbf{f}\left(\mathbf{x}, \mathbf{u}\right)\right)^T \left(\hat{\mathbf{f}}_{\mathbf{a}}\left(\mathbf{x}, \mathbf{u}\right) - \mathbf{f}\left(\mathbf{x}, \mathbf{u}\right)\right) p\left(\mathbf{x}, \mathbf{u}\right) d\mathbf{x} d\mathbf{u} + \Sigma$$

Model Error

– exact and approximate discrete-time process feedbacks



Control Design

$$J = E[\ell(\mathbf{x}_t, \mathbf{u}_t)]$$

$$J = \int_{\mathcal{D}_{\mathbf{x}}} \ell\left(\mathbf{x}_t, \mathbf{k}\left(\mathbf{x}_t\right)\right) p\left(\mathbf{x}_t\right) d\mathbf{x}_t$$

P.R. Latour (1996), T.J. Harris (1992)

Control Design

- parameterize control law: $\mathbf{u}_t = \mathbf{k}\left(\mathbf{x}_t\right) \simeq \hat{\mathbf{k}}\left(\mathbf{x}_t; \mathbf{a}\right)$
- parameterize stationary PDF: $p\left(\mathbf{x}\right) \simeq \hat{p}\left(\mathbf{x}; \mathbf{c}\right)$
- find parameters to optimize objective
- implicit constraint on parameters from $(*)$

PDF-Shaping

- in previous design, shape of PDF comes out of optimization
- instead of using objective function, pick a good PDF
- find corresponding controller parameters

– L.G. Crespo and J.Q. Sun (2002), M. Karny (1996)

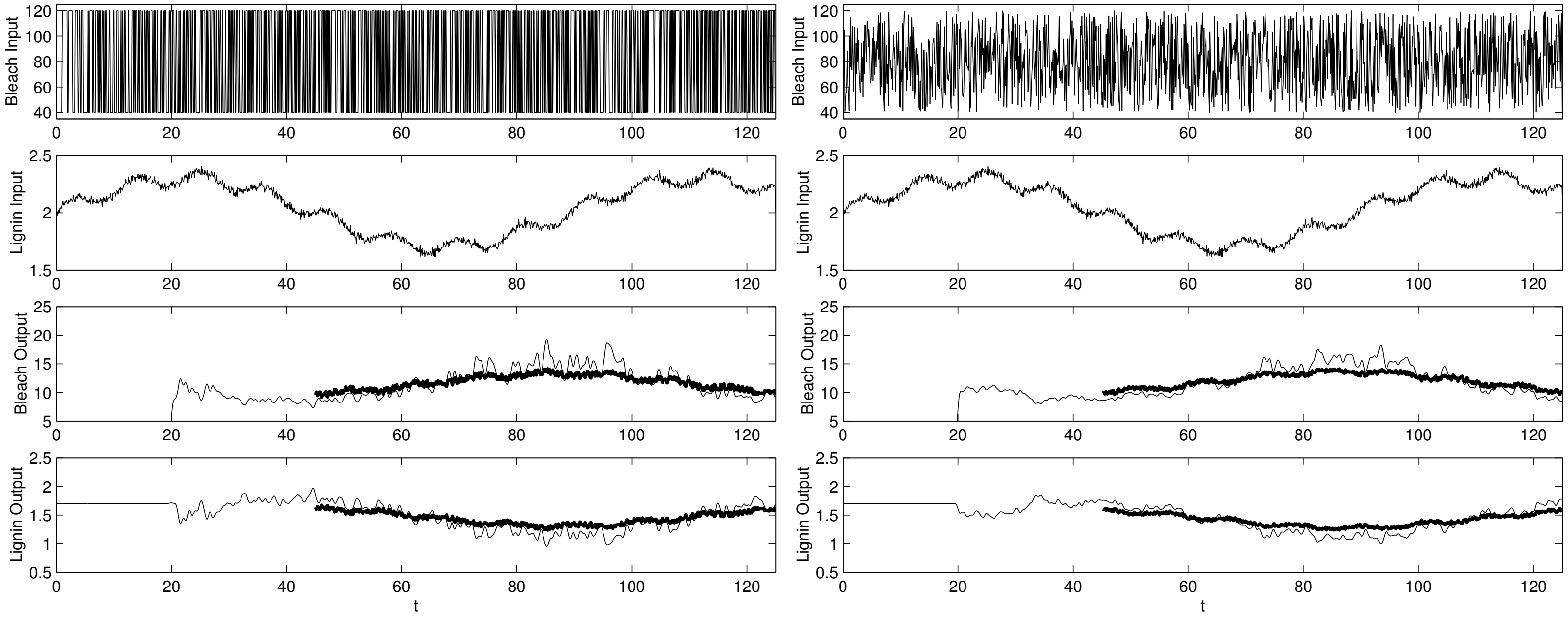
The Bleaching Reactor

– bleach mixed with pulp, mixture flows through reactor, lignin removed

- PDE's for lignin and bleach
- boundary conditions, initial conditions
- include convection, dispersion and reaction terms
- model of S. Renou, M. Perrier, D. Dochain and S. Gendron (2003)

$$\frac{\partial C}{\partial t} = -v\frac{\partial C}{\partial z} + D\frac{\partial^2 C}{\partial z^2} - k_C C^3 L^3$$

Time Series

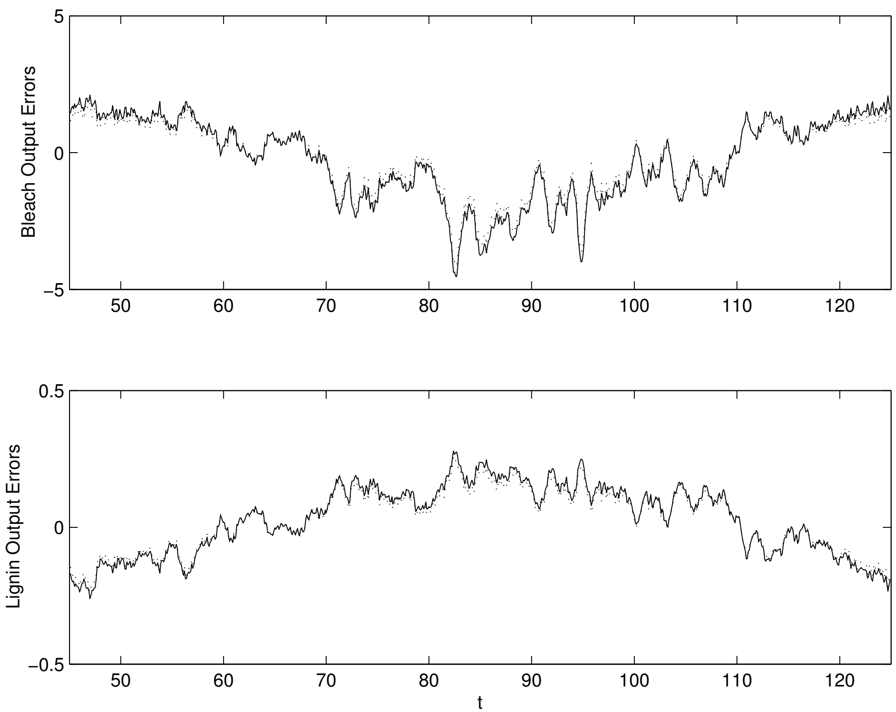


Model Building from Time Series

– tried different orders, nonlinearities

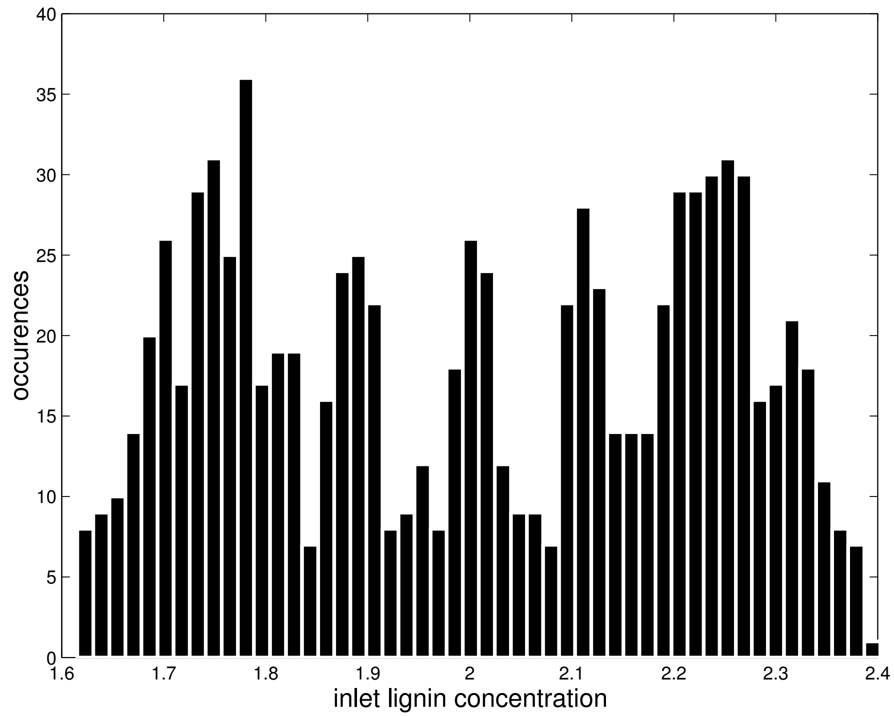
– plots of model errors

– improvements in range of $28 - 29\%$



Model Building

inlet lignin concentration is measured feedforward disturbance variable, needs probabilistic description



Control

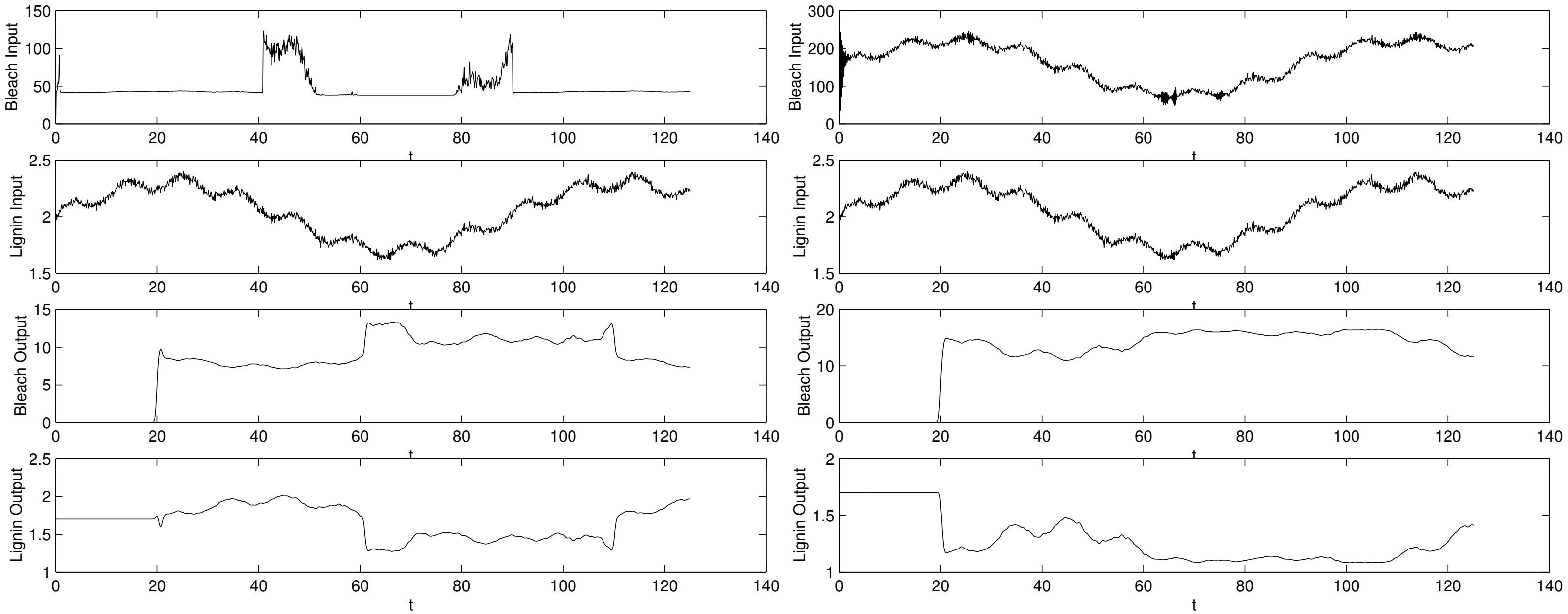
$$J = E\left[\ell\left(L_{k+b}^{out}\right) + \frac{\left(C_k^{in} - 40\right)^2}{4000}\right]$$

where:

$$\ell\left(L_{k+b}^{out}\right) = \begin{cases} 1.4 - L_{k+b}^{out} & L_{k+b}^{out} \leq 1.4 \\ 5 + \left(L_{k+b}^{out} - 1.4\right)^2 & L_{k+b}^{out} > 1.4 \end{cases}$$

– one design based on removing input loss from expectation, one design based on full, unconditional expectation

Control Results



– Summations of errors are $J = 3221$ and $J = 2497$.

Discussion

- very different behaviours
- first bounded, ‘gives up’ sometimes, tends to be myopic
- second strategy more aggressive, keeps lignin concentration lower
- 22% decrease in the cost for the second control law

Summary

- demonstrated use of probabilistic concepts to benefit identification and control in a process control setting
- ideas based on probabilistic concepts lead to improved experimental designs for the generation of identification data – improved process models
- use of unconditional expectation in regulatory controller design leads to better long term performance

Future Work

Probabilistic techniques for processing and manufacturing industries is an open area of research:

- continued interested in shaping distribution of identification data
- impact of random feedforward variables on PDF of process can be analyzed in general
- investigating full output feedback case

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Questions?