MINLP Optimization of Cooling Towers

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Abstract

This paper presents a mixed-integer nonlinear programming (MINLP) model for the optimal design of mechanical counter-flow cooling towers, subjected to standard design constraints. The objective function consists of the minimization of the total annual cost, which includes the capital cost of the cooling tower (that depends on the filling material and the air flowrate) and the operating cost (that includes the costs for the water makeup and for the power consumed by the tower fan). Merkel's method is used to set the dimensions of the tower, and empirical correlations are used to estimate mass transfer coefficients for the packed section of the tower. The design variables to be optimized are the water-to-air flow ratio, the water flow rate, the water inlet and outlet temperatures, the operating temperature approach, the type of packing, the type of draft, the volume of packing material, the total pressure drop due to the air flow rate, the fan power, and the water consumption.

The MINLP model was solved with the GAMS software package using the DICOPT solver. The performance of the proposed model is shown with six examples. From the results of the examples, it can be observed that cooling towers with low temperature approaches are not suitable because driving forces become a limiting factor. On the other hand, dry-bulb temperature variations have a negligible effect on the tower size. From the set of design parameters analyzed, the temperature approach is shown to be a critical factor for the optimal design of cooling towers.

Introduction

Closed-cycle cooling water systems are widely used to dissipate the low grade heat of chemical and petrochemical process industries, electric-power generating stations, refrigeration and air conditioning plants. In closed-cycle systems, water cools the hot process streams, then it is cooled by evaporation and direct contact with air in a wet-cooling tower and recycled to the cooling network.

The wet or evaporative cooling towers may be classified as natural draft and mechanical draft types. In both, warm water from the cooling network of a plant enters the top of the tower and always flows downward over an internal labyrinth-like packing, called fill, and air can flow upward (counter flow) or horizontally (cross flow). The fill distributes the water flow uniformly and provides a large air-water interface area for the simultaneous heat and mass transfer processes. As a result of the direct contact between the water and the air in the packing region, part of the water is vaporized and the water temperature is reduced while the air enthalpy is increased. The cooled water is then collected in a cold water basin below the fill from which it is returned to the cooling network. The blow-down water and the water lost through evaporation and drift are replaced with fresh make-up water. The moist air leaves the top of the tower.

In the mechanical draft cooling tower, air is circulated through the tower by means of electrically driven fans. On the other hand, the natural draft cooling tower uses the natural buoyancy of the warmed air to circulate it through the tower. Mechanical draft towers can be either induced draft (fan located at the top of the tower) or forced draft (fan located at the bottom of the tower).

In practice, large natural draft cooling towers are used in power plants for cooling the water supply to the condenser. Mechanical draft cooling towers are preferred for oil refineries and other process industries, as well as for central air-conditioning systems, due to they cover a much wider range of sizes, can be made more compact, give more uniform cooling and have lower water loss than natural draft towers.

Though the fundamentals of mechanical draft counter flow cooling towers are presented in a number of references, very few studies are available for the optimal economic design of this type of cooling towers. In this paper, an MINLP formulation is developed for optimizing mechanical draft counter flow cooling towers, which takes into consideration the above mentioned group of independent variables in conjunction with the constraints imposed on the problem. The objective is to minimize the total annualized cost of the tower. The Merkel's method is used for estimating the size and performance of cooling towers. The design of cooling towers also requires the optimal selection of the type of packing and, in this work, the choice is limited to film, splash, and tickle type fills. The mass transfer and pressure drop characteristics of these types of packing are presented in the form of empirical correlations given by Kloppers and Kröger.[1, 2] In addition, mechanical cooling towers can be either induced or forced. Disjunctive programming is used to formulate the discrete choices of type of packing and type of draft. In thermodynamic property modeling, the enthalpy of saturated air-water vapor mixture is expressed as an exponential function of the local bulk water temperature. For developing this correlation, the ASHRAE[3] property table was used as the source of the enthalpy of saturated air data from 8ºC to 55ºC for standard atmospheric pressure. The fitted equation reproduces the data from ASHRAE[3] with 0.047% of average absolute deviation. The optimization problem is solved with GAMS, using the solver DICOPT.[4]

Cooling tower model

Figure 1 shows the general arrangement and variables employed for the representation of a counter flow cooling tower.

The temperature difference between the water outlet temperature and the wet-bulb temperature of the air entering the tower is called the tower approach.

$$Approach = TW_{out} - TWB_{in} \tag{1}$$

The temperature difference between the cooling tower water inlet and outlet temperatures is termed the range of the tower.

$$Range = TW_{in} - TW_{out} \tag{2}$$

The integrated form the Merkel's equation is presented as:

$$Me = \int_{TW_{out}}^{TW_{in}} \frac{c_{pw} dTW}{(hsa - ha)} \tag{3a}$$

where the Merkel number, *Me*, is given by

$$Me = \frac{h_d a_{fi} A_{fr} L_{fi}}{m_w} = \frac{h_d a_{fi} L_{fi}}{G_w} \tag{3b}$$

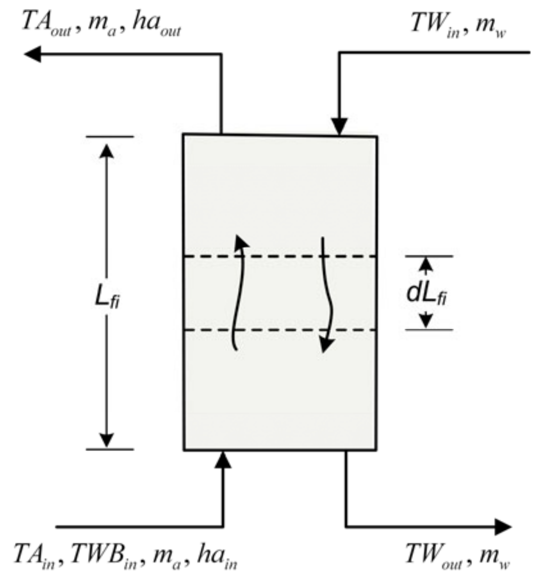

Figure 1. A differential control volume of a counter flow cooling tower

The operating line, which results from an overall energy balance on the fluid streams entering and leaving the tower is given as follows,

$$c_{pw}m_w\left(TW_{in} - TW_{out}\right) = m_a\left(ha_{out} - ha_{in}\right) \tag{4}$$

MINLP model formulation

Using the Merkel method, the optimal mechanical draft counter flow cooling tower problem is formulated as an MINLP optimization problem as follows.

Heat load

$$Q = c_{pw}m_w\left(TW_{in} - TW_{out}\right) \tag{5}$$

Required Merkel number

$$Me = \int_{TW_{out}}^{TW_{in}} \frac{c_{pw}dTW}{(hsa - ha)} = 0.25c_{pw}\left(TW_{in} - TW_{out}\right)\sum_{i=1}^{4} 1/\Delta h_i \tag{6}$$

For each temperature increment, the local enthalpy difference (Δh_i) is calculated from

$$\Delta h_i = hsa_i - ha_i, \quad i = 1,\ldots,4 \tag{7}$$

and the algebraic equations for calculating the water temperature and air enthalpies corresponding to each Chebyshev point are given by

$$TW_i = TW_{out} + TCH_i\left(TW_{in} - TW_{out}\right), \quad i = 1,\ldots,4 \tag{8}$$

$$ha_i = ha_{in} + \frac{c_{pw}m_w}{m_a}\left(TW_i - TW_{out}\right), \quad i = 1,\ldots,4 \tag{9}$$

$$hsa_i = -6.38887667 + 0.86581791 * TW_i + 15.7153617\exp\left(0.05439778 * TW_i\right), \quad i = 1,\ldots 4 \tag{10}$$

Packing performance: Transfer and loss coefficients.

The measured transfer coefficients are correlated in terms of the *available Merkel number* for a particular fill type as,

$$Me = c_1\left(\frac{m_w}{A_{fr}}\right)^{c_2}\left(\frac{m_a}{A_{fr}}\right)^{c_3}\left(L_{fi}\right)^{1+c_4}\left(TW_{in}\right)^{c_5} \tag{11}$$

The following disjunction is the one associated with the discrete choice of the type of packing. This is accomplished with the Boolean variable Y^k, which can be true or false depending on whether type of packing k is selected or not. The disjunction is as follows:

$$\begin{bmatrix} Y^1 \\ (\text{splash fill}) \\ c_j = c_j^1, \quad j = 1,...,5 \end{bmatrix} \vee \begin{bmatrix} Y^2 \\ (\text{trickle fill}) \\ c_j = c_j^2, \quad j = 1,...,5 \end{bmatrix} \vee \begin{bmatrix} Y^3 \\ (\text{film fill}) \\ c_j = c_j^3, \quad j = 1,...,5 \end{bmatrix}$$

This disjunction is transformed using the convex hull reformulation as following:

$$y^1 + y^2 + y^3 = 1 \tag{12}$$

$$c_j = c_j^1 + c_j^2 + c_j^3, \quad j = 1,...,5 \tag{13}$$

$$c_j^k = a_j^k \, y^k, \quad k = 1,...,3. \quad j = 1,...,5 \tag{14}$$

where y^k is the set of binary variables that decides the type of packing.

The tower fill performance is described not only by the transfer coefficient, but also by the loss coefficient per meter depth of fill, K_{fi}, that is required to calculate the pressure drop through a fill. For the splash, trickle, and film type fills, the loss coefficient correlations can be expressed in the following form[1]

$$K_{fi} = \left[d_1 \left(\frac{m_w}{A_{fr}} \right)^{d_2} \left(\frac{m_a}{A_{fr}} \right)^{d_3} + d_4 \left(\frac{m_w}{A_{fr}} \right)^{d_5} \left(\frac{m_a}{A_{fr}} \right)^{d_6} \right] L_{fi} \tag{15}$$

The corresponding disjunction is given by,

$$\begin{bmatrix} Y^1 \\ (\text{splash fill}) \\ d_l = d_l^1, \quad l = 1,...,6 \end{bmatrix} \vee \begin{bmatrix} Y^2 \\ (\text{trickle fill}) \\ d_l = d_l^2, \quad l = 1,...,6 \end{bmatrix} \vee \begin{bmatrix} Y^3 \\ (\text{film fill}) \\ d_l = d_l^3, \quad l = 1,...,6 \end{bmatrix}$$

This disjunction using the convex hull reformulation is expressed as follows

$$d_l = d_l^1 + d_l^2 + d_l^3, \quad l = 1,...,6 \tag{16}$$

$$d_l^k = b_l^k \, y^k, \quad k = 1,...,3. \quad l = 1,...,6 \tag{17}$$

Tower pressure drop

In mechanical draft cooling towers, the total pressure drop along the air path, ΔP_t, is the sum of the static pressure drop and the velocity pressure.[5] The static pressure drop includes the pressure drop through the fill, ΔP_{fi}, and the sum of miscellaneous pressure losses, ΔP_{misc}, e.g., in drift eliminator, air inlet, water distribution piping, etc.

The pressure drop through the fill matrix is coupled to the loss coefficient by the following relation[1]

$$\Delta P_{fi} = K_{fi} L_{fi} \frac{mav_m^2}{2 \rho_m A_{fr}^2} \tag{18}$$

where

$$mav_m = \frac{mav_{in} + mav_{out}}{2} \tag{19}$$

and

$$\rho_m = 1/\left(1/\rho_{in} + 1/\rho_{out}\right) \tag{20}$$

The air-vapor flow at the fill inlet and outlet mav_{in} and mav_{out} are calculated as follows

$$mav_{in} = m_a + w_{in} m_a \tag{21}$$

$$mav_{out} = m_a + w_{out} m_a \tag{22}$$

As pointed out by Mills[6], the miscellaneous pressure losses in components such drift eliminator, air inlet, water distribution piping, column supports, etc., are calculated by using a velocity head equation:

$$\Delta P_{mcl} = K_{mcl} \frac{\rho_{mcl} v_{mcl}^2}{2}$$

Some information on typical values for K_{mcl} is given by Li and Priddy[5] and Mills[6].

Total miscellaneous pressure losses can be represented by

$$\Delta P_{misc} = 6.5 \frac{\rho_m v_m^2}{2} = 6.5 \frac{mav_m^2}{2 \rho_m A_{fr}^2} \tag{23}$$

Another source of the total pressure drop in cooling towers is the velocity pressure, ΔP_{vp}, which is calculated as follows

$$\Delta P_{vp} = (2/3)(\Delta P_{fi} + \Delta P_{misc}) \tag{24}$$

By summing equations (22), (23), and (24), one can express the total pressure drop of the air stream for counter flow cooling towers as

$$\Delta P_t = 1.667(\Delta P_{fi} + \Delta P_{misc}) \tag{25}$$

Power consumption

The required power for a cooling tower fan can be determined by multiplying the total pressure drop with the air volume flow rate at the fan location. Hence, the power consumption is in terms of the air volume flow rate at the packing inlet for forced draft towers or at the packing outlet for induced draft towers. Consequently, the location of the air fan has effect on the power requirement. According to this feature, the following disjunction is required for the discrete decision associated with the optimal selection of draft type:

$$\begin{bmatrix} Y_{CT}^{mfd} \\ (\text{mecanical forced draft}) \\ P = \dfrac{mav_{in} \Delta P_t}{\rho_{in} \eta_f} \end{bmatrix} \vee \begin{bmatrix} Y_{CT}^{mid} \\ (\text{mecanical induced draft}) \\ P = \dfrac{mav_{out} \Delta P_t}{\rho_{out} \eta_f} \end{bmatrix}$$

This disjunction can be described by the following set of equations:

$$P \le \frac{mav_{in} \Delta P_t}{\rho_{in} \eta_f} + M_P^{\max} \left(1 - y_{CT}^{mfd}\right) \tag{26}$$

$$P \ge \frac{mav_{in} \Delta P_t}{\rho_{in} \eta_f} - M_P^{\max} \left(1 - y_{CT}^{mfd}\right) \tag{27}$$

$$P \le \frac{mav_{out} \Delta P_t}{\rho_{out} \eta_f} + M_P^{\max} \left(1 - y_{CT}^{mid}\right) \tag{28}$$

$$P \ge \frac{mav_{out} \Delta P_t}{\rho_{out} \eta_f} - M_P^{\max} \left(1 - y_{CT}^{mid}\right) \tag{29}$$

$$y_{CT}^{mfd} + y_{CT}^{mid} = 1 \tag{30}$$

Water Consumption

Makeup water (m_{mw}) is constantly added to the cooling tower basin to compensate for the loss of water to evaporation, drift, and blowdown.

From the conservation of mass, the rate of water evaporated (m_{wev}) into the air stream is related to the dry air flow rate and the air-inlet and outlet water vapor contents as follows

$$m_{wev} = m_a (w_{out} - w_{in}) \tag{31}$$

Since the evaporation water is essentially free of dissolved solids and impurities, all those introduced in the makeup water must be equal to that in blowdown and drift losses. Thus, the amount of blowdown can be expressed as

$$m_{bw} = \frac{m_{mw}}{n_{cycles}} - m_{wd} \tag{32}$$

In a well designed tower the drift loss is no more than 0.2 percent of the total water circulating flow[7]

$$m_{wd} = 0.002 m_w \tag{33}$$

The makeup water must equal the sum of the water losses to maintain a steady circulating water flow, that is, $m_{mw} = m_{mev} + m_{bw} + m_{wd}$,

$$m_{mw} = \frac{n_{cycles} m_{wev}}{n_{cycles} - 1} \tag{34}$$

Physical properties

For total pressure of 1 atm, the enthalpy of the air entering the tower, ha_{in}, is equal to the enthalpy of saturated air at a temperature equal to the inlet air wet-bulb temperature, TWB_{in} (Singham, 1983). Thus, ha_{in} can be calculated as follows

$$ha_{in} = -6.38887667 + 0.86581791 * TWB_{in} + 15.7153617 \exp(0.05439778 * TWB_{in}) \tag{35}$$

The mass fraction humidity of the air stream at tower inlet, w_{in}, is a function of the inlet air dry- and wet-bulb temperatures and is given by[8]

$$w_{in} = \left(\frac{2501.6 - 2.3263\,(TWB_{in})}{2506 + 1.8577\,(TA_{in}) - 4.184\,(TWB_{in})} \right) \left(\frac{0.62509\,(PV_{wbin})}{P_{tot} - 1.005\,(PV_{wbin})} \right) - \left(\frac{1.00416\,(TA_{in} - TWB_{in})}{2506 + 1.8577\,(TA_{in}) - 4.184\,(TWB_{in})} \right) \tag{36}$$

Eq. (4) can be rearranged to give

$$hsa_{out} = ha_{in} + \frac{c_{pw} m_w}{m_a} (TW_{in} - TW_{out}) \tag{37}$$

The outlet air temperature can be represented by the following functional relationship

$$TA_{out} = f_1(hsa_{out}) \tag{38}$$

The mass fraction humidity of the saturated air stream at tower outlet is given by (Kröger, 2004)

$$w_{out} = \frac{0.62509\, PV_{out}}{P_{tot} - 1.05\, PV_{out}} \tag{39}$$

The vapor pressure of water corresponding to a specified temperature is calculated from the correlation given by Hyland and Wexler[9],

$$\ln(PV) = \sum_{n=-1}^{3} c_n T^n + 6.5459673\, \ln(T) \tag{40}$$

The density of air-water vapor mixtures can be calculated from the ideal gas law as

$$\rho = \frac{P_{tot}}{287.08\, T} \left[1 - \frac{w}{w + 0.62198} \right] [1 + w] \tag{41}$$

Feasibility constraints

In this section, inequality constraints are written to define the feasible region of the optimization problem as well as feasible operation of the optimal cooling tower as follows,

$$TW_{out} - TWB_{in} \geq 2.8 \tag{42}$$

$$TW_{out} \leq TMPO - \Delta T_{min} \tag{43}$$

$$TW_{in} \leq TMPI - \Delta T_{min} \quad (44)$$

$$TW_{in} \leq 50^{o} C \quad (45)$$

$$TW_{in} > TW_{out} \quad (46)$$

$$TA_{out} > TA_{in} \quad (47)$$

$$hsa_{i} - ha_{i} > 0 \quad i = 1,...,4 \quad (47)$$

$$0.5 \leq \frac{m_w}{m_a} \leq 2.5 \quad (48)$$

$$2.90 \leq \frac{m_w}{A_{fr}} \leq 5.96 \quad (49)$$

$$1.20 \leq \frac{m_a}{A_{fr}} \leq 4.25 \quad (50)$$

$$m_w > 0 \quad (51)$$

$$m_a > 0 \quad (52)$$

Objective function

The objective function involves the minimization of the total annualized cost, *TAC*, which is the sum of the annualized capital cost of the cooling tower and annual operating costs C_{op}:

$$\min imize\, TAC = K_f C_{cap,CT} + C_{op} \quad (53)$$

The annual operating costs are determined by the makeup water consumed and the fan power required for a given application:

$$C_{op} = H_Y c_w m_{mw} + H_Y c_e P \quad (54)$$

The formula for the installed capital cost of cooling towers, given below, is from Kintner-Meyer and Emery[10]:

$$C_{cap,CT} = C_{CTF} + C_{CTV} A_{fr} L_{fi} + C_{CTMA} m_a \quad (55)$$

The cost coefficient C_{CTV} depends on the type of packing and is represented by the following disjunction:

$$\begin{bmatrix} Y^1 \\ (splash\ fill) \\ C_{CT,V} = C^1_{CT,V} \end{bmatrix} \vee \begin{bmatrix} Y^2 \\ (trickle\ fill) \\ C_{CT,V} = C^2_{CT,V} \end{bmatrix} \vee \begin{bmatrix} Y^3 \\ (film\ fill) \\ C_{CT,V} = C^3_{CT,V} \end{bmatrix}$$

This disjunction can be described by the following equations:

$$C_{CT,V} = C^1_{CT,V} + C^2_{CT,V} + C^3_{CT,V} \quad (56)$$

$$C^k_{CT,V} = e^k y^k, \quad k = 1,...,3 \quad (57)$$

Numerical examples

The software DICOPT included in the GAMS[4] optimization package was used to solve the proposed MINLP model given by Eqs. (5) to (57). In Table 1, the design specifications and process constraints for six examples serving as test problems are presented. Example 1 is the base case. In the other examples, only one input variable or design constraint varies while the other input variables or design constraints maintain at the base values. For all examples, the cooling tower is required to remove 3400 kW at an ambient air pressure of 101 325 Pa. The c_{pw} for the water is taken as 4.187 kJ/kgºC, the H_Y parameter is assumed as 2.934×10^7 s/year, and the annualizing factor for the capital cost as 0.2983/year. The values of c_w, c_e, C_{CTF}, C_{CTMA}, η_f and n_{cycles}, are taken as US$5.283x$10^{-4}$/kg, US$0.085/kWh, US$31,185, US$1,097.5/(kg of dry air/s), 0.75 and 4, respectively.

Table 1. Design specifications and process Constraints for the examples

	Examples					
	1	2	3	4	5	6
Q (kW)	3400	3400	3400	3400	3400	3400
TA_{in} (ºC)	22	17	22	22	22	22
TWB_{in} (ºC)	12	12	7	12	12	12
$TMPI$ (ºC)	65	65	65	55	65	65
$TMPO$ (ºC)	30	30	30	30	25	30
ΔT_{min} (ºC)	10	10	10	10	10	5

The results of the total annualized cost minimization for the six examples are presented in Table 2. For each example, the selected type of filling material is the film packing that offers the best combination of heat transfer and pressure losses, enabling the lowest total annualized cost. Also, the selected type of draft is the forced draft in all examples. The water and air loadings of all examples are in the ranges given by constraints (49) and (50), respectively. It should be noted that the total annualized cost is dominated by the operating costs for all examples. In general, the cost of makeup water is greater than the cost of electric power required for the fan performance.

The effect of variation of the inlet air dry-bulb temperature, TA_{in}, on the total annualized cost can be established from the Examples 1 and 2. As TA_{in} is decreased from 22 to 17ºC, the optimum total annualized cost shifts from US$66,065.14/yr to US$64,604.64/yr. It causes a 2.21% decrease in the total annualized cost. Thus, the total annualized cost is fairly insensitive to the inlet air dry-bulb temperature for the range of conditions tried, and decreases slightly with TA_{in}.

However, as shown in Table 2, decreasing the tower approach has a pronounced effect on the optimum value of total annualized cost. By taking the optimum solution of Example 1 as reference, we see that, when the optimal tower approach is decreased from 8 to 3 (Example 5), the total annualized cost is increased by 77.52%. In general, as the approach is reduced, tower size increases exponentially if other conditions remain unchanged, and it follows that total annualized cost of the tower also increases exponentially. This is because the driving forces become more limiting as the tower approach decreases and so a larger Me or tower size is required for rejecting the same heat load. In contrast to this behavior, Examples 3 and 6 show that as the tower approach increases the total annualized cost decreases significantly. Thus, it is clear that the tower approach is an important optimization variable.

Examples 3 and 6 show that optimal tower approach increases when TWB_{in} and ΔT_{min} decrease. On the other hand, Example 5 shows that, as $TMPO$ decreases, the optimal tower approach also decreases. Since TWB_{in} is site-specific and $TMPO$ is limited by the temperature conditions of the process, these variables are taken as input parameters. Thus, only the value of ΔT_{min} could be determined by optimization. This decision variable could be minimized to maximize the tower approach and, therefore, minimize the size of the cooling tower. However, this is offset by increases in the size of the coolers in the cooling network due to lower log-mean temperature differences. Thus, this optimization problem must take into account the interaction between tower performance and cooling network performance to select the optimum value of ΔT_{min} in the context of the entire system.

The effect of the cooling range on the cooling tower cost is illustrated in Examples 3 and 6 for a constant tower approach and inlet water temperature. It was found that a reduced cooling range results in a higher water outlet temperature, with the corresponding increase in driving forces for cooling. Thus, as the cooling range is decreased, the tower size (or Merkel number) and tower cost become smaller.

Table 2. Optimization Results for the Examples

	Examples					
	1	2	3	4	5	6
m_w (kg/s)	25.720	25.794	25.700	30.973	22.127	30.749
m_a (kg/s)	31.014	31.443	28.199	36.950	32.428	27.205
m_w/m_a	0.829	0.820	0.911	0.838	0.682	1.130
m_{mw} (kg/s)	1.541	1.456	1.564	1.547	1.542	1.540
m_{wev} (kg/s)	1.156	1.092	1.173	1.160	1.157	1.155
m_{bw} (kg/s)	0.334	0.312	0.340	0.325	0.341	0.323
m_{wd} (kg/s)	0.051	0.052	0.051	0.062	0.044	0.061
TW_{in} (ºC)	50	50	50	45	50	50
TW_{out} (ºC)	20	20	20	20	15	25
TA_{out} (ºC)	37.077	36.871	36.998	34.511	36.411	39.083
Range (ºC)	30	30	30	25	35	25
Approach (ºC)	8	8	13	8	3	13
L_{fi} (m)	2.294	2.239	1.858	2.154	6.299	1.480
A_{fr} (m^2)	8.869	8.894	8.862	10.680	7.630	9.296
K_{fi}	21.946	21.950	21.926	21.942	22.066	22.639
ΔP_{fi} (Pa)	280.331	277.727	186.621	254.540	1139.529	131.908
ΔP_{misc} (Pa)	36.189	36.740	29.782	35.011	53.288	25.596
ΔP_t (Pa)	527.640	524.216	360.744	482.683	1988.425	262.560
P (hp)	24.637	24.474	15.205	26.852	97.077	10.754
Type of packing	Film	Film	Film	Film	Film	Film
Type of draft	Forced	Forced	Forced	Forced	Forced	Forced
Me	3.083	3.055	2.466	2.923	7.335	1.858
C_{mw}	23885.109	22566.366	24239.785	23983.449	23901.657	23865.877
C_{power}	12737.595	12653.677	7861.037	13882.754	50190.495	5559.875
C_{op}	36622.703	35220.043	32100.822	37866.203	74092.153	29425.752
$KC_{cap,CT}$	29442.436	29384.597	26615.995	32667.705	43186.526	25030.274
TAC	66065.139	64604.640	58716.817	70533.909	117278.68	54456.026

In addition, Examples 2 and 4 (or 1 and 4) show the effect of cooling range for fixed tower approach and fixed water outlet temperature. In this case, we see that as cooling range increases, the inlet water temperature also increases. This results in decreased water and air mass flow rate, which gives smaller operating costs and capital cost.

It should be noted that the best optimal solution (Example 6) has a range and an approach temperature of 25ºC and 13ºC, respectively. This result is in agreement with above discussion. The associated operating and capital costs are US$29,425.752/yr and US$25,030.274/yr. This constitutes a cost reduction of 14.98% for the capital cost and 19.65% for the operating cost when compared to the base case.

Conclusions

This paper presents an MINLP formulation for the optimal design of mechanical counter flow cooling towers. The Merkel method is used for sizing cooling towers due to it is usually adopted in the practice to design these industrial units.[2] This method relates quantities associated with the specified cooling requirement to quantities associated with the heat transfer performance of a given packing.[11] The *required Merkel number* is calculated using the Chebyshev integration technique. The film, splash, and trickle type fills are the three different types of packing arrangements considered in this paper. Thus, the MINLP model has different empirical correlations for calculating the *available Merkel number* and packing pressure drop of air depending on the type of packing. Also, the model considers that mechanical cooling towers could be induced or forced. These design decisions are represented by disjunctive programming models.

For a given heat load of the tower and inlet air conditions, the solution yields the optimal geometric dimensions of fill height, and cross sectional area of the tower and the optimal operating parameters of water mass flow rate, air mass flow rate, water consumption, power consumption, water outlet temperature, and water inlet temperature, as well as the optimal type of packing and type of draft. As shown in the examples, cooling towers with low approaches are more expensive since driving forces become more limiting. The reverse is also true. On the other hand, dry-bulb temperature variations have negligible effect on tower size. The conclusion is that approach is critical for the optimal design of cooling towers.

References

1. Kloppers, J.C., and Kröger, D.G. (2003). Loss coefficient correlation for wet-cooling tower fills. *Applied Thermal Engineering,* 23(17): 2201-2211.
2. Kloppers, J.C., and Kröger, D.G. (2005). Refinement of the transfer characteristic correlation of wet-cooling tower fills. *Heat Transfer Engineering,* 26(4): 35-41.
3. ASHRAE. (2001). *ASHRAE Handbook – Fundamentals* (American Society of Heating, refrigerating and Air-Conditioning Engineers, Inc., USA).
4. Brooke, A., Kendrick, D., and Meeraus, A. (2006) *GAMS User's Guide* (The Scientific Press, USA).
5. Li, K.W., and Priddy, A.P. (1985). *Power Plant System Design* (John Wiley & Sons, USA).
6. Mills, A.E. (1999). *Basic Heat and Mass Transfer* (Prentice Hall, USA).
7. Kemmer, F.N. (1988). *The NALCO water handbook (*McGraw-Hill, USA).
8. Kröger, D.G. (2004). *Air-Cooled Heat Exchangers and Cooling Towers* (PenWell Corp., USA).
9. Hyland, R.W. and Wexler, A. (1983). Formulation for the thermodynamic properties of the saturated phases of H O from 173.15 K and 473.15 K, *ASHRAE Transactions*, 89(2A): 500-519.
10. Kintner-Meyer, M. and Emery, A.F. (1995). Cost-optimal design for cooling towers, *ASHRAE Journal*, 4: 46-55.
11. Singham, J.R. (1983). *Heat Exchanger Design Handbook* (Hemisphere Publishing Corporation, USA).