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Accurate Model Identification for Non-Invertible MIMO Sandwich Block-Oriented Processes

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Abstract

In the previous work our research group introduced an exact solution to the Hammerstein system and is known as H-BEST (Hammerstein Block-Oriented Exact Solution Technique). It was later expanded to multiple input, multiple output (MIMO) Hammerstein and Wiener (W-BEST) systems. This work is extends this approach to more complicated block-oriented systems, namely a Hammerstein-Wiener process. By exploiting a closed form solution, the proposed method has the major advantages of being simple to build and implement the model, and does not require a complex algorithm or solution to a difficult optimization problem. The method is evaluated and compared to another method. With a slower sampling rate, the proposed method is shown to provide a higher level of predictive accuracy.

I. Introduction

A continuous-time method, which results in a "grey-box" or semi-empirical modeling has been first proposed by Rollins et al.[1] by introducing an exact solution to the Hammerstein system and it is known as H-BEST (Hammerstein Block-Oriented Exact Solution Technique). Rollins et al.[2] and Bhandari and Rollins[3] extended this work into multiple input, multiple output (MIMO) Hammerstein and Wiener system (W-BEST), respectively. Because of the limitation of the proposed solutions, another algorithm is also developed to give a more accurate prediction to the process and a detail explanation can be found in Chin et al.[5] In order to widen the scope of "BEST", the work here is to extent and apply it to a more complicated block-oriented system, namely a Hammerstein-Wiener process.

According to Brillinger[6], Greblicki and Pawlak[7], Pearson and Ogunnaike[8], Hammerstein-Wiener process is a type of a general "sandwich model" and in this case the linear dynamic block is "sandwiched" between two static nonlinearities models as shown in Figure 1. Thus, both Hammerstein and Wiener process are special cases of this general model. The work done on system identification for Hammerstein-Wiener system can be found in the literatures.[9-15] For example, Bai[9] proposed a two-stage identification approach by first making assumptions of the model structure, then estimating an oversized parameter vector and finally using singular vector decomposition to reduce its dimension. He later improved his method by using a general structure of the nonlinearities instead and calling this a blind approach.[10] However, the output nonlinear static function has to be an invertible function. This is also the key assumption for the case for Bloemen et al.[11], Zhu[12], Crama and Schoukens[13], Lee et al.[14] and Park et al.[4]. Bolkvadze [15] applied recursive identification method in the context of stochastic systems but it was limited to discrete model.

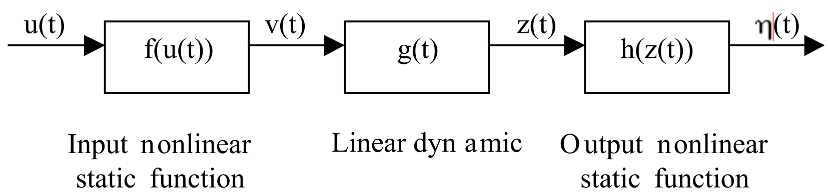

Figure 1. Description of a single input, single output Hammerstein-Wiener process.

But because of the nature of BEST method, one will not need to make any assumptions of the model structures and also not limited to using invertible nonlinear static functions. Thus, this work here presents the application of BEST to Hammerstein-Wiener process. In order to show its superiority, the case studies from Park et al.[4] will be re-simulated so that comparison can be made between their method and the BEST method but only one case study will be presented here due to space limitation. Also, not many of the work done in this area had dealt with MIMO case except for Bloemen et al.[11] and Lee et al.[14]. Bloemen et al.[11] presented an example of two parallel SISO Hammerstein-Wiener system which they considered as a MIMO case. In Lee et al.[14], even though they included MIMO problem in the scope of work, the assumption of invertiblity is still very crucial in their method. Therefore the main advantage of BEST is that not only it is not restricted to using invertible nonlinear static functions but it can easily be extended to MIMO case. So this paper is organized as follows. In the next section (Section II), an overview of BEST is given followed by its extension to the Wiener-Hammerstein process. In Section IV, a brief summary of the method introduced by Park et al.[4] will be given and Section V consist of one study case in order to illustrate the application of the proposed method as well as to compare to method by Park et al.[4]. Finally, concluding remarks will be presented in Section VI.

II. Overview of BEST

BEST is a comprehensive model building approach that utilizes Statistical Design Of Experiment (SDOE) to provide optimal and complete information, uses a two-stage identification procedure, and exploits an exact solution to both Hammerstein and Wiener system. The SDOE not only serves as an optimal experimental design method that maximizes information but it also provides the necessary information to accurately estimate all the parameters[3]. This can be done because the static nonlinear block can be modeled separately from the dynamic block.

In the presence of noise, the measurement model used for the scope of this work is presented below:

$$y_i(t) = \eta_i(t) + \varepsilon_i(t) \tag{1}$$

where

$$\varepsilon_i(t) \sim N\left(0,\ \sigma_i^2\right)\ \forall\ i \tag{2}$$

$y_i(t)$ is the measurement of output i taken at time t, and $\varepsilon_i(t)$ is the corresponding error term, with the covariance of $\varepsilon_i(t)$ and $\varepsilon_i(t)$ equal to zero for $i \neq j$. For the exact solutions to these systems, the authors[1-3,5] have developed 3 different solutions based on different conditions and only one will be used in this paper. For a Wiener system with no restrictions on the input vector $\boldsymbol{u}^{px1}(t)$, a general solution known as the classical algorithm (CA) can be expressed as[3]

$$\eta_i(t) = f_i\left[v_i(t)\right], \qquad j = 1,\ \ldots,\ q \tag{3}$$

$$v_{ij} = \mathcal{L}^{-1}\left\{G_{ij}(s) \cdot U_j(s)\right\}, \quad i = 1, \ldots., p \tag{4}$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform operator.

The development of these solutions enables one to do system identification for Hammerstein and Wiener processes. The specific steps for model building are as follow:[2]

1. Determine the statistical experimental design

2. Run the experimental design as a series of step tests, allowing steady state to occur after each change and collecting the data dynamically over time.

3. Use the steady state data to determine the nonlinear static function, f(u).

4. Use all data to determine the dynamic function.

For a Hammerstein-Wiener system (refer to Figure 1), the classical solution written in terms of matrix variable, is given in the following equations

$$\mathbf{V} = f(\mathbf{U}) \tag{5}$$

$$\mathbf{Z} = \mathcal{L}^{-1}\left\{G(s) \cdot \mathbf{V}(s)\right\} \tag{6}$$

$$\mathbf{H} = \mathrm{h}(\mathbf{Z}) \tag{7}$$

The steps for the model building in Hammerstein-Wiener system will be explained in the next section.

III. Modeling Approach

In Wiener (or Hammerstein) process, the nonlinear static function can be modeled separately from the linear dynamic model because as the process reach steady state the following equation is true.

$$u_i(t) = v_{ij}(t) \tag{8}$$

when $g_{ij}(t) \approx 1$. Therefore,

$$\eta_i(t) = f_i(v_{ij}(t)) = f_i(u_i(t)) \tag{9}$$

This concept is then applied to the Hammerstein-Wiener process. Therefore when the process reach approximately steady state, base on Figure 1, equations (9) and (10) are true because $g(t) \approx 1$.

$$f(u(t)) = v(t) = z(t) \tag{10}$$

when $g_{ij}(t) \approx 1$. Therefore,

$$\eta(t) = h(z(t)) \tag{11}$$

In order to obtain the estimates of these equations, a proposed method is to minimize the sum of square error (SSE) (as given in equation (11)) by directly estimating f(u(t)), h(z(t)) and g(t) simultaneously.

$$SSE = \underset{\text{steady state data}}{\left(y_i^{\infty} - \hat{\eta}_i^{\infty}\right)^2} + \underset{\text{all data}}{\left(y_i - \hat{\eta}_i\right)^2} \tag{12}$$

y_i and y_i^{∞} is the true response and true response at steady state, respectively. $\hat{\eta}_i$ and $\hat{\eta}_i^{\infty}$ is the predicted response produce by BEST and the predicted response produce by BEST at steady state, respectively. Unlike the current method where the steady state model and the dynamic model are done in two stages, these two models have to be estimated simultaneously. The reason is because even though *f(u)* describe the steady state behavior, *h(z)* no longer only does that but also include the dynamic behavior. This is due to the fact that *h(z)* comes from z which describes the dynamic structure of the Hammerstein-Wiener system. Therefore the steps for model building for Hammerstein-Wiener system need to be modified from the original approach and is given as bellow:

1. Determine the statistical experimental design

2. Run the experimental design as a series of step tests, allowing steady state to occur after each change and collecting the data dynamically over time.

3. Use the steady state data to determine the nonlinear static functions and use all data to determine the dynamic function, simultaneously.

IV. Method By Park et al.

Because of the non-convergence, high computational time of a multidimensional nonlinear optimization problem (unless using the right initial values), Park et al.[4] proposed three special input tests signals that enable Hammerstein-Wiener process to be estimated sequentially and we will use PSL as the abbreviation for their method. Each step involves at most solving one-dimensional problem which makes it a simpler problem. Two of the three test signals are of the random binary sequence (RBS) (a type of pseudo random sequence design (PRSD)) with varying switching time and the third one is of the uniform distribution sequence. In our method (BEST), it uses an input change sequence in the form of sequential step tests for model development derived from the statistical design of experiment (SDOE). Studies had been performed which concluded that SDOE is more superior that PRSD in terms of a quantitative measure of information content based on D-optimality criterion (Rollins et al.[15]) as well as in a study of a continuous stirred-tank reactor (Bhandari and Rollins[3]).

As mentioned earlier, one of the important requirements in this method is to assume the output nonlinear static function to be strictly monotone and invertible so that it can be estimated first. But in BEST such requirement is not necessarily because of the simultaneous estimation of all the parameters. Even though it is a multidimensional nonlinear optimization problem but because the algorithm used are an exact classical solution to the Hammerstein-Wiener system, only a small number of parameters needed to be estimated, therefore it is not difficult to reach a convergence point and does not take long to compute.

Lastly, PSL method is applicable to both of the continuous-time and discrete-time model which is also true for BEST method.

V. Comparison Studies

In order to compare with the method proposed by Park et al.[4], four different study cases have been performed, three of which are taken from the paper and one multiple input, multiple output case but only one will be presented here. All of the cases taken from the paper were sampled at every 0.01 minutes running at 100 units of time for both the design and test data. As part of the design data in their method, two of the test signals were running simultaneously from 0 to 50 minutes and it went from time 0 to 100 minutes. So by using the same total amount of the time, our SDOE is to run in length of 150 minutes. In all of these cases, we sampled at 1 minute for design data and are still able to perform much better than the PSL method.

1. *Case Study 1*

The first case study is a fourth order dynamic Hammerstein-Wiener process with the exponential input and output nonlinear functions. It is a single-input, single output system as described by equations (13) to (16).

$$y(t) = 0.18z(t) + 0.5\left(e^{2.1z(t)} - 1\right) \tag{13}$$

where

$$z(t) = \mathcal{L}^{-1}\left\{G(s)V(s)\right\} \tag{14}$$

$$G(s) = \frac{e^{-s}}{s^4 + 4s^3 + 6s^2 + 4s + 1} \tag{15}$$

$$V(s) = \frac{1 - e^{(-3u(t))}}{s} \tag{16}$$

The theoretical process is shown in Figure 2, along with the application of the classical solution using the true parameters. Since the lines agree exactly, this confirms that the classical solution is an exact solution to the Hammerstein-Wiener process. The next step is to test how well the proposed method fit this theoretical process assuming that the true process in unknown. A variety of the input and output nonlinear steady state functions as well as the dynamic structures are fitted. The R^2 values are given in the Table 1 for each fitted model. The same test sequence obtain from Park et al.[4] is used to see how well the models performance in comparison to theirs. In order to estimate the accuracy of the different prediction models quantitatively, we used a measure that is called the sum of squared prediction error (SSPE), which is defined as

$$SSPE = \sum_{i=1}^{N} \left(y_i(t) - \hat{\eta}_i(t)\right)^2 \tag{17}$$

where N is the total number of equally spaced sampling points used over the testing interval. Park et al.[4] used integral of the square error (ISE) to quantify its model performance.

$$ISE = \int \left(y(t) - \hat{y}(t)\right)^2 dt \tag{18}$$

By writing ISE in a discrete form,

$$\text{ISE} \approx \sum_{i=1}^{N} \left(y_i(t) - \hat{y}_i(t)\right)^2 \Delta t \tag{19}$$

where Δt is the width of the interval time. The $\hat{y}_i(t)$ in the above equation is equivalent to $\hat{\eta}_i(t)$ in our definition of SSPE. Therefore, under the same sampling time (i.e. same dt value), $\text{ISE} = \text{SSPE} \cdot \text{dt}$. The values of SSPE and ISE are computed under the estimated models and are given Table 2 where FOCPD, TOCPD, TOOPD, SOCPD, SOOPD are abbreviations of fourth order critical plus dead time, third order critical plus dead time, third order overdamped plus dead time, second order critical plus dead time and second order overdamped plus dead time, respectively and θ is the dead time.

Table 1. R^2 values for the different fitted nonlinear static models and dynamic models for case study 1

Nonlinear static functions	Dynamic models				
	FOCPD (θ=1)	TOCPD (θ=2)	TOOPD (θ=1.5)	SOCPD (θ=3)	SOOPD (θ=2)
5^{th} order	0.99997*	0.9993	0.9999*	0.9926	0.9995*
4^{th} order	0.9995*		0.9995*		
3^{rd} order	0.9940		0.9966		

* Values that are higher than those obtained by Park et al.[4]

The authors in Park et al.[4] chose a fifth order polynomial for both the nonlinear static functions and a third order plus time delay (θ=1.496) for the linear dynamic structure with R^2 value of 0.9992 and the model validation shows an ISE value of 0.2945. For the best accuracy, BEST produces a 0.99997 R^2 value under a FOCPD dynamic model and fifth order polynomial static nonlinear functions with ISE value of 0.0104 for the test sequence. Even for a simpler dynamic model or a lower order nonlinear static functions, BEST is still able to perform better than PSL method. Compare BEST TOOPD with PSL fitted model, BEST's R^2 value is higher and ISE value for the test sequence is 17% of their fitted model's ISE value which shows the superiority of our method.

Table 2. SSPE (the top values) and ISE (the bottom values) values for the different fitted nonlinear static models and dynamic models for the test sequence in case study 1

Nonlinear static functions	Dynamic models				
	FOCPD (θ=1)	TOCPD (θ=2)	TOOPD (θ=1.5)	SOCPD (θ=3)	SOOPD (θ=2)
5^{th} order	1.037* (0.0104)	53.430 (0.5343)	5.0783* (0.0508)	287.27 (2.873)	28.8205* (0.2882)
4^{th} order	4.6160* (0.0462)		11.4604* (0.1146)		

3rd order	65.164 (0.6516)		221.001 (2.21)		

* Values that are lower than those obtained by Park et al.[4]

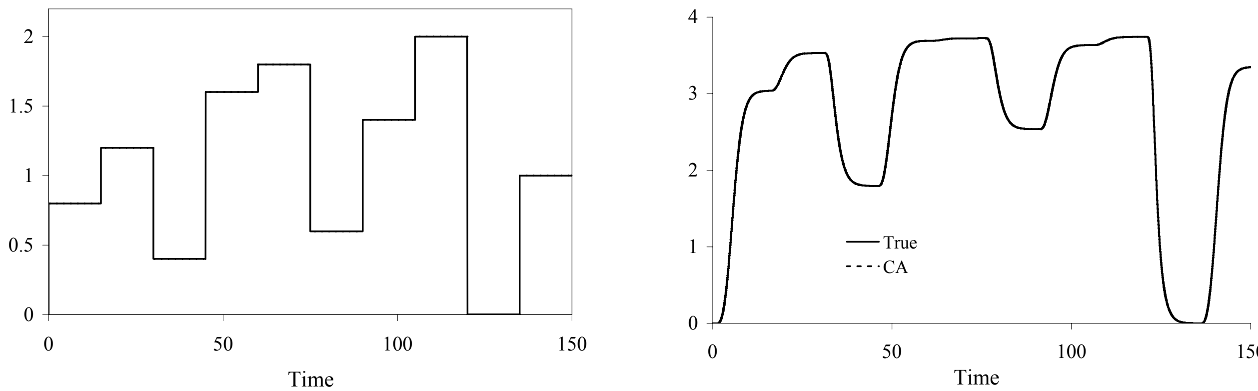

Figure 2. (a) Step input changes used for identification of BEST model. (b) Comparison of the theoretical (i.e. true) solution with the classical solutions for a Hammerstein-Wiener system with a sequence of step input changes.

V. Closing Remarks

The work here is to extent the existing BEST method to a Hammerstein-Wiener process, a more intricate block-oriented system. A proposed estimation method to this system is to simultaneously estimate all of the parameters which include all of the nonlinear static and dynamic parameters. Because this is a semi-empirical technique, the numbers of parameters are in the reasonable numbers, therefore it is not difficult to reach a convergence point. Its higher performance can be seen in the example shown. Also unlike the PSL method where the output nonlinear static function has to be strictly monotone and invertible so that it can be estimated first, BEST has no such requirement. Finally, BEST incorporation with as SDOE allows the user obtain more accurate information content in order to do parameter estimation.[16]

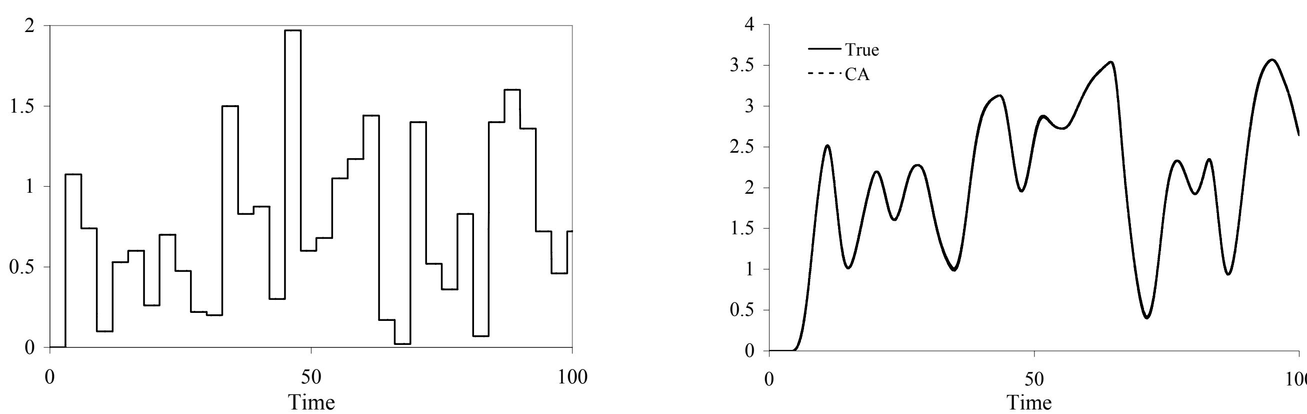

Figure 3. (a) Input test sequence for case study 1. (b) Comparison of the theoretical (i.e. true) solution with the classical solution for a Hammerstein-Wiener system using the estimated fifth order polynomial nonlinear static functions and a fourth order dynamic structures with time delay.

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