Robust Scheduling of Crude Oil Operations under Demand and Ship Arrival Uncertainty

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1 Introduction

Scheduling of crude oil operations is an important and complex routine task in a refinery. It involves crude oil unloading, tank allocation, storage and blending of crudes, and CDU charging. Optimal crude oil scheduling can increase profits by exploiting cheaper but poor quality crudes, minimizing crude changeovers, avoiding ship demurrage, and managing crude inventory optimally. In our previous work (Li et al. 2005), we have developed robust algorithms for obtaining optimal schedules for operations without any uncertainty. However, in a practice, uncertainties are unavoidable. Some common and frequent uncertainties in refinery operations include ship arrival delays, demand fluctuations, equipment malfunction, etc. In the face of these uncertainties, an optimal schedule obtained using nominal parameter values may often be suboptimal or even become infeasible. Thus, it is critical to develop algorithms that can consider future uncertainty at the scheduling stage to improve schedule feasibility and robustness.

So far, scheduling and planning under uncertainty has been studied in specific fields such as capacity expansion, production planning, batch plant scheduling, etc. Demand and processing time uncertainties have been the focus of most existing work in the batch area. However, little work exists on refinery planning and scheduling under uncertainty. Arief et al. (2004) proposed a heuristic approach to reschedule operations of a given schedule to accommodate disruptions. Neiro and Pinto (2005) proposed a production-planning model incorporating product price and demand uncertainties in a refinery. Li et al. (2004) addressed the problem of refinery planning under demand or other economic parameters uncertainties with two-stage stochastic programming approach. Li et al. (2005) developed a planning model for refinery under correlated and truncated price and demand uncertainties. However, no work has so far addressed the development of robust schedules for crude oil scheduling in the face of uncertainties.

Therefore, in this paper, we modify the deterministic MILP approach of Reddy et al. (2004b) to address two important uncertainties in crude operations, namely product demand and ship arrival uncertainties and develop some strategies to obtain robust schedules.

2 Problem Statement

Figure 1 shows the schematic configuration of crude oil scheduling in a typical marine access refinery. The configuration consists of crude unloading facilities such as an SBM or SPM station and/or one or more jetties, storage facilities such as storage tanks and/or charging tanks and processing facilities such as crude distillation units (CDUs). Crude oil arrives in either large multi-parcel tankers or small single-parcel vessels at the regulated date determined in planning stage. The whole operation involves unloading crudes into multiple storage tanks from VLCCs (very large crude carriers) or small vessels via SBM (single buoy mooring), SPM (single point mooring) or jetties at various times, mixing different types of

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crudes at storage tanks or charging tanks and charging CDUs from storage tanks or charging tanks at various rates over time.

Other detailed description about this problem can be referred to the paper of Reddy et al. (2004b) and Li et al. (2005).

3 Basic Deterministic Formulation

We employ the formulation proposed by Reddy et al. (2004b). In their formulation, three binary variables are defined as follows to model parcel to SBM/Jetties connection, tank to SBM/jetties connection and tank to CDU connection.

$$XP_{pt} = \begin{cases} 1 & \text{if a parcel } p \text{ is connected to the SBM/Jetty line for unloading during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$XT_{it} = \begin{cases} 1 & \text{if tank } i \text{ is connected to the SBM line during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{iut} = \begin{cases} 1 & \text{if tank } i \text{ feeds CDU } u \text{ during period } t \\ 0 & \text{otherwise} \end{cases}$$

The details about their formulation including the definition of periods, variables and parameters can be found in the paper of Reddy et al. (2004b).

4 Robustness Definition and Evaluation

Gan and Wirth (2004) defined schedule effectiveness, performance predictability and rescheduling stability. Based on effectiveness, predictability and stability, they proposed a cost function to determine the robustness of an initial schedule. According to their idea, we propose the following function to calculate the real objective function as follows.

$$\text{Real\_Profit} = \text{Profit} - COST[\Delta m]$$

Where, *Cost* [·] is the cost of the Δm, while Δm is the cost for schedule changes. In this function, we impose different penalties for different schedule changes. Thus, we include all into the objective function.

So far, some robustness criteria in the literature have been addressed such as absolute robustness, robust deviation and relative robustness and standard deviation. Because absolute robustness, robust deviation and relative robustness are defined based on worst-case scenario, we use standard deviation as the robustness metric defined as follows,

$$SD = \sqrt{\sum_{s} \frac{(\text{Real\_profit}_{s} - \text{Real\_profit}_{opt})^2}{(s-1)}}$$

Where, Real_profit_s corresponds to the real profit of scenario s

Real_profit_{opt} corresponds to the optimal real profit of scenario s.

SD is the standard deviation from the deterministic real profit.

To evaluate the robustness of schedules obtained, we propose the following procedure:

(1) Simulate a series of random disruptions
(2) Adjust each schedule to accommodate each disruption
(3) Calculate the penalty function for these adjustments
(4) Calculate the standard deviation to measure the robustness of schedules.

5 Methodology for infeasibility

When dealing with demand and ship arrival uncertainty, infeasibility may happen for some scenarios. To account for scenarios that may be infeasible, we proposed the following method:

(1) For demand uncertainty, we just add a positive slack variable u1u to demand constraints and penalize this slack variable in the objective function. Thus, we can obtain the correct objective.
(2) For ship arrival uncertainty, we assume we buy crudes from the spot market with a high price if there is not enough crudes to feed CDU. We also add a positive slack variable $IFBC_{ut}$ to the constraint accounting for the total throughput of one CDU during any period and also minimize this slack variable in the objective.

6 Demand Uncertainty

As mentioned above, demand uncertainty is one of the most important uncertainties. To improve the schedule robustness, we consider the demand uncertainty at the scheduling stage. We formulate the problem involving different demand scenarios within the expected range of demand variability as follows.

6.1 Scenario-Based Formulation

Most of objective in the literature is to minimize or maximize the expected values. Here, our objective is to maximize the total profit over all scenarios. We develop the following formulation in which the binary variables are treated as here and now.

Max $\text{Profit} = \sum_i \sum_u \sum_c \sum_t \sum_s FCTU_{iucts} CP_c - \sum_v DC_v - COC \sum_u \sum_t CO_{ut} - \sum_t \sum_s SC_{ts}$

$(i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC}$

Subject to

$$XP_{pt} = XP_{p(t-1)} + XF_{pt} - XL_{p(t-1)} \qquad (p, t) \in \boldsymbol{PT} \qquad (1)$$

$$XP_{pt} \geq XL_{pt} \qquad (p, t) \in \boldsymbol{PT} \qquad (2)$$

$$\sum_t XF_{pt} = \sum_t XL_{pt} = 1 \qquad (p, t) \in \boldsymbol{PT} \qquad (3,4)$$

$$TF_p = \sum_t (t-1) \cdot XF_{pt} \qquad (p, t) \in \boldsymbol{PT} \qquad (5)$$

$$TL_p = \sum_t t \cdot XL_{pt} \qquad (p, t) \in \boldsymbol{PT} \qquad (6)$$

$$\sum_p XP_{pt} \leq 2 \qquad (p, t) \in \boldsymbol{PT} \qquad (7)$$

$$TF_{(p+1)} \geq TL_p - 1 \qquad (8)$$

$$TF_p \geq ETA_p \qquad (9)$$

$$\sum_i XT_{it} \leq 2 \qquad (10)$$

$$X_{pit} \geq XP_{pt} + XT_{it} - 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (11)$$

$$\sum_i X_{pit} \leq 2 \cdot XP_{pt} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (12)$$

$$\sum_p X_{pit} \leq 2 \cdot XT_{it} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (13)$$

$$\sum_p \sum_i X_{pit} \leq 2 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (14)$$

$$\sum_u Y_{iut} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (15)$$

$$\sum_i Y_{iut} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (16)$$

$$2XT_{it} + Y_{iut} + Y_{iu(t+1)} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (17)$$

$$FPT^L_{pi} X_{pit} \le FPT_{pits} \le FPT^U_{pi} X_{pit} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (18)$$

$$\sum_p \sum_i \frac{FPT_{pits}}{FPT^U_{pi}} \le 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (19)$$

$$\sum_{i,t} FPT_{pits} = PS_p \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (20)$$

$$FTU_{iuts} = \sum_{(i,c)\in IC} FCTU_{iucts} \qquad (i, u) \in \boldsymbol{IU} \qquad (21)$$

$$Y_{iut} FTU^L_{iu} \le FTU_{iuts} \le Y_{iut} FTU^U_{iu} \qquad (i, u) \in \boldsymbol{IU} \qquad (22)$$

$$FU_{uts} = \sum_{(i,u)\in IU} FTU_{iuts} \qquad (23)$$

$$FU^L_{ut} \le FU_{uts} \le FU^U_{ut} \qquad (24)$$

$$FU_{uts} \cdot xc^L_{cu} \le \sum_i FCTU_{iucts} \le FU_{ut} \cdot xc^U_{cu} \qquad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \qquad (25)$$

$$xk^L_{ku} FU_{uts} \le \sum_i \sum_c FCTU_{iucts} xk_{kc} \le xk^U_{ku} FU_{uts} \qquad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \qquad (26)$$

$$YY_{iut} \ge Y_{iut} + Y_{iu(t+1)} - 1 \qquad (i, u) \in \boldsymbol{IU} \qquad (27a)$$

$$YY_{iut} \le Y_{iu(t+1)} \qquad (i, u) \in \boldsymbol{IU} \qquad (27b)$$

$$YY_{iut} \le Y_{iut} \qquad (i, u) \in \boldsymbol{IU} \qquad (27c)$$

$$CO_{ut} \ge Y_{iut} + Y_{iu(t+1)} - 2YY_{iut} \qquad (i, u) \in \boldsymbol{IU} \qquad (28)$$

$$M[2 - \sum_i YY_{iut}] + FTU_{iuts} \ge FTU_{iu(t+1)s} \qquad (i, u) \in \boldsymbol{IU} \qquad (29a)$$

$$M[2 - \sum_i YY_{iut}] + FTU_{iu(t+1)s} \ge FTU_{iuts} \qquad (i, u) \in \boldsymbol{IU} \qquad (29b)$$

$$VCT_{icts} = VCT_{ic(t-1)s} + \sum_{(p,c)\in PC, (p,t)\in PT} FPT_{pits} - \sum_{(i,u)\in IU} FCTU_{iucts} \qquad (i, c)\in \boldsymbol{IC} \qquad (30)$$

$$V_{its} = \sum_{(i,c)\in IC} VCT_{icts} \qquad (31)$$

$$V^L_i \le V_{its} \le V^U_i \qquad (32)$$

$$xt^L_{ic} V_{its} \le VCT_{icts} \le xt^U_{ic} V_{its} \qquad (33)$$

$$FCTU_{iucts} = f_{icts} \cdot FTU_{iuts} \qquad (34)$$

$$VCT_{icts} = f_{icts} \cdot V_{its} \qquad (35)$$

$$\sum_t FU_{uts} = D_{us} \qquad (36)$$

$$DC_v \ge (TL_p - ETA_p - ETD_v) SWC_v \qquad (p, v) \in \boldsymbol{PV} \qquad (37)$$

$$SC_{ts} \ge SSP(SS - \sum_t V_{its}) \qquad (38)$$

In order to extend the above formulation to jetties, they make some modifications to the above formulation. *J* is the number of jetties.

Drop Equations 8 and 10 and modify equations 7 and 14 as follows,

$$\sum_p XP_{pt} \le J \qquad (p, t) \in \boldsymbol{PT} \qquad (40)$$

$$\sum_p \sum_t X_{pit} \le 2J \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (41)$$

Replacing equation 19 by,

$$\sum_{i} \frac{FPT_{pits}}{FPT_{pi}^{U}} \leq 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (42)$$

$$\sum_{p} \frac{FPT_{pits}}{FPT_{pi}^{U}} \leq 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (43)$$

For both SBM pipeline and Jetties, they use the following constraints, instead of equations. 19, 42 and 43,

$$\sum_{i} \frac{FPT_{pits}}{FPT_{pi}^{U}} \leq 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (44)$$

$$\sum_{p \in SP} \frac{FPT_{pits}}{FPT_{pi}^{U}} + \sum_{p \in JP} \frac{FPT_{pits}}{FPT_{pi}^{U}} \leq 1 \qquad (45)$$

From the above scenario-based formulation, the size of the problem greatly increases with the number of scenarios. To reduce the size of the problem and make the problem easy to solve, we start from the following example.

6.2 Example 1

A refinery has one SBM pipeline, four storage tanks (T1, T2, T3, and T4), two CDUs (CDU 1 and CDU 2), and processes four crudes (C1 to C4). The scheduling horizon is 9 days. The nominal demands of CDU 1 and CDU 2 are 400 kbbl and 400 kbbl, respectively. These two demands can vary uniformly within [250kbbl, 550kbbl]. Five scenarios are considered involving the four vertexes and the nominal demand. We solve the model presented in the previous section over the five scenarios. Then, we then evaluate the robustness of the schedule obtained following the procedure proposed in the previous section using a uniform grid of 49 points distributed within the range of demand uncertainty ([250, 550] for both CDUs). We also use the proposed procedure with these 49 points to evaluate the robustness of the initial schedule obtained by solving the basic deterministic model with nominal demand. We compare the robustness of the schedule obtained by scenario-based model named the new schedule with the initial schedule. The result is given in Table 1. The result shows that the schedule by scenario-based model is more robust than the initial schedule. Moreover, that schedule is feasible over the entire demand uncertainty range, while the initial schedule is infeasible over some part of the uncertainty range.

6.3 Minimum Number of Scenarios

As mentioned before, the size of the problem greatly increasing with the number of scenarios makes difficult to solve. Therefore, we should find the minimum numbers of scenarios to be included in the formulation in order to improve the schedule performance in the face of demand uncertainty. We solve Example 1 with different numbers of scenarios and evaluate these schedules obtained with our proposed evaluation procedure. The result is illustrated in Table 2. From Table 2, we can see that there are minor differences among the results with different numbers of scenarios. Thus, two scenarios corresponding to the maximum and the nominal demand values are enough to increase schedule robustness. This schedule obtained with those two scenarios is more robust than the initial schedule.

7 Ship Arrival Uncertainty

Ship arrival delay often happens in the real crude oil scheduling operation. In the following, we develop model and propose methodology to handle this uncertainty.

7.1 Scenario-Based Formulation

Similar to the demand uncertainty, we also present the scenario-based formulation in which we treat all variables as wait and see compared to the demand uncertainty. To model the schedule changes, we introduce the following constraints:

$$PPXP_{pts} \geq XP_{ptn} - XP_{pts} \qquad (p, t) \in \boldsymbol{PT} \qquad (46a)$$

$$PPXP_{pts} \geq XP_{pts} - XP_{ptn} \qquad (p, t) \in \boldsymbol{PT} \qquad (46b)$$

$$PPXT_{its} \geq XT_{its} - XT_{itn} \qquad (46c)$$

$$PPXT_{its} \geq XT_{itn} - XT_{its} \qquad (46d)$$

$$PPY_{iuts} \geq Y_{iuts} - Y_{iutn} \qquad (i, u) \in \boldsymbol{IU} \qquad (46e)$$

$$PPY_{iuts} \geq Y_{iutn} - Y_{iuts} \qquad (i, u) \in \boldsymbol{IU} \qquad (46f)$$

Where, index *n* means the nominal scenario.

The above six constraints model the schedule changes of other scenarios to the nominal scenario. The objective of this model is to maximize the expected profit and minimize the schedule changes simultaneously.

7.2 Approximation Decomposition Strategy

As discussed above, the model size increases with the number of scenarios and makes the problem hard to solve. Thus, we develop algorithms to solve this problem.

As mentioned before, we have developed robust algorithms for obtaining optimal schedules for operations without any uncertainty in our previous work (Li et al. 2005). In other words, we can solve the problem for each scenario. Based on this, we proposed an approximation decomposition strategy stated as follows. First, we solve the problem for each scenario except the nominal scenario. Then we solve the nominal scenario with maximizing the expected profit and minimizing schedule changes simultaneously. The objective is stated as follows,

$$\begin{aligned}\text{Profit} = {} & PROP_n\Big(\sum_i\sum_u\sum_c\sum_t FCTU_{iuct}CP_c - \sum_v DC_v - COC\sum_u\sum_t CO_{ut} - \sum_t SC_t\Big) \\ & \sum_i\sum_u\sum_c\sum_t\sum_{s\neq n} FCTU_{iucts}CP_c - \sum_v\sum_{s\neq n} DC_v - COC\sum_u\sum_t\sum_{s\neq n} CO_{uts} - \sum_t\sum_{s\neq n} SC_{ts} \\ & - \text{penalty1}\sum_p\sum_t\sum_s RPROP_s PPXP_{pts} - \text{penalty2}\sum_i\sum_t\sum_s RPROP_s XT_{pts} \\ & - \text{penalty3}\sum_i\sum_u\sum_t\sum_s RPROP_s Y_{iuts}\end{aligned}$$

Where, $PROP_s$ is the probability of scenario *s*. $RPROP_s$ is the relative probability of scenario *s* compared to nominal scenario.

Example 3

In this example, we consider one SBM, six tanks and three CDUs with 15-periods scheduling horizon. Only one VLCC is involved. The nominal arrival time for the VLCC is at period 5. We assume that the VLCC can arrive between [0 9]. We consider 5 scenarios including 1, 3, 5, 7, 9 with the probability 0.01, 0.12, 0.52, 0.35 and 0.009 respectively. We solve this problem with our approximation decomposition strategy and evaluate the robustness of the schedule obtained with our proposed procedure. To compare the performance of this strategy, we also evaluate the robustness of the initial schedule. Then we compare the result of the schedule got with the approximation decomposition strategy with the initial schedule, shown in Table 3. Table 3 shows the schedule obtained with the approximation decomposition strategy is more robust than the initial schedule although the profit of the obtained schedule is less than the initial schedule.

8 Conclusion

In this paper, we addressed two important uncertainties in crude operations, namely those related to product demands and ship arrivals. First, we defined schedule robustness and a penalty function for schedule effectiveness, predictability, and stability. We also proposed a simulation procedure to evaluate the robustness of a schedule. Then, we developed scenario-based models for addressing demand and ship arrival uncertainties separately. For demand uncertainty, we show that the schedule thus obtained was more robust and more feasible than the "average-demand" schedule over the entire expected range of uncertainty. For ship arrival uncertainty, we proposed an approximate decomposition strategy in which we first solved each scenario independently and then solved the nominal scenario by maximizing the expected profit and penalizing schedule changes to other scenarios. The result shows that the resulting schedules were superior to the original schedules. In the future, we will consider these two uncertainties simultaneously and incorporate more uncertainties such as tank unavailable and pipeline malfunction.

References

Adhitya, A., Srinivasan, R., Karimi, I. A., A Heuristic Reactive Scheduling Strategy for Recovering from Refinery Supply Chain Disruptions, AIChE Annual Meeting, Austin, TX, Nov 7-12, 2004.

J. Li, W. Li, I. A. Karimi and R. Srinivasan, Robust and Efficient Algorithm for Optimizing Crude Oil Operations, Presented in AIChE annual meeting, Cincinnati, OH, Oct 30- Nov 4, 2005.

Li, W. K., Hui, C. W., Li, P., Li, A. X., Refinery Planning under Uncertainty, Industrial and Engineering Chemistry Research, 2004, 43, 6742-6755.

Neiro, S., Pinto, J. M., Multiperiod Optimization for Production Planning of Petroleum Refineries, Chemical Engineering Communications, 2005, 192 (1): 62-88.

Reddy, P. C. P., Karimi, I. A., Srinivasan, R., A Novel Solution Approach for Optimizing Scheduling of Crude Oil Operations, AIChE Journal*,* 2004, 50(6), 1177-1197.

Table 1 Comparison of the results for the new schedule obtained and initial schedule

Items	The initial schedule	The new schedule
Nominal scenario (kbbl)	400, 400	400, 400
Objective to meet nominal demand ($)	3774.037	3774.037
SD ($)	336.7853	16.19

Table 2 Results of different numbers of scenarios

Items	The initial schedule	The new schedule			
		2 scenarios	3 scenarios	4 scenarios	5 scenarios
Nominal scenario (kbbl)	400, 400	400, 400	400, 400	400, 400	400, 400
Objective to meet nominal demand ($)	3774.037	3774.037	3774.037	3774.037	3774.037
SD ($)	336.7853	16.19	16.19	16.19	16.19
Items	The initial schedule	The new schedule			
		9 scenarios	25 scenarios	40 scenarios	49 scenarios
Nominal scenario (kbbl)	400, 400	400, 400	400, 400	400, 400	400, 400
Objective to meet nominal demand ($)	3774.037	3774.037	3774.037	3774.037	3774.037
SD ($)	336.7853	16.19	16.19	16.19	16.19

Table 3 Comparison of the results for the new schedule obtained and initial schedule

Items	The initial schedule	The new schedule
Nominal scenario (kbbl)	5	5
Objective to meet nominal ship arrival ($)	27511.085	26931.077
SD ($)	3378.679209	1161.219323

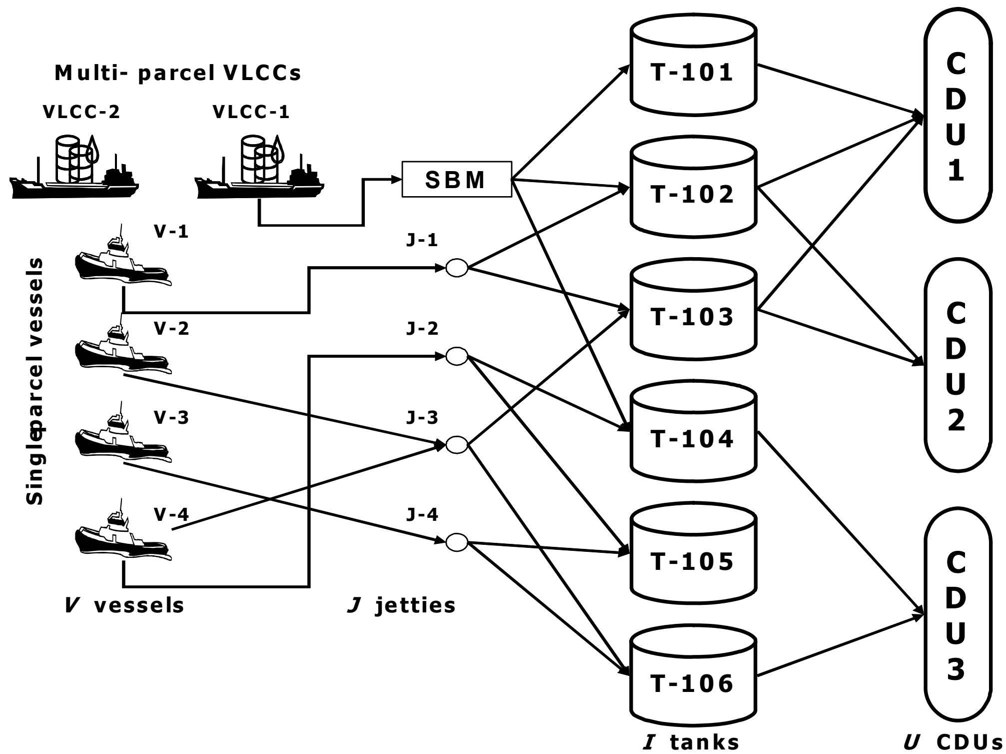

Figure 1 Schematic of oil unloading and processing