Optimal Control Theory for Sustainable Environmental Management

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Abstract

The available resources being rapidly diminishing, and the harmful effects of various pollutants being realized, research focus in environmental management has shifted towards sustainability. The main thrust of sustainability is the consideration of long terms effects, benefits and drawbacks in all decisions relevant to the society as a whole. The goal of a sustainable management strategy is to promote the structure and operation of the human component of a system (society, economy, technology, etc.) in such a manner as to reinforce the persistence of the structures and operation of the natural component (i.e., the ecosystem). Since this is a highly multi-disciplinary task, involving interactions of systems on multiple temporal and spatial scales, a systematic approach, based on sound mathematical techniques, is essential. Two important aspects of this approach are: formulation of sustainability based objectives and development of the management strategies. Fisher information based sustainability hypotheses can be used to formulate mathematical objectives relevant to the disparate systems. Once the objectives are developed, systems theory based methods and tools provide the means to derive the appropriate management strategies. This work proposes to use optimal control theory to derive time dependent management strategies. This paper presents a proof of concept for this approach using a model representing an ecosystem that includes humans, a very rudimentary industrial process, and a very simple agricultural system. To formulate the control problem, it is important to identify the right control variables in the model, which is achieved through partial correlation coefficient analysis. The results help in identifying the effective management options. The results also emphasize that management using multiple parameters of different nature can be distinctly effective.

1 Introduction

The available resources being rapidly diminishing, and the harmful effects of various pollutants being realized, research priorities in the field of ecology and environment have evolved. The focus has shifted from achieving short term goals to envisioning long term sustenance, also known as sustainability. In its simplest terms, it calls for the consideration of long terms effects, benefits and drawbacks in all decisions relevant to the society as a whole. As a consequence, management strategies targeting sustainability are sought.

The goal of a sustainable management strategy is to promote the structure and operation of the human component of a system (society, economy, technology, etc.) in such a manner as to reinforce the persistence of the structures and operation of the natural component (i.e., the ecosystem) [1]. This will be a highly multi-disciplinary approach, involving interactions of systems on multiple temporal and spatial scales. A systematic approach, based on sound mathematical techniques, is essential to communicate between such diverse systems. Two important aspects of this approach are: formulation of sustainability based objectives and development of the management strategies. Fisher information based sustainability hypotheses, based on information theory, allow the formulation of mathematical objectives relevant to disparate systems [2]. Once the objectives are developed, systems theory based methods and tools provide the means to derive the appropriate management strategies. This work proposes to use optimal control theory to derive time dependent management strategies. The ideas are presented through a case study application on a twelve compartment food web model. The model system represents an ecosystem that includes humans, a very rudimentary industrial process, and a very simple agricultural system. Since the model is quite complicated, identification of the appropriate manipulated parameters (to achieve the objective) is based on partial correlation coefficient analysis.

The article is arranged as follows. The next section correlates sustainability with systems theory approach. Section 3 presents control theory basics while section 4 discusses Fisher information based sustainability index. Section 5 discusses the model in detail, starting with the basic model description, followed by the correlation coefficient analysis. Sections 6 and section 7 report the control problem formulation and simulation cases for the model. 8 presents the important results for the model. The article ends with summary of results in section 9 and concluding remarks in section 10.

2 Sustainability and systems theory approach

Sustainable development is defined as “the development that meets the needs of the present without compromising the ability of the future generations to meet their own needs” [3]. Sustainability must embody, in some form, elements of physics, engineering, ecology, law, economics, sociology, and politics and cannot be investigated successfully within a confines of a single discipline. The concept goes beyond industrial ecology and brings in the complex interactions between a single industrial production plant with the larger scales of an integrated industrial park, community, and natural systems. It also brings in the time dependent nature of the ecosystem. Consequently, the scope of available management options and decision making widens, and decisions regarding regulations, human interactions with ecosystem come in picture. Further, green engineering concepts change the single goal of engineering design from profitability to include a number of different and often conflicting objectives that can define sustainability in the end. Sustainability therefore calls for a synergic approach to technological, social and regulatory decision making.

However, this integration poses challenging problem of discrete and continuous decisions, and nonlinear models. It involves dealing with various time scales and time dependent uncertainties. Time independent as well as time dependent decision need to be taken simultaneously. As the envelope is extended, the uncertainties in the model increase. To achieve these goals, appropriate modelling of the objectives, risks and uncertainties is necessary. Decisions based on logic and heuristics might be sub-optimal due to the complex interactions between various sub-systems, and hence a systematic approach is needed to resolve these issues. Systems theory approach offers a suitable methodology to tackle such complex issues.

Systems theory is defined as the transdisciplinary study of the abstract organization of phenomena, independent of their substance, type, or spatial or temporal scale of existence. It investigates both the principles common to all complex entities, and the (usually mathematical) models which can be used to describe them. Systems analysis applies systems principles to aid a decision-maker with problems of identifying, reconstructing, optimizing, and controlling a system (usually a socio-technical organization), while taking into account multiple objectives, constraints and resources. It aims to specify possible courses of action, together with their risks, costs and benefits. Modeling forms the basis of this approach. Efficient models translate a given system into the mathematical domain and allows effective communication with other systems. Model reliability is therefore an important issue in this approach. The model development is followed by systems analysis to achieve the desired objective such as parameter optimization, benefit maximization as well as time dependent control. Here, mathematical theories such as optimization, optimal control and real options theory are employed to come up with efficient management strategies.

The discussion of systems theory and systems analysis clearly indicate that there is a close correlation between systems approach and sustainability. Like sustainability, systems theory aims to connect various disciplines through a common base and account for the interdependencies between these disciplines. Hence it is prudent to apply this theory towards the proposed sustainability analysis.

3 Control theory approach

The tools based on systems approach are again founded on the theory of mathematics. Two widely established techniques in the field of engineering are optimization and optimal control theory. Optimization leads to static decisions that do not change with time (e.g. design parameters). However, natural systems, such as those often encountered in sustainability related studies, are dynamic. Hence, static decision might be sub-optimal and an effective approach is to use time dependent management decisions. For example, limiting nutrient or pollutant input into a lake at a constant level without giving proper credence to the environmental cycles and aquatic life cycles in the lake can affect species diversity and life expectancy. Such effects are not evident immediately and manifest themselves only over a long time period. Recent examples of such applications include Shastri and Diwekar [4, 5], Ludwig et al. [6], Chukwu [7] and Kolosov [8] among others. This work proposes to use control theory to achieve time dependent liming of a lake.

In general, control refers to a closed loop system, where the desired operating point is compared with an actual operating point and a knowledge of difference is fed back to the system to take appropriate corrective actions. Different types of control techniques applied to a control problem (e.g. optimal control, model predictive control, linearized control etc.). This work proposes to use optimal control theory. The fundamentals of optimal control theory have been well established. The main advantages of using optimal control theory are: it does not make any assumption about the structure of the controller (control law), and it theoretically works for all types of systems, including nonlinear. Due to these advantages, most of the control theory applications in natural systems use optimal control theory. The theoretical details are skipped here for brevity and interested readers are referred to texts such as Kirk [9] and Lewis [10].

Application of optimal control theory necessitates the knowledge of the objective and the control variable. The selection of a control variable depends on the particular systems being analyzed. Formulation of sustainability based objective is though a difficult task. This work proposes to use Fisher information based sustainability hypothesis to formulate the appropriate objectives. This aspect is discusses in the next section.

4 Sustainability and Fisher information

Cabezas and Fath [2] have proposed to use information theory in ecology to derive a measure for the sustainability of a system, the hypothesis being based on the argument that information is a fundamental quantity of any system, irrespective of the discipline [11]. Various previous applications of information theory in ecology include using Shannon information [12] as an index of biodiversity; using entropy of information to investigate evolutionary processes [13]; measuring the distance of a system from the thermodynamic equilibrium (based on exergy) [14] and developing the concept of Ascendency [15]. Cabezas and Fath though use Fisher information as the quantity for their hypothesis.

Fisher information (FI), introduced by Ronald Fisher [16], is a statistical measure of indeterminacy. One of its interpretations, relevant for this work, is as a measure of the state of order or organization of a system or phenomenon [11]. The Fisher information, I, for one variable is given as [2]

$$I = \int \frac{1}{p(x)} \left(\frac{dp(x)}{dx} \right)^2 dx \tag{1}$$

where p is the probability density function (pdf) of variable x. This definition can be extended to a system of n variables. When the n variables constitute the state variable vector of a system, it gives the Fisher information of that system. Fisher information, being a local property, dependent on the derivative of the state vector density function, is sensitive to perturbations that affect the density function and therefore can be used as an indicator of the self-organization of the system. Here self-organization refers to the distribution of states in which the system exists. A detailed discussion of the various aspects of Fisher information can be found in [2] and [17].

The objective for the control problem is based on the sustainability hypothesis. The hypothesis therefore states that: the time-averaged Fisher information of a system in a persistent regime does not change with time. Any change in the regime will manifest itself through a corresponding change in Fisher information value [2]. A additional corollary in Cabezas and Fath [2] states that if the Fisher information of a system is increasing with time, then the system is maintaining a state of self-organization. This leads to following two possible objectives for the control problem:

- Minimization of the Fisher information variance over time
- Maximization of the time averaged Fisher information:

Shastri and Diwekar [4, 5] have used these objectives on a three species predator prey model to compare different control options. They conclude that although maximization of Fisher information leads to aggressive changes, minimization of Fisher information variance is usually the better objective since it ensures the stability of the system.

The preceding sections discusses the aspect of sustainable management, highlighted the importance of using systems theory based approach, and control theory as the decision making tool and Fisher information as the sustainability quantification tool. This form the basic approach that is being proposed through this work. The ideas are now presented through a case study example of a twelve compartment food web model. Various issues specific to the model are also discussed in the subsequent article. The next section presents the food web model.

5 Twelve compartment food web model

5.1 Basic model

The food web model considered in this work is presented in Cabezas et al. [1]. The model represents an ecosystem that includes humans, a very rudimentary industrial process, and a very simple agricultural system. The model, shown in Fig. 1, tracks the flow of mass resources (biomass, nutrients, water, etc.) within a closed system (i.e. the cumulative sum of masses of all the system compartments is constant). In Fig. 1, the solid arrows represent transfers of mass due solely to biological or geological drivers, with no interference from humans. The dashed arrows represent a transfer of mass from the nutrient pool to the inaccessible nutrient pool that occurs as a byproduct of human activities. This is the infrastructure and organizational overhead necessary for extraction of natural resources as well as technological enhancement of ecosystem services. The dotted arrows represent mass transfers that the human population can increase or decrease according its own criteria. The model is divided into two characteristic branches: domesticated (representing agricultural and livestock activities) comprising compartments P1, P2, and H1 on the left, and non-domesticated (representing species hunted, gathered, and species not consumed by humans) consisting of compartments P3, P4, H2, H3, C1, and C2 on the right. The model has four trophic levels (plants, herbivores, carnivores, and humans as a top omnivore) and two resource pools (RP and IRP), one of which is an "inaccessible" pool (IRP) with a slow "background" rate of mass moving out to the body of accessible resources. Humans (HH) rely on the non-domesticated branch for both resources and for the recycling of mass from the inaccessible resource pool back into the rest of the system. The industrial process is meant to represent at a very elementary level a generic human industrial activity that offers a benefit to the human population. The industrial process simply takes mass from three compartments (P1, RP, and H3) in different proportions and combines it to form a product. Consumption of the product reduces the mortality rate of the human compartment (HH). The mathematical equations governing the model are reported in Cabezas et al. [1].

The goal is to devise time dependent management strategies for the sustainability of this food web model. To formulate any control problem, it is important to first identify the model parameters that can be used as control variables. Equally important is the identification of model variables that are indicators of the model sustainability. This is achieved in this work through the partial correlation coefficient analysis which is explained in the next section.

5.2 Model PRCC analysis

The food web model has multiple inputs and outputs complicating the identification of the appropriate control variable. Those model parameters that have a major impact on the model output should be selected as control variables. One way of identifying this is to do parametric sensitivity analysis. However, this typically involves perturbing parameters around some specific points. Therefore, this sensitivity is restricted to a local region of operation. This work uses stochastic analysis [18] based on sampling to obtain partial correlation and partial rank correlation coefficients (PCC and PRCC), thus providing overall sensitivity.

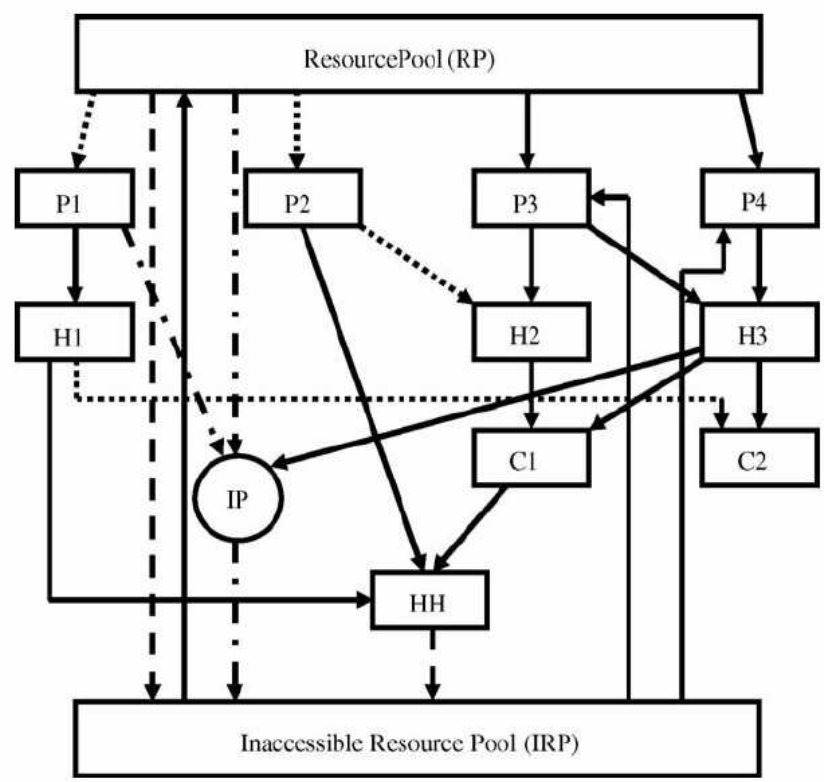

Figure 1: Food web model

PCC corresponds to Partial Correlation Coefficient and provides a major or unique or unshared contribution of each variable and explains the unique relationship between two variables that cannot be explained in terms of the relations of these variables with any other variable. PCCs are useful when the model is relatively linear. However, the food web model also has nonlinearities. For nonlinear models, Partial Rank Correlation Coefficients (PRCC) is supposed to provide equivalent information as PCCs. Therefore, in this work, we are calculating both PCCs and PRCCs. This results in identifying the parameters that have the most impact overall and on individual compartments. It also helps in identifying the model compartments that are more sensitive to these parameters. If the absolute value of the correlation coefficient between two variables is closer to one, then the given input and output variables are strongly correlated. If the value of the coefficient is positive, then change in the input variable has a positive effect on the output variable. A negative value on the contrary reflects an inverse response between the input and output variables.

The computation of the PRCCs is carried out through a sampling analysis. The model input parameters of interest are sampled in the neighborhood of the base case parameter value, and the model is simulated of each sample set. The simulation results at a particular time step of the simulation horizon along with the parameter samples are used to compute the coefficients. For the analysis to be exhaustive, it is important to cover the complete range of the possible parameter values. This calls for an efficient sampling technique. This work uses the Hammersley Sequence sampling (HSS) technique which has been shown to be more efficient than the other techniques [19, 20].

The model parameters analyzed here are:

- $g(1)$: Coefficient of mass transfer from RP to P1
- $g(2)$: Coefficient of mass transfer from RP to P2

- $g(3)$: Coefficient of mass transfer from RP to P3
- $g(4)$: Coefficient of mass transfer from RP to P4
- $gH1C2$: Coefficient of mass transfer from H1 to C2 (influenced by humans)
- W: Constant for the waste term associated with human consumption
- α: Constant reflecting the effectiveness or efficiency of the industrial process in reducing the human mortality rate
- η: Organizational or infrastructure costs associated with the industrial process, rendering inaccessible a portion of the resource pool in proportion to the flow through the industrial process.

The base values of these parameters are presented in [1] and samples are taken considering a $\pm 10\%$ variation in the model parameter values. The sample size used for this work is 500, and the simulation time horizon is 2500 time units.

The values of the partial rank correlation coefficients are reported in Table 1. The coefficients having absolute values higher than 0.5 are represented in bold letters. To rank the considered model parameters according to their importance in model dynamics, the absolute values of the partial rank correlation coefficients for each parameter are summed and compared. The parameters in the decreasing order of importance are reported in Table 2. Thus, $g(2)$ representing the coefficient of mass transfer from RP to $P2$ is the most important parameter, while $gH1C2$ representing the mass transfer coefficient from $H1$ to $C2$ is the least important. The PRCC analysis results are also used to rank the different model compartments according to their sensitivities, the results for which are also reported in Table 2.

6 Optimal control problem formulation

The theory presents three possible methodologies to derive the optimal control law: dynamic programming (Hamilton-Jacobi-Bellman equation), calculus of variation (Euler-Lagrange equation) and Pontryagin's maximum principle [9, 21]. In this work, Pontryagin's maximum principle has been used. This technique results is a set of algebraic and ordinary differential equations which needs to be solved as a boundary value problem. A detail explanation of the methodology is skipped here. Given here is the summary of the final equations.

Consider a system represented by the following set of differential equations, called the state equations in the language of control theory:

$$\dot{x} = f(x, u, t) \tag{2}$$

where, x is the state variable vector ($x(t) \in R^n$). The initial condition of the state variable vector is $x(t_0) = x_0$, while the final condition is $x(T)$. u is the control variable vector ($u(t) \in R^m$). In optimal control, there is a time dependent performance index. It is represented here as:

$$J(t_0) = \int_{t_0}^{T} F(x(t), u(t), t)dt \tag{3}$$

where, F is the function to be optimized over the time interval $[t_0, T]$.

Using the Pontryagin's maximum principle, the optimal control law is given by the solution of

	P1	P2	P3	P4	H1	H2	H3	C1	C2	HH	RP	IRP
$g(1)$	**0.833**	0.141	**-0.634**	**-0.743**	**0.916**	**-0.779**	**-0.786**	**-0.740**	0.218	**-0.782**	**0.764**	**-0.780**
$g(2)$	**-0.695**	**0.707**	**0.653**	**-0.638**	**-0.894**	**0.850**	**0.817**	**-0.655**	-0.106	**0.815**	**-0.999**	**0.874**
$g(3)$	-0.014	0.001	0.053	0.035	0.011	0.153	0.004	0.035	0.004	0.003	-0.028	0.002
$g(4)$	0.059	-0.133	-0.068	0.189	0.053	-0.314	0.027	0.193	0.038	0.029	-0.003	-0.311
$gH1C2$	-0.040	-0.007	0.017	0.015	-0.006	0.012	0.014	0.013	0.040	0.013	-0.048	0.012
W	-0.475	**-0.721**	0.004	**0.875**	-0.206	-0.137	-0.153	**0.878**	-0.121	-0.160	0.006	-0.142
α	**0.948**	**-0.699**	-0.171	**0.684**	**0.804**	**0.706**	**0.734**	**0.690**	**0.606**	**0.745**	-0.491	**0.708**
η	-0.007	-0.003	-0.056	0.025	-0.031	-0.012	-0.017	0.025	-0.023	-0.017	0.024	-0.017

Table 1: Food web model: Partial rank correlation coefficients

the following set of equations [9]:

$$\text{State Equations}$$
$$\dot{x} = \frac{\partial H}{\partial \lambda} = f \qquad t \geq t_0 \tag{4}$$
$$\text{Adjoint Equations}$$
$$-\dot{\lambda} = \frac{\partial H}{\partial x} = \frac{\partial f'}{\partial x}\lambda + \frac{\partial F}{\partial x} \qquad t \leq T \tag{5}$$
$$\text{Optimality Conditions}$$
$$0 = \frac{\partial H}{\partial u} = \frac{\partial F}{\partial u} + \frac{\partial f'}{\partial u}\lambda \tag{6}$$

where, H is the Hamiltonian defined as:

$$H(x,u,t) = F(x,u,t) + \lambda' \, f(x,u,t) \tag{7}$$

and λ is a set of the costate or adjoint variables (R^n) (λ' represents the matrix transpose).

This is a set of $2n$ ordinary differential equations (state and adjoint equations) and m algebraic equations (optimality or stationarity condition), to be solved as a boundary value problem. The state variables are known at the initial time t_0 (x_0), while the adjoint variables are known at the final time T ($\lambda(T)$). The boundary values of the adjoint variables depend on the problem specification [9]. The control trajectory obtained using the optimality condition is optimal for the considered objective function and the starting conditions.

FI bases sustainability hypothesis is used to formulation the time dependent objectives. This work uses the objective of FI variance minimization over time. Accordingly, if I_t represents the time averaged Fisher information for a system with n states, then the objective is mathematically represented as:

$$J = \text{Min} \int_0^T (I_t - I_{constant})^2 \, dt \tag{8}$$

Here, T is the total time horizon under consideration. I_t is time averaged FI for one system cycle, and $I_{constant}$ is the constant around which the Fisher information variation is to be minimized.

The exact objective function formulation depends on the model variables that are being used

	Independent model parameters	Model compartments
1	$g(2)$	$C1$
2	$g(1)$	$P4$
3	α	$P1$
4	W	$H2$
5	$g(4)$	$H1$
6	$g(3)$	IRP
7	η	HH
8	$gH1C2$	$H3$
9		$P2$
10		RP
11		$P3$
12		$C2$

Table 2: Food web model: Ranking of the model parameters and output compartments

as the sustainability indicators. This is decided using the results from the PRCC analysis and also based on the dynamics of the uncontrolled system.

Owing to the mathematical complexity of the resulting control problem, it is solved using the numerical technique of steepest ascent of Hamiltonian.

7 Simulation cases

The aim is to explore and rank different control possibilities for the food web model listed in section 5.2. For this, different cases of the food web model showing undesirable dynamics are simulated. This simulates the situations when the system needs external intervention to avoid imbalance. The objective is to recover the system from the disturbance in a sustainable manner, i.e. achieving dynamic stability. The analysis assesses the ability of different control options to perform this task. The cases simulated for this study are based on the work presented in [1]. These cases are:

- Minor reduction in the effectiveness of the industrial process in reducing the human mortality (α)
- Doubling the flow through the industrial process (IPF)
- Changing the input composition for the industrial process
- Increasing the constant for waste term associated with human consumption (W)
- Major reduction in the effectiveness of the industrial process in reducing the human mortality (α), simulating a major disaster or regime shift

In each case, the total simulation time horizon is 5000 units, where the cycle time for the model (based on the cyclic input) is 500 units. The parameters to be disturbed are linearly ramped to the new values in 500 time steps between 1000 and 1500 time units. Control is exercised only after the disturbance is introduced in the model, i.e. only after 1000 time unit. The control variables stay at the base values till 1000 time unit. The objective function requires a constant FI value around which the variance is to be minimized ($I_{constant}$). This value is taken as the average FI for the first two undisturbed simulation cycles (till time step 1000). For each of these cases, the control problem is formulated and solved for all possible control variables listed before. The important results are presented in the following sections.

8 Results and discussion

In the following sections, representative results for each case are presented and discussed. The important conclusions from all the cases are summarized in the last section.

8.1 Case A: Minor reduction in α

In this case, the constant reflecting the effectiveness of the industrial process in reducing the human mortality rate, α, is reduced from its base case value by 15%. This results in the uncontrolled case with: significant instability in compartments P1, H1 and C2. The mass in these compartment is continuously decreasing. The effect on the other compartments is not severe. Since α is changed to simulate the case, it is not used as the control variable.

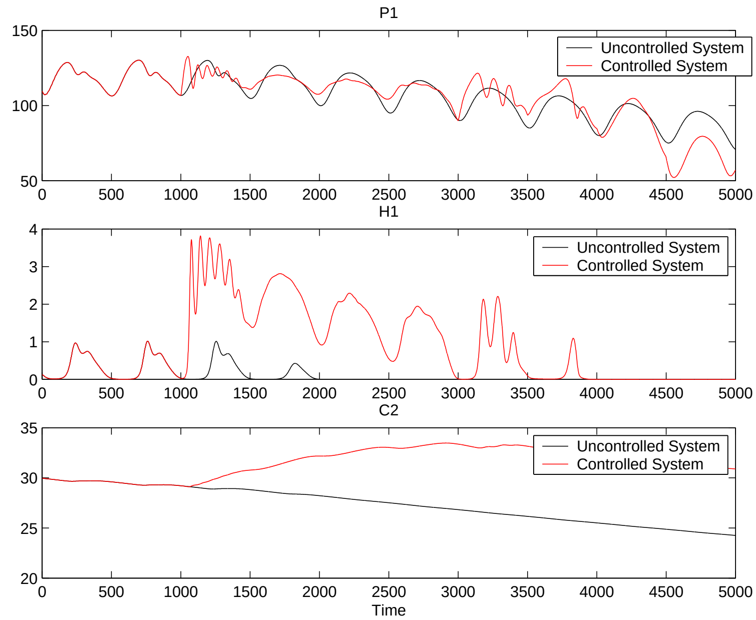

Figure 2: Case A: Controlled system using $g(1)$ as control variable

The uncontrolled dynamics for P1, H1 and C2, along with the controlled dynamics using $g(1)$ and $g(2)$ as control variables are shown in Figures 2 and 3, respectively. The results show that $g(1)$ has a stronger effect than $g(2)$ on the model dynamics. Although extinction in H1 can not be eliminated, it is considerably delayed using $g(1)$ as control variable. $g(2)$ has a significant impact only on compartment C2. The results also show that changes in $g(2)$ lead to disturbances in other compartments of the model. The use of W as control variable had insignificant impact on all the compartments, and hence the results are not reported here.

8.2 Case B: Doubling the industrial process flow IPF

In this case, the uncontrolled case is simulated by doubling the parameter IPF in the model. It represents the flow through the industrial process and hence a change in this parameter represents a change in the size of the industrial sector. This results in an unstable system with: significant instability in compartments P3, H2 and H3; extinction in compartment H1; onset of extinction in compartment C2 and onset of instability in compartments P2, P4, HH and IRP.

The uncontrolled profile for some of the compartments and the results using $g(1)$ as control variable are shown in Figures 4 and 5. The results using α as the control variable are shown in figures 6 and 7. A comparison between the two results suggests that $g(1)$ is effective in avoiding instability in the model, while α is more effective in avoiding compartment extinction. The results using $g(2)$ as control variable are qualitatively similar, however inferior, to those using $g(1)$ as control variable. Using W as control variable does not result in significant change in the dynamics of most compartments.

8.3 Case C: Change in IS input composition

The industrial process (IS) takes mass from three compartments (P1, RP, and H3) in different proportions and combines it to form a product which is used by the humans. The base case input composition to IS is: 2% P1, 7% RP and 91% H3. To simulate this case, the input configuration

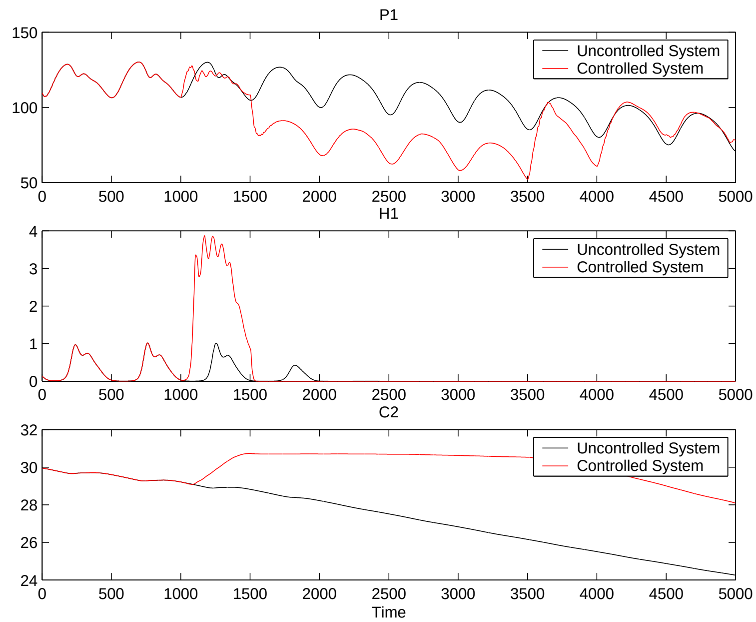

Figure 3: Case A: Controlled system using $g(2)$ as control variable

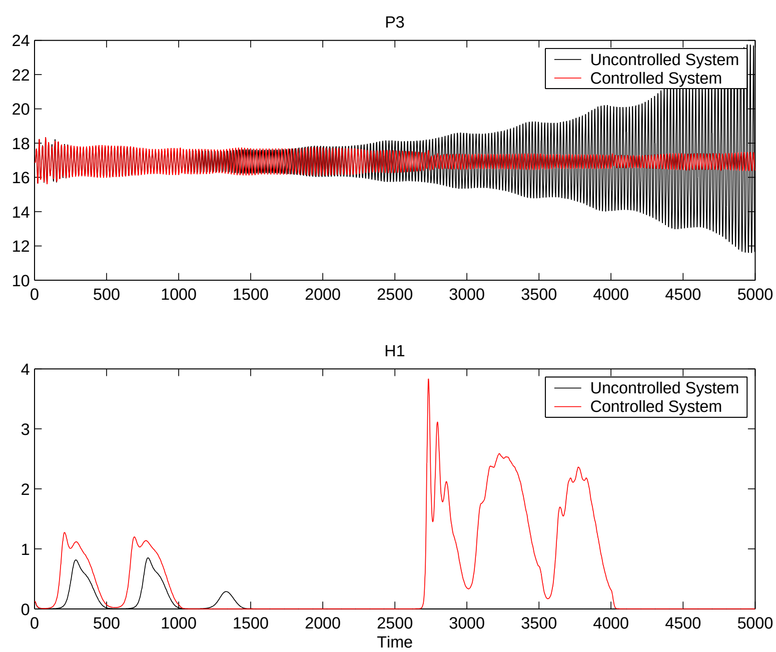

Figure 4: Case B: Controlled system using $g(1)$ as control variable

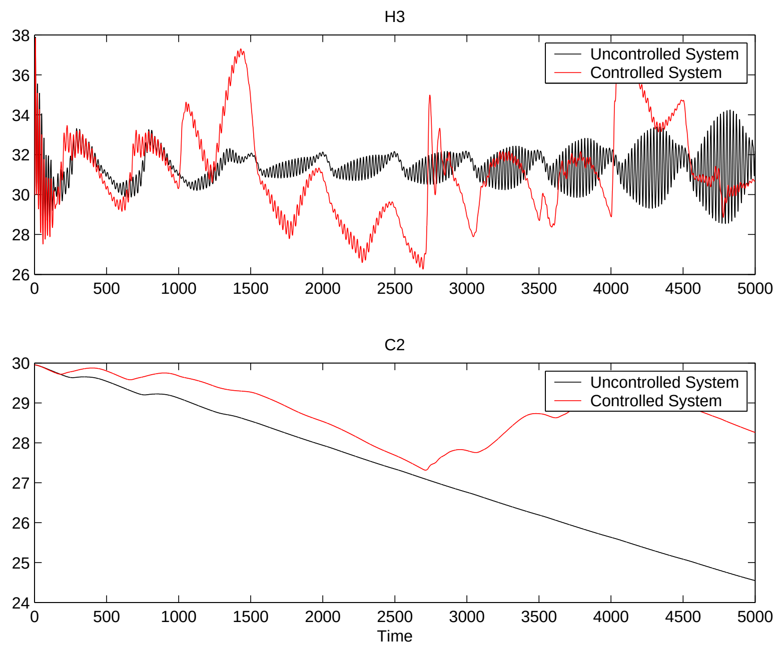

Figure 5: Case B: Controlled system using $g(1)$ as control variable

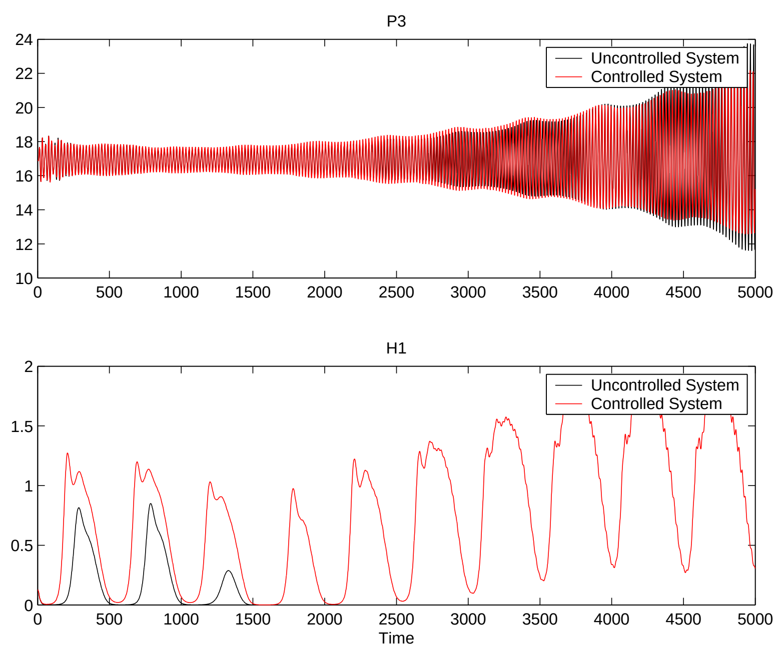

Figure 6: Case B: Controlled system using α as control variable

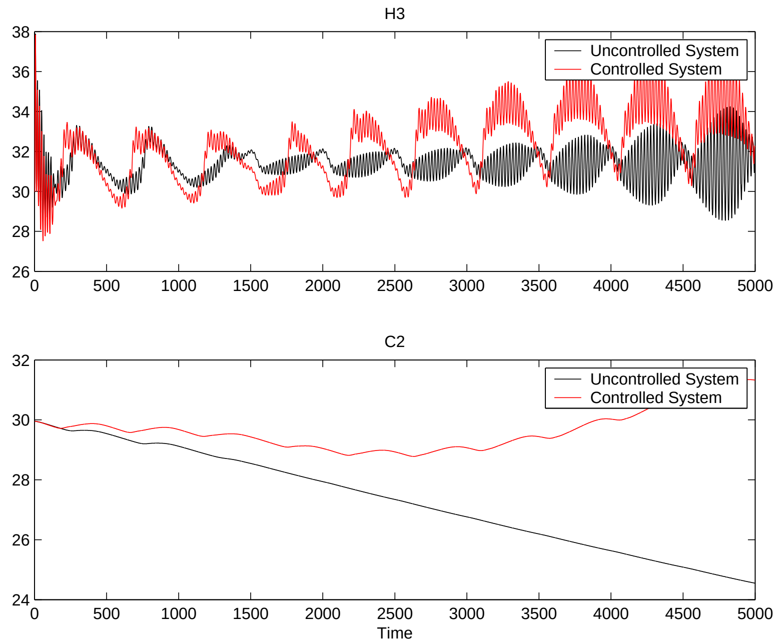

Figure 7: Case B: Controlled system using α as control variable

is altered from its base case. The new input composition is: 40% P1, 55% RP and 5% H3. This change affects all the compartments of the model. The compartments P3, H1, H2, C2, HH and IRP are particularly perturbed.

The control problem solution results show that using W and α as control variables does not lead to favorable results. On the contrary, some of the compartments become unstable after enforcing control. Using $g(2)$ as the control variable also results in significant disturbance in some of the compartments and the overall controlled response is not satisfactory. Some representative results while using $g(1)$ as control variable are shown in Figures 8 and 9. It can be observed that most of the compartments show excellent dynamics which are very similar to those observed for the undisturbed model.

8.4 Case D: Change in W

In this case, the constant for the waste term associated with human consumption, W, is increased from its base case value by 70%. This results in the uncontrolled case with: slight change in the dynamics of compartments P1, P2, P4, C1; temporary disturbance in the dynamics of compartment H1, H2, H3, HH, IRP; and a sustained decrease in the mass of compartment C2. The degree of disturbance caused by this change is less severe than the other cases considered in this work. Since W is changed to simulate the case, use of W as the control variable is omitted.

A comparison of the results (not reported here) indicates that changes in α lead to most effective control of the compartments showing temporary extinction. Changes in $g(1)$ and $g(2)$ also result in favorable model dynamics. It is again observed that $g(1)$ and $g(2)$ have similar qualitative effects, however, $g(1)$ has a stronger impact on the model dynamics than $g(2)$.

8.5 Case E: Severe reduction in α

Drastic changes in the parameters of an ecosystem can cause shifts in the dynamic regimes of these systems, which can be stable or unstable. Such changes are caused by the natural disasters such

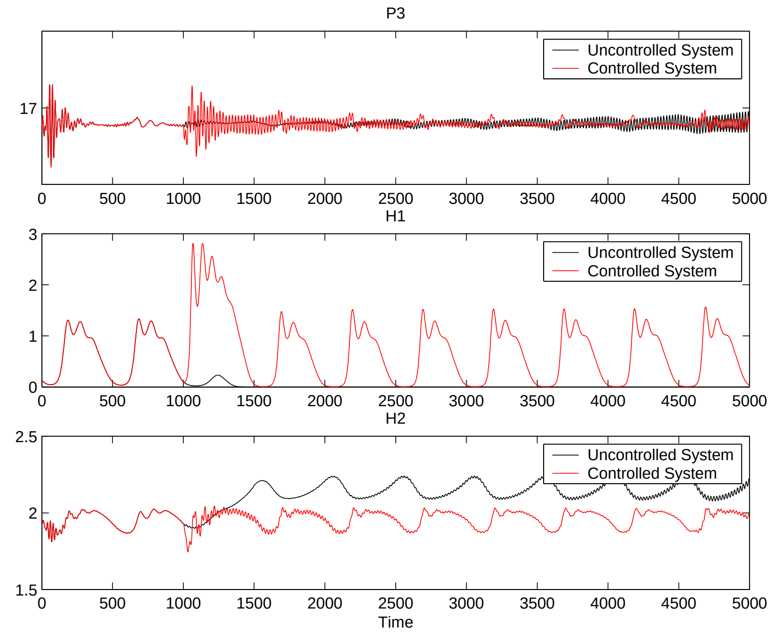

Figure 8: Case C: Controlled system using $g(1)$ as control variable

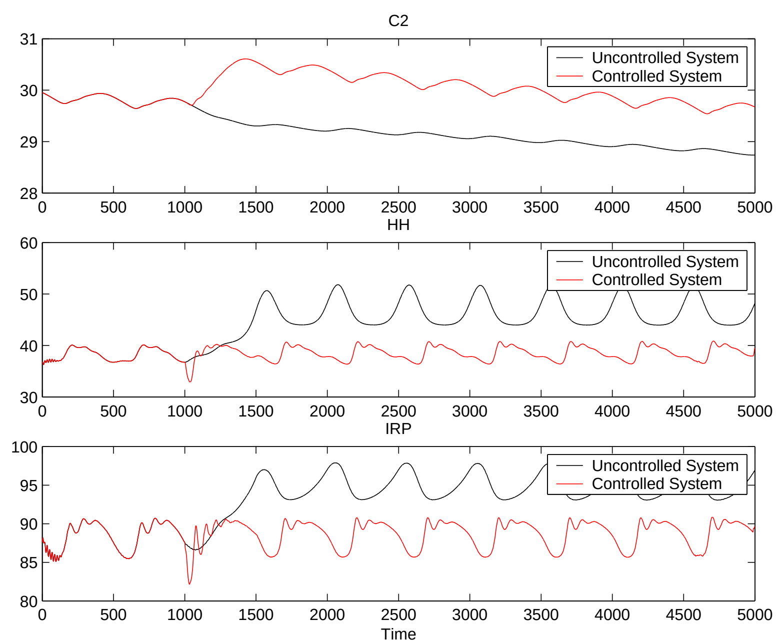

Figure 9: Case C: Controlled system using $g(1)$ as control variable

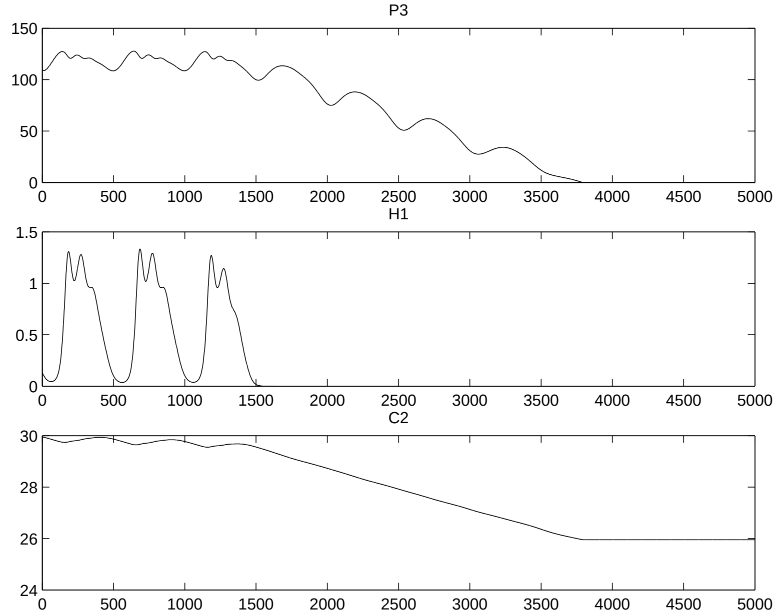

Figure 10: Case E: Uncontrolled system due to major change in α

as floods, hurricanes, or dramatic changes in the global climatic conditions [22]. Quite often, these regime shifts are nonlinear, exhibiting phenomenon like hysteresis, meaning that the restoration of the original regime is complicated. The purpose of modelling a major change in α is to simulate such a case of severe disturbance and then assess the ability of various control options. Here, the value of α is reduced by 50% for the uncontrolled system. This results in fast decrease in the compartmental mass of P1 and it goes to extinction towards the end of the considered time horizon. This causes other compartments to show undesirable variations. Other compartments that are independently disturbed include H1 and C2. Figure 10 shows the uncontrolled trajectories for these compartments.

After solving the control problem for $g(1)$, $g(2)$ and W, it is observed that none of these are able to recover the system from the disaster. While W has very little favorable impact, changes in $g(1)$ lead to marginal improvement in model dynamics. These observations illustrate that if the disturbance is too large, it may not be possible to recover the system using the given control options.

8.6 Multiple control analysis

The results presented so far use only one model parameter as the control variable. It was observed that the relative effect on each model compartment is different using different control variables. The effect depends on the location of a particular compartment in the food web. Moreover, some variables are more effective in controlling model instability, while others are more effective in avoiding extinction. This prompts one to think about multivariable control as an option to further improve the model dynamics. Multivariable control refers to the case when more than one model parameters are simultaneously varied. However, multivariable control is complicated due to the interactive effects (coupling) of control variables on model dynamics. Literature on multivariable control discusses various aspects of such interactions and suggests methods to decouple the model parameters.

Owing to these complications, this work looks at a variant of the actual multivariable control. Here, the idea is to solve two different single variable control problems sequentially for the model. The model is first subjected to one control action (primary control action), using one parameter as the control variable, referred to as the primary control variable (CV-1). The results presented in the preceding sections constitute the results for primary control action. In the next step, this controlled model is taken as the starting condition, and another control problem with a different control variable, referred as the secondary control variable (CV-2), is solved. The time dependent profile of CV-1 is based on the primary control problem solution and does not change during the second control problem solution. The solution of the second control problem thus has time dependent profiles for CV-1 as well as CV-2. The representative result for one case using this method is presented in the following section.

8.6.1 Doubling the industrial process flow IPF

The uncontrolled case is simulated by doubling the parameter IPF in the model. The results using single variable control for this case are already discussed in one of the preceding sections. The uncontrolled system shows instability with: significant instability in compartments P3, H2 and H3; extinction in compartment H1; onset of extinction in compartment C2 and onset of instability in compartments P2, P4, HH and IRP.

For this case, $g(1)$ effectively controls the instability of the compartments while α effectively eliminates species extinction. Hence, these two parameters are selected as the control variables for the model. α is the primary control variable (CV-1), while $g(1)$ is the secondary control variable (CV-2). The results are plotted in Figures 11 and 12. The results clearly indicate that the model dynamics using multiple controls are much better than those with single variable control. Using only α as the control variable, instability in some of the compartments was not avoided. However, additional control using $g(1)$ resulted in effective control of the model instability.

These results lead to a very interesting argument that to achieve sustainability goals for a complicated system like this model, use of multiple control pertaining to different aspects of the system is effective. Performing the multiple control analysis using $g(1)$ and $g(2)$ as control variables does not improve the dynamics. This might be due to the fact that both these parameters are similar (controlling resource flow in the food web) and are at the same trophic level. $g(1)$ and α however pertain to distinctly different things in the model and hence are less likely to suffer from coupling effects. This leads to substantial improvement in model dynamics. This argument should be considered while deciding possible control options in other models.

9 Summary of results

The previous sections discusses the results for various cases on the food web model. The important conclusions to be drawn from those results are:

- $g(1)$, the coefficient of mass transfer from RP to P1, is overall the most effective control variable among the considered options.
- $g(2)$, the coefficient of mass transfer from RP to P2, is also an effective option to exercise control on the model, although it is not as effective as $g(1)$
- The relative success of $g(1)$ and $g(2)$ in the control of various compartments depends on the position of the compartment in the food web. Thus, $g(2)$ is observed to be more effective in manipulating compartment C2 while $g(1)$ is more effective is manipulating compartment H1.
- α is moderately effective in controlling model dynamics.
- W is found to be the least effective control option. This might be because W affects compartments that are towards the end of the food chain and hence does not exert a strong effect on the compartments at the bottom of the food chain.

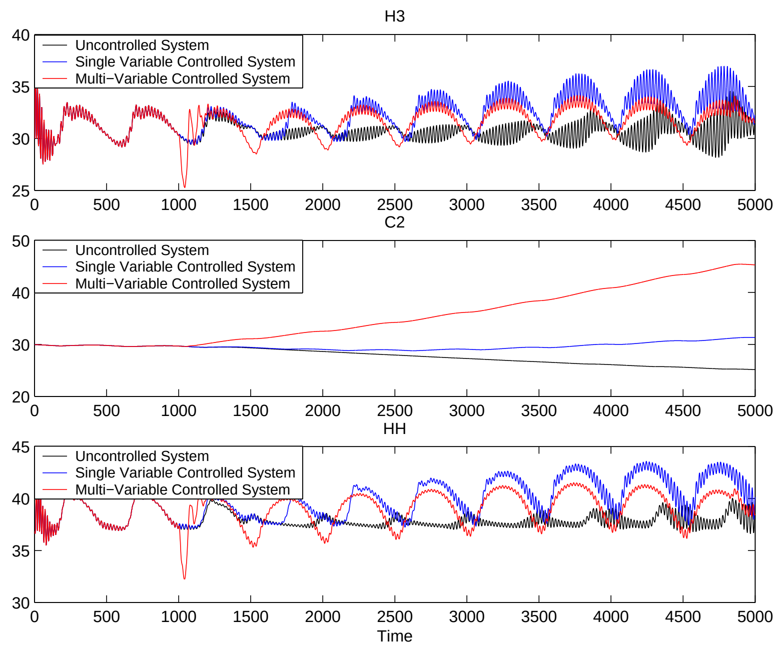

Figure 11: Multiple control solution

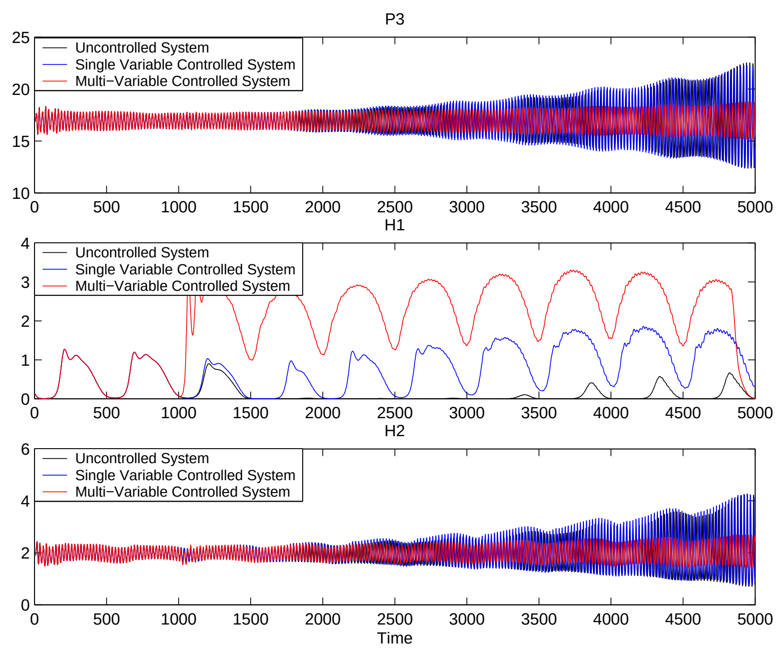

Figure 12: Multiple control solution

- It is observed that the performance of $g(1)$ and $g(2)$ is excellent in controlling the instability of the system. On the other hand, α is observed to be very effective is averting the extinction of mass in various compartments.
- The analysis of control profiles shows that $g(1)$ and $g(2)$ have opposite effects on many compartments. This might be due the sharing of RP by these two streams.
- The use of multiple control variables leads to improved dynamics in some of the cases simulated for the model. This suggests that appropriate combination of multiple control variables can be an effective solution to control the model.

10 Concluding remark

Sustainable management of the human and natural systems, taking into account their interactions, has become paramount. To achieve this complex multidisciplinary objective, systems theory based techniques prove useful. The proposed work is a step in that direction. Taking a food web model incorporating the essential aspects of the complete spectrum, it uses Fisher information based sustainability objectives and optimal control theory to derive sustainable management strategies. For the considered model, the results highlight the important parameters and variables. However, on broader perspectives, the results should be viewed as the proof of concept for the application of systems theory based techniques in sustainability. The important conclusions and results from this study should form the basis for the use of such approaches for more complicated models.

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