Coupled Resonances on the QCM and The Lumped Equivalent Model

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Abstract—QCM interactions with a discretely resonant load much greater in frequency than the quartz resonator's shows negative frequency changes with increasing mass loading. However, a positive frequency increase of the coupled system occurs when the load resonant frequency is less than the quartz resonator's. A brief review of the method by which a resonant load can be coupled into the electromechanical equations for the quartz resonator is given. Though this yields exact results, the purpose of this communication is to show the derivation of an equivalent circuit for the resonance following the acoustic impedance approach. This equivalent circuit replicates the characteristics of the exact solutions.

I. INTRODUCTION

The analysis for the coupled resonance within the electromechanical model was initially done for a specific application, namely the behavior of small spheres on the QCM[1]. However, the model is quite robust and can represent any general load on the QCM in which the load has its own resonance. Here we discuss the load's resonance and its on the behavior of the QCM resonance. We introduce an equivalent circuit which may be useful in pointing towards the origins of that resonance.

The initial notion for the effect on a resonant system of a load which is itself resonant was first discussed by Dybwad[2]. This was done in terms of a spring and mass system in which the load itself was another spring and mass. We have been successful in taking this phenomenon to the case of a load on the QCM, including its effects on higher harmonics. Under "Exact Approach", we outline this procedure. Under "Equivalent Circuits" we extend this approach to obtain an equivalent circuit representation using the approach given by the Sandia group[3].

II. EXACT APPROACH

The key to including the effects of a self-resonant load into the complete electromechanical equations is to express the stress π and displacement of the load on the surface of the quartz, permitting matching of boundary conditions to the quartz surface. The model for the coupled resonance is that of a set of identical spheres, each of mass M coupled via a spring to a massless rigid rod implanted on the quartz surface. Writing the amplitude of the motion of the mass away from the rod as W, and the amplitude of the motion of the rod coupled to the quartz as U, the equation of motion for this system can be written:

$$(\omega_R^2 - \omega^2 + j\omega\frac{\omega_R}{Q_R})W = (\omega_R^2 + j\omega\frac{\omega_R}{Q_R})U \quad (1)$$

The equation has been written in terms of the self-resonant frequency ω_R and its quality Q_R . For a collection of these non-interacting close packed spheres each of diameter D, the stress can be written:

$$\pi = \frac{M}{D^2}\omega^2 W \ . \qquad (2)$$

Solving for the boundary conditions in a manner similar to that detailed previously[3], the parameters of the quartz and the sphere motions are determined..

While the availability of the exact analysis would permit the determination of the frequency and dissipative changes in the resonance, it was felt that a reduction of that analysis to an equivalent circuit representation would be helpful in interpreting the influence of experimental changes. For this purpose, we used the analysis described by the Sandia group[4]. They had earlier shown[5] that the electrical equivalent circuit can be divided into a portion dependent only on the unloaded quartz parameters and with added series elements representing the load parameters. For example, in the case of a pure liquid load on the quartz, the equivalent circuit was:

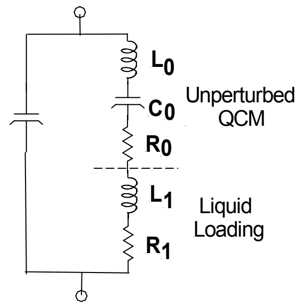
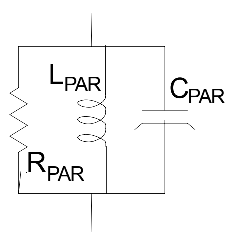


Figure 2. Equivalent circuit representation for a QCM loaded with a liquid.

Their expression takes the form:

$$\frac{N\pi}{4K^2\omega C_P}\left(\frac{Z_{AC}}{Z_Q}\right) = Z_{ADD} \qquad (3)$$

Here, N is the harmonic number, K^2 is the electromechanical coupling constant for the quartz, Cp is its dielectric capacitance, Z_Q the quartz acoustic impedance, and Z_{AC} is the load's acoustic impedance. This latter is given by the ratio of the stress to the velocity of the load at the quartz surface. The impedance Z_{ADD} can be interpreted either as a series combination or a parallel combination of an R, L and C. We were able to obtain explicit expressions for the parallel combination. For the example shown in Figure 1, we obtained R_{PAR}=1.287x10^8 ohms, C_{PAR}= 0.5735 pf, and L_{PAR}=5.252 mH. It is gratifying that the parallel resonant frequency comes to 2.9 MHz, the postulated resonant frequency of the load. It was also interesting that the inductance value was independent of the load resonance parameters of ω_R and Q_R, depending only on the strength of the coupling. It may serve as a measure of the coupling strength.



The inverse relation between the QCM frequency changes with the harmonic number are in accord with this equivalent circuit, as is the return to Sauerbrey-like behavior when the load resonance frequency is very high.

ACKNOWLEDGMENT

This work was supported in part by the MRSEC Program of the National Science Foundation under Award Number DMR-1213618.

REFERENCES

[1] J.N. D'Amour, J.J.R. Stalgren, K.K. Kanazawa, C.W. Frank, M. Rodahl and D. Johannsmann, "Capillary Aging of the Contacts between Glass Spheres and a Quartz Resonator Surface, Phys Rev. Lett, *vol 96, p.058301, Feb. 2006*

[2] G.L. Dybwad, "A sensitive new method fr the determination of adhesive bonding between a particle and a substrate", J. Appl. Phys., vol 58, p 2789, Oct. 1985

[3] K.K.Kanazawa, "Mechanical Behavior of Films on the quartz Microbalance, Faraday Discuss. Vol 107, p 77, 1997

[4] H.L. Bandey, S.J. Martin R.W. Cernosek and A.R. Hillman, "Modeling the Responses of Thickness-Shear Mode Resonators under Various Loading Conditions". Anal. Chem. Vol 71, p 2205, June

[5] S.J. Martin, V.E. Branstaff and G.C. Frye, "Characterization of a Quartz Crystal Microbalance with Simultaneous Mass and Liquid Loading", Anal. Chem., vol 63, p.2272, October 1991