GLOBAL OPTIMIZATION OF MULTIPHASE FLOW NETWORKS IN OIL AND GAS PRODUCTION SYSTEMS

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Abstract

A mathematical program for finding the optimal oil production rates in an oil production system is developed. Each well may be manipulated by injecting lift gas and adjusting a production choke. The oil production from the wells may be restricted with multiple constraints in the maximum oil flow rate, water flow rate, liquid flow rate, and gas flow rate. The wells may also be restricted with a maximum total lift gas rate. In oil production systems with sub sea wells, flow lines are often shared between two or more wells. The pressure in the production manifold will in such configurations be affected by the flow from the wells. The commonly used models based on gas lift performance curves (GLPC) no longer apply to these problems due to changing pressure conditions in the production manifold. Because of this, a model of the flow line is also required to get more accurate results. This work incorporates such a model. A piecewise linear approximation is proposed. This makes it possible to find a proven global optimum, within the approximation, for the optimization problem. The problem is formulated as a mixed integer linear program and solved with a commercial branch and cut solver.

Introduction

In the daily operation of oil production systems many decisions have to be taken that affects the volumes of oil produced. One of them is which settings to use for the chokes to somehow maximize the oil production. Because of limited processing capacity, the optimal solution may be to choke back some wells with high production of water or gas relative to oil production.

To increase the production of oil, gas lift has been installed in many wells. Gas lift reduces the pressure drop in the raiser by reducing the average density of the fluid. The effect of gas lift reduces with the amounts used because the gas also increases the friction. Furthermore, the gas has to be processed by the production system's compressors which are limited. A challenging optimization problem then has to be solved in order to maximize the production. The problem has been studied by many people, including Fang and Lo [1]. In that paper, a scheme for solving the problem using gas lift performance curves was proposed using linear programming. They pointed out that the method might not give a correct solution if a well was not able to flow naturally, i.e. a well was not able to produce with zero lift gas. They therefore proposed to use a mixed integer solver in such cases. This was later studied by people including Wang [2]. The problem was formulated as

$$\max_{\lambda} \sum_{i \in W} \sum_{k \in K_i} q^{\mathrm{o}}_{i,k} \lambda_{i,k} \tag{1}$$

subject to:

$$\sum_{i \in W} \sum_{k \in K_i} q^{\mathrm{gl}}_{i,k} \lambda_{i,k} \leq q^{\mathrm{gl,M}} \tag{2}$$

$$\sum_{k\in K_i} \lambda_{i,k} = 1 \forall i \in W \quad (3)$$

$$\lambda_{i,k} \geq 0 \forall i \in W, k \in K_i \quad (4)$$

For each i, at most two $\lambda_{i,k}$ may be positive, and they must be adjacent. (5)

The pair $(q^{\mathrm{o}}_{i,k}, q^{\mathrm{gl}}_{i,k})$ is the oil and gas lift rate which makes up a point in the gas lift performance curve, W is the set of wells, K_i is the set of points for well i, and $q^{\mathrm{gl,M}}$ is the maximal total gas lift rate. (4)-(5) forms a special ordered set of type two [3], which is directly supported by most modern mixed integer solvers [4]. It may also be formulated as a pure mixed integer program [5, 6].

However, Wang [2] pointed out an important drawback with the methods using gas lift performance curves. It assumed that the production from each well was independent. In some sense this is often true for some offshore installations. Here, the blending point, called the production manifold, is placed on the production platform itself. Due to the short distance between the production manifold and the pressure controlled production separator, the production manifold pressure is assumed to be fixed.

However, the introduction of new sub sea technologies has changed this for the offshore oil production platforms. Wells far from the production platforms are connected to a sub sea template at the sea bed, in which well streams are blended. The blended well streams are sent through a flow line to the production platform. Because of the long distance of this shared flow line, the pressure drop may be large. Furthermore, the pressure drop will typically be sensitive to the volumes flowing through the flow line. Thus, adjusting the production from one well by changing the lift gas rates or the production choke, will most certainly affect the other wells.

The optimization of such a flow line network has therefore been studied by several people. In [7] the optimal lift gas rates for one, two, and three identical wells sharing a flow line was compared. Also, larger field-wide flow line networks were studied. By the use of Successive Quadratic Programming (SQP) it was found that the optimal lift gas rates for each well reduced as the number of wells increased. SQP was also used by Wang [2] to solve a similar flow line network.

Instead of using SQP, Successive Linear Programming (SLP) was proposed by [8]. The pressure drops in the flow lines were modeled using standard nonlinear equations. In each iteration of the SLP algorithm, the pressure drop is linearized in the flow lines were found. The inflow performance of each well was modeled as a piecewise linear surface using linear inequalities, similar to [1]. According to the author, this reduced the number of SLP iterations required.

All the above proposed solutions use only local algorithms that at best may guarantee that a local optimum is found. Because all the problems formulated in general are non-convex, the algorithm may not find the global optimum solution. Even worse, a feasible solution is not guaranteed to be found even if the problem is feasible for sure. Some parts of the physics itself make the problem feature multiple local optima. For fixed boundary pressures on a flow line, there may exist two different flow rates satisfying the conditions; one low flow rate and one high flow rate. If the wrong initial solution is used in the simulator, then the wrong solution will be found. In an optimizer, the problem will be the same, but in a larger scale.

To be able to escape from only local optima, a genetic algorithm was used by Stoisits et al. [9] to give near global optimum solution in a similar problem. Unfortunately, genetic algorithms still have some draw backs. They do not include a guarantee for neither a local nor global optimum. Furthermore, the computational load is very high because little structure of the problem is utilized.

The drawbacks for the above methods motivates for a new method for solving such flow line networks that is able to find a proven global optimum, do not require an initial solution to be provided, and has a reasonable computational load in the optimization. In this work such a method will be presented.

Methodology

The work of Fang and Lo [1] allowed a global optimum to be found by modifying the problem into a mixed integer problem. Unfortunately, it can only be used for the simplest oil production system configuration due to the missing support for flow lines shared by multiple wells. In this work this model will be extended to include pressure drop in shared flow lines.

Well

The well model relates the oil rates, gas lift rates, and outlet pressure of the well, i.e. the production manifold pressure, in some way. It is possible to argue for different choices of independent and dependent variables, but in this work the oil and gas lift rates were used as independent variables, while the production manifold pressure was the dependent (calculated) variable. This is because the flow in a pipe is calculated using integration of the partial derivative of pressure. Thus, the pressure may be found using a single integration, while a nonlinear equation set (including integration) would have to be solved to find the flow rates if the outlet pressure was an independent variable. Using a mixed integer framework, the outlet pressure equation

$$p_i = p_i\,(q_i^{\mathrm{o}}, q_i^{\mathrm{lg}}) \forall i \in W \tag{6}$$

for well i will be modeled, where q_i^{o} is oil flow rate, q_i^{lg} is the lift gas rate, and p_i is the outlet pressure of the well. Similar to the gas lift performance curve, each of the independent variables will be defined into a finite number of break points, e.g. point in which a function evaluation of $p_i^{\mathrm{O}}(\cdot)$ will be performed. Lets call the break points $q_{i,k^{\mathrm{o}}}^{\mathrm{o}}$ and $q_{i,k^{\mathrm{lg}}}^{\mathrm{lg}}$ for oil and lift gas rates, respectively. For the oil rate, the set K_i^{o} will define the indices of the break points, while K_i^{lg} will have the same role for lift gas (for each well i). A function evaluation $p_{i,k^{\mathrm{o}},k^{\mathrm{lg}}} := p_i\,(q_{i,k^{\mathrm{o}}}^{\mathrm{o}}, q_{i,k^{\mathrm{lg}}}^{\mathrm{lg}})$ of the outlet pressure will be performed in each combination of those points, thus

$$p_i = \sum_{k^{\mathrm{o}} \in K_i^{\mathrm{o}}} \sum_{k^{\mathrm{lg}} \in K_i^{\mathrm{lg}}} p_{i,k^{\mathrm{o}},k^{\mathrm{lg}}} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{lg}}} \forall i \in W \tag{7}$$

The model should also include the oil and lift gas rates. To add them, some auxiliary variables are defined

$$\lambda_{i,k^{\mathrm{o}}}^{\mathrm{o}} = \sum_{k^{\mathrm{lg}} \in K_i^{\mathrm{lg}}} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{lg}}} \forall i \in W, k^{\mathrm{o}} \in K_i^{\mathrm{o}} \tag{8}$$

$$\lambda_{i,k^{\mathrm{lg}}}^{\mathrm{lg}} = \sum_{k^{\mathrm{o}} \in K_i^{\mathrm{o}}} \lambda_{i,k^{\mathrm{o}}k^{\mathrm{lg}}} \forall i \in W, k^{\mathrm{lg}} \in K_i^{\mathrm{lg}} \tag{9}$$

Using them, the oil and lift gas rates can be included

$$q_i^{\mathrm{o}} = \sum_{k^{\mathrm{o}} \in K_i^{\mathrm{o}}} q_{i,k^{\mathrm{o}}}^{\mathrm{o}} \lambda_{i,k^{\mathrm{o}}} \forall i \in W, k^{\mathrm{o}} \in K_i^{\mathrm{o}} \tag{10}$$

$$q_i^{\mathrm{lg}} = \sum_{k^{\mathrm{lg}} \in K_i^{\mathrm{lg}}} q_{i,k^{\mathrm{lg}}}^{\mathrm{lg}} \lambda_{i,k^{\mathrm{lg}}} \forall i \in W, k^{\mathrm{lg}} \in K_i^{\mathrm{lg}} \tag{11}$$

The gas and water rates will also be handy, so they will also be defined here

$$q_i^{\mathrm{g}} = \sum_{k^{\mathrm{o}} \in K_i^{\mathrm{o}}} r_i^{\mathrm{g}} q_{i,k^{\mathrm{o}}}^{\mathrm{o}} \lambda_{i,k^{\mathrm{o}}} \forall i \in W, k^{\mathrm{o}} \in K_i^{\mathrm{o}} \tag{12}$$

$$q_i^{\mathrm{w}} = \sum_{k^{\mathrm{o}} \in K_i^{\mathrm{o}}} r_i^{\mathrm{w}} q_{i,k^{\mathrm{o}}}^{\mathrm{o}} \lambda_{i,k^{\mathrm{o}}} \forall i \in W, k^{\mathrm{o}} \in K_i^{\mathrm{o}} \tag{13}$$

where r_i^{g} is the gas oil ratio and r_i^{w} is the water oil ratio (i.e. $r_i^{\mathrm{w}} := \mathrm{WC}_i/(1-\mathrm{WC}_i)$ where WC_i is the water cut of the well). Furthermore, the convexity constraints are added similarly as in [1],

$$\sum_{k^{\mathrm{o}} \in K_i^{\mathrm{o}}} \sum_{k^{\mathrm{lg}} \in K_i^{\mathrm{lg}}} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{lg}}} = 1 \forall i \in W \tag{14}$$

$$\lambda_{i,k^{o},k^{\mathrm{lg}}} \geq 0 \forall i \in W, k^{\mathrm{o}} \in K_i^{\mathrm{o}}, k^{\mathrm{lg}} \in K_i^{\mathrm{lg}} \tag{15}$$

To ensure that neighbors are used in the interpolation, two more constrains have to be added

For each *i*, at most two $\lambda_{i,k^{\mathrm{o}}}^{\mathrm{o}}$ may be positive, and they must be adjacent. (16)

For each *i*, at most two $\lambda_{i,k^{\mathrm{lg}}}^{\mathrm{lg}}$ may be positive, and they must be adjacent. (17)

The model of the well has now been completed.

Flow line

The well model was an extension of previous work by Fang and Lo [1]. No similar model piecewise linear model of a flow line or pipe has been found in the literature. The closest match was some work done by Litvak and Darlow [10], who used a look up table of the pressure drop in a flow line. They parameterized it in four independent variables: oil rate, gas rate, water rate, and pressure. Thus, they assumed that the stream consisted of only three linearly independent fluid compositions, and that the temperature at the inlet was fixed. The same assumptions will be used in this work. Tests done using a flow simulator for a real field showed little change in temperature. However, it should be noted that the method itself does not restrict the inclusion of temperature/enthalpy. The assumption is done to reduce the computational requirement. The temperature was included in a similar model [11] by the use of enthalpy.

As for the well, the outlet pressure will be described by piecewise linear functions that is approximated. Thus,

$$p_i = p_i(q_i^{\mathrm{o}}, q_i^{g}, q_i^{\mathrm{w}}, p_i^{\mathrm{I}}) \forall i \in F \tag{18}$$

has to be modeled, where q_i^{g} is the gas rate, q_i^{w} is the water rate, and p_i^{I} is the inlet pressure of the flow line. Using the same notation as for wells, the outlet pressure can be defined as

$$p_i = \sum_{k^{\mathrm{o}} \in K_i^{\mathrm{o}}} \sum_{k^{\mathrm{g}} \in K_i^{\mathrm{g}}} \sum_{k^{\mathrm{w}} \in K_i^{\mathrm{w}}} \sum_{k^{\mathrm{p}} \in K_i^{\mathrm{p}}} p_{i,k^{\mathrm{o}},k^{g},k^{\mathrm{w}},k^{\mathrm{p}}} \lambda_{i,k^{\mathrm{o}},k^{g},k^{\mathrm{w}},k^{\mathrm{p}}} \forall i \in F \tag{19}$$

Auxiliary variables are then defined

$$\lambda^{\mathrm{o}}_{i,k^{\mathrm{o}}} = \sum_{k^{\mathrm{g}}\in K^{\mathrm{g}}_i} \sum_{k^{\mathrm{w}}\in K^{\mathrm{w}}_i} \sum_{k^{\mathrm{p}}\in K^{\mathrm{p}}_i} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{g}},k^{\mathrm{w}},k^{\mathrm{p}}} \forall i\in F, k^{\mathrm{o}}\in K^{\mathrm{o}}_i \quad (20)$$

$$\lambda^{\mathrm{g}}_{i,k^{\mathrm{g}}} = \sum_{k^{\mathrm{o}}\in K^{\mathrm{o}}_i} \sum_{k^{\mathrm{w}}\in K^{\mathrm{w}}_i} \sum_{k^{\mathrm{p}}\in K^{\mathrm{p}}_i} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{g}},k^{\mathrm{w}},k^{\mathrm{p}}} \forall i\in F, k^{\mathrm{g}}\in K^{\mathrm{g}}_i \quad (21)$$

$$\lambda^{\mathrm{w}}_{i,k^{\mathrm{w}}} = \sum_{k^{\mathrm{o}}\in K^{\mathrm{o}}_i} \sum_{k^{\mathrm{g}}\in K^{\mathrm{g}}_i} \sum_{k^{\mathrm{p}}\in K^{\mathrm{p}}_i} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{g}},k^{\mathrm{w}},k^{\mathrm{p}}} \forall i\in F, k^{\mathrm{w}}\in K^{\mathrm{w}}_i \quad (22)$$

$$\lambda^{\mathrm{p}}_{i,k^{\mathrm{p}}} = \sum_{k^{\mathrm{o}}\in K^{\mathrm{o}}_i} \sum_{k^{\mathrm{g}}\in K^{\mathrm{g}}_i} \sum_{k^{\mathrm{w}}\in K^{\mathrm{w}}_i} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{g}},k^{\mathrm{w}},k^{\mathrm{p}}} \forall i\in F, k^{\mathrm{p}}\in K^{\mathrm{p}}_i \quad (23)$$

Using them, oil, gas, water, and inlet pressure can be included

$$q^{\mathrm{o}}_i = \sum_{k^{\mathrm{o}}\in K^{\mathrm{o}}_i} q^{\mathrm{o}}_{i,k^{\mathrm{o}}} \lambda^{\mathrm{o}}_{i,k^{\mathrm{o}}} \forall i\in F, k^{\mathrm{o}}\in K^{\mathrm{o}}_i \quad (24)$$

$$q^{\mathrm{g}}_i = \sum_{k^{\mathrm{g}}\in K^{\mathrm{g}}_i} q^{\mathrm{g}}_{i,k^{\mathrm{g}}} \lambda^{\mathrm{g}}_{i,k^{\mathrm{g}}} \forall i\in F, k^{\mathrm{g}}\in K^{\mathrm{g}}_i \quad (25)$$

$$q^{\mathrm{w}}_i = \sum_{k^{\mathrm{w}}\in K^{\mathrm{w}}_i} q^{\mathrm{w}}_{i,k^{\mathrm{w}}} \lambda^{\mathrm{w}}_{i,k^{\mathrm{w}}} \forall i\in F, k^{\mathrm{w}}\in K^{\mathrm{w}}_i \quad (26)$$

$$p^{\mathrm{I}}_i = \sum_{k^{\mathrm{lg}}\in K^{\mathrm{lg}}_i} p^{\mathrm{I}}_{i,k^{\mathrm{p}}} \lambda^{\mathrm{p}}_{i,k^{\mathrm{p}}} \forall i\in F, k^{\mathrm{p}}\in K^{\mathrm{p}}_i \quad (27)$$

Furthermore, the convexity constraints are added similarly to before,

$$\sum_{k^{\mathrm{o}}\in K^{\mathrm{o}}_i} \sum_{k^{\mathrm{g}}\in K^{\mathrm{g}}_i} \sum_{k^{\mathrm{w}}\in K^{\mathrm{w}}_i} \sum_{k^{\mathrm{p}}\in K^{\mathrm{p}}_i} \lambda_{i,k^{\mathrm{o}},k^{\mathrm{g}},k^{\mathrm{w}},k^{\mathrm{p}}} = 1 \forall i\in F \quad (28)$$

$$\lambda_{i,k^{\mathrm{o}},k^{\mathrm{g}},k^{\mathrm{w}},k^{\mathrm{p}}} \geq 0 \forall i\in F, k^{\mathrm{o}}\in K^{\mathrm{o}}_i, k^{\mathrm{g}}\in K^{\mathrm{g}}_i, k^{\mathrm{w}}\in K^{\mathrm{w}}_i, k^{\mathrm{p}}\in K^{\mathrm{p}}_i \quad (29)$$

To ensure that neighbors are used in the interpolation, four more constrains have to be added

For each i, at most two $\lambda^{\mathrm{o}}_{i,k^{\mathrm{o}}}$ may be positive, and they must be adjacent. (30)

For each i, at most two $\lambda^{\mathrm{g}}_{i,k^{\mathrm{g}}}$ may be positive, and they must be adjacent. (31)

For each i, at most two $\lambda^{\mathrm{w}}_{i,k^{\mathrm{w}}}$ may be positive, and they must be adjacent. (32)

For each i, at most two $\lambda^{\mathrm{p}}_{i,k^{\mathrm{p}}}$ may be positive, and they must be adjacent. (33)

The model of the flow line has now been completed.

Choke

Wang [2] investigated how the pressure drop of the choke increased when closing the choke for fixed flow rates. He utilized it to remove an explicit model of the choke in the model used for optimization. In his work it was attractive because some non-convex features of a typical choke model.

For the piecewise linear model a model of the choke would introduce more independent variables in the pressure drop equations (6) and (18), thus requiring many new decision variables.

Fortunately, because of the property observed by Wang, this is not required. Instead, the minimal pressure drop of the choke will be included in the outlet pressure p_i of the wells and/or flow lines. This minimal pressure drop is found by including the choke model in the calculation of the pressure drop in the well and/or flow line with a choke opening set to it maximal opening, typically position 1.0. Any reduction of the choke opening will give a higher pressure drop, thus for any well or flow line $i \in W \cup F$ with a choke

$$p_{\mathrm{i}}^{\mathrm{O}} \le p_i \tag{34}$$

And if a choke does not exist, then just use pressure equality

$$p_{\mathrm{i}}^{\mathrm{O}} = p_i. \tag{35}$$

It should be noted that the above statement is only true if the flow direction is given. If the flow changes direction, then the additional pressure drop will have opposite sign.

Outlet Boundary

A model of the outlet boundary of the system is included. This can be the production manifold or the production separator. Nevertheless, it is assumed that this outlet boundary i has a fixed inlet pressure p_i^{I} for all $i \in O$ where O is the set of outlet boundary nodes.

Connection

The flow lines have to be connected to other flow lines or wells at the inlet. This is done by enforcing mass balance to be satisfied

$$q_i^{\mathrm{o}} = \sum_{j \in \Omega_i} q_j^{\mathrm{o}} \forall i \in F \cup B, \tag{36}$$

$$q_i^{\mathrm{g}} = \sum_{j \in \Omega_i} q_j^{\mathrm{g}} \forall i \in F \cup B, \tag{37}$$

$$q_i^{\mathrm{w}} = \sum_{j \in \Omega_i} q_j^{\mathrm{w}} \forall i \in F \cup B. \tag{38}$$

And the pressure equality at the node,

$$p_i^{\mathrm{I}} = p_j^{\mathrm{O}} \forall i \in F \cup B, j \in \Omega_i \tag{39}$$

where Ω_i is the set of flow lines and wells connected to the inlet of flow line or outlet boundary i.

Objective

The objective is to maximize the total oil production rate, which can be formulated as

$$\max \sum_{i \in B} \mathrm{q}_i^{\mathrm{o}}. \tag{40}$$

This assumes that all production ends in an outlet boundary node $i \in B$.

Constraints

The stated optimization problem can easily be incorporated with constraints on flow rates and pressures. This is done by

$$q_i^{\mathrm{o}} \le \overline{q}_i^{\mathrm{o}} \forall i \in W \cup F \cup B, \tag{41}$$

$$q_i^{\mathrm{g}} \le \overline{q}_i^{\mathrm{g}} \forall i \in W \cup F \cup B, \tag{42}$$

$$q_i^{\mathrm{w}} \le \overline{q}_i^{\mathrm{w}} \forall i \in W \cup F \cup B, \tag{43}$$

$$q_i^{\mathrm{w}} + q_i^{\mathrm{o}} \le \overline{q}_i^{\mathrm{l}} \forall i \in W \cup F \cup B. \tag{44}$$

And for pressure there is an upper bound

$$p_i^{\mathrm{o}} \le \overline{p}_i^{\mathrm{o}} \forall i \in W \cup F \cup B \tag{45}$$

where $\overline{p}_i^{\mathrm{o}}$ denote the maximal outlet pressure and a lower bound

$$p_i^{\mathrm{o}} \ge \underline{p}_i^{\mathrm{o}} \forall i \in W \cup F \cup B. \tag{46}$$

Case Study

The proposed method was applied to data from a real oil field in the North Sea. The oil field consists of a flow line/raiser configuration with two sub sea templates. Each sub sea template has two wells. The topology is shown in Figure 1.

Each independent variable in the piecewise functions was modeled using 10 break points. This gave each flow line about 10,000 break points. Each well was represented by 100 break points.

Various cases were studied where the constraints were varied. The computational times were in a range from less than 1 second to just below 100 seconds on a standard personal computer.

The evaluation of the pressure drops in the flow lines and the wells were done using the simulation capabilities of the commercial available virtual flow metering software Well Monitoring Software [12].

A comparison was made on the total oil flow rates for the same choke openings using the simulation software and the approximated model in the optimization model. It showed a difference in the range of 1-3 %. This can be further be reduced by including more break points in the optimization problem.

Conclusions

In this work a method of calculating the optimal oil production rates for an oil production system was developed. The method uses a piecewise linear model to approximate the pressure drops in wells and flow lines. By using this, it is possible to find the global optimal production rates for each well in the oil field. Furthermore, the global optimum is found, unlike other methods, without requiring the user to provide an initial solution. In combination, this makes the method robust for the user.

The method does, however, require the user or implementation to be able to decide on ranges for some of the independent variables. Furthermore, the distance between each point in the approximation must be carefully chosen.

The inclusion of a pressure drop equation of the flow lines in the model extends earlier work on piecewise linear gas lift performance curves, and allows handling of cases where two or more wells shares a flow line.

The optimization itself was done within reasonable time (about 10 seconds). However, the generation of the lookup tables for the cases studied consumed about a day. Fortunately, generation of new curves is only required when changing geometry of the pipes, reservoir pressure, or temperatures.

The proposed method satisfied the accuracy required for production by being in the range of 1-3 % of the rates predicted by the original model. The accuracy can easily be further improved at the expense of computational load.

Further Work

The proposed method requires a large number of calculations in advance to build pressure tables used in the optimization. Further work should focus on how to reduce this number while maintaining the accuracy of the model.

Currently, the method does not include any rules for defining the bounds on the independent variables used, and the distance used when creating the grid. Such a method should be developed.

The proposed method includes much structure. This structure can be utilized to generate valid inequalities in order to provide tighter bounds for the branch and cut/bound solver.

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Nomenclature

W	Indexes of wells
F	Indexes of flow lines
K_i	Indexes of the points in GLPC for well i
K_i^{o}	Indexes of the points in oil direction for well i
K_i^{lg}	Indexes of the points in lift gas direction for well i
K_i^{g}	Indexes of the points in gas direction for well i
K_i^{w}	Indexes of the points in water direction for well i
K_i^{p}	Indexes of the points in inlet pressure direction for well i
$\lambda_{i,k}$	Weight of point k in GLPC of well i
$\lambda_{i,k^{\mathrm{o}}}$	Weight of $q^{\mathrm{o}}_{i,k^{\mathrm{o}}}$ of well i
$\lambda_{i,k^{\mathrm{lg}}}$	Weight of $q^{\mathrm{lg}}_{i,k^{\mathrm{lg}}}$ of well i
$\lambda^{\mathrm{o}}_{i,k^{\mathrm{o}}}$	Weight of $q^{\mathrm{o}}_{i,k^{\mathrm{o}}}$ of flow line i
$\lambda^{\mathrm{g}}_{i,k^{\mathrm{g}}}$	Weight of $q^{\mathrm{g}}_{i,k^{\mathrm{g}}}$ of flow line i
$\lambda^{\mathrm{w}}_{i,k^{\mathrm{w}}}$	Weight of $q^{\mathrm{w}}_{i,k^{\mathrm{w}}}$ of flow line i
$\lambda^{\mathrm{p}}_{i,k^{\mathrm{p}}}$	Weight of $p^{\mathrm{I}}_{i,k^{\mathrm{p}}}$ of flow line i

$\lambda_{i,k^{o},k^{lg}}$ Weight of point $(q^{o}_{i,k^{o}}, q^{lg}_{i,k^{lg}})$ of well i

$\lambda_{i,k^{o},k^{g},k^{w},k^{p}}$ Weight of point $(q^{o}_{i,k^{o}}, q^{g}_{i,k^{g}}, q^{w}_{i,k^{w}}, p_{i,k^{p}})$ of flow line i

q^{o}_{i} Oil rate for well or flow line i

$q^{o}_{i,k}$ Oil rate for point k in GLPC of well i

q^{lg}_{i} Gas lift rate for well or flow line i

q^{w}_{i} Water rate for well or flow line i

$q^{g}_{i,k^{o},k^{lg}}$ Gas rate for $(q^{o}_{i,k^{o}}, q^{lg}_{i,k^{lg}})$ of well i

$q^{w}_{i,k^{o},k^{lg}}$ Water rate for $(q^{o}_{i,k^{o}}, q^{lg}_{i,k^{lg}})$ of well i

q^{g}_{i} Gas rate for well or flow line i

$q^{gl}_{i,k}$ Gas lift rate for point k in GLPC of well i

$q^{gl,M}$ Maximal available total gas lift rate

q Volumetric flow rate

p_{i} Outlet pressure of well i, with open choke

$p_{i}(\cdot)$ Evaluate outlet pressure of well or flow line i, with open choke

p^{I}_{i} Inlet pressure of well i

p^{O}_{i} Outlet pressure of well i

$p_{i,k^{o},k^{lg}}$ Outlet pressure at $(q^{o}_{i,k^{o}}, q^{lg}_{i,k^{lg}})$ of well i, open choke

$p_{i,k^{o},k^{g},k^{w},k^{p}}$ Outlet pressure at $(q^{o}_{i,k^{o}}, q^{g}_{i,k^{g}}, q^{w}_{i,k^{w}}, p_{i,k^{p}})$ of flow line i, open choke

Ω_{i} Set of wells and/or flow lines at inlet of flow line

i Index of well or flow line

j Index of well or flow line

k Index of point in GLPC

k^{o} Index of point in oil direction

k^{lg} Index of point in gas lift direction

k^{g} Index of point in gas direction

k^{w} Index of point in water direction

k^{p} Index of point in inlet pressure direction

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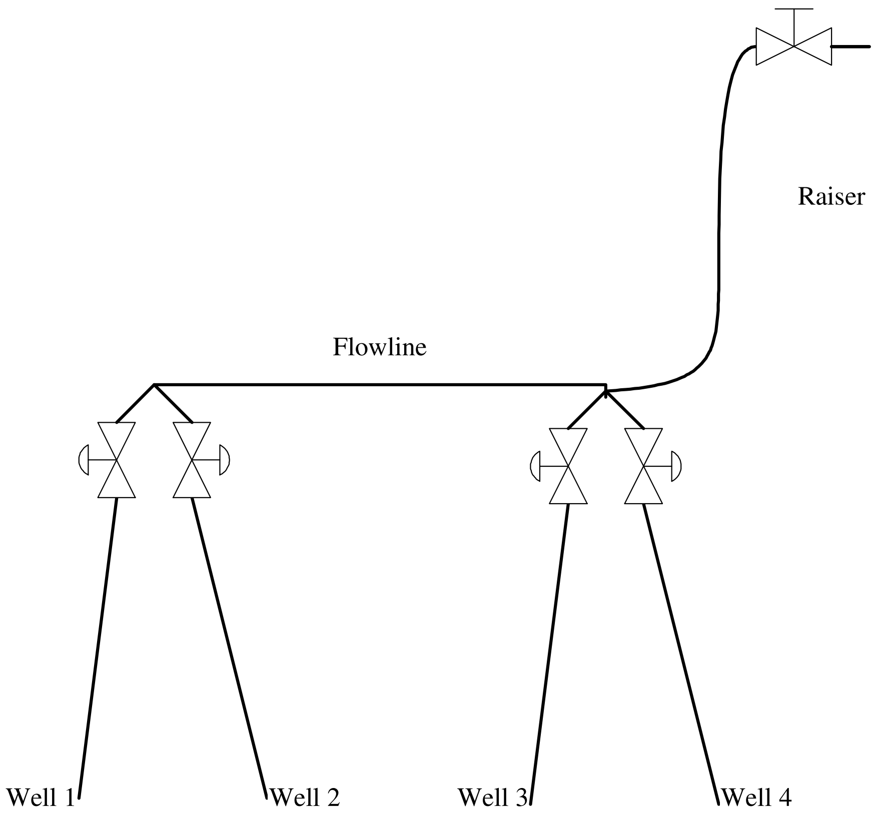

Figure 1. Well topology of field studied.