

# Sensitivity Analysis of Mammalian Circadian Clocks

Henry Mirsky<sup>\*</sup>, Rudiyanto Gunawan<sup>†</sup>, Stephanie Taylor<sup>‡</sup> and Francis J. Doyle III<sup>\*§¶</sup>

<sup>\*</sup>Program in Biomolecular Science and Engineering  
University of California, Santa Barbara  
Santa Barbara, CA 93106

<sup>†</sup>Department of Chemical and Biomolecular Engineering  
National University of Singapore  
Singapore 119077

<sup>‡</sup>Department of Computer Science  
University of California, Santa Barbara  
Santa Barbara, CA 93106

<sup>§</sup>Department of Chemical Engineering  
University of California, Santa Barbara  
Santa Barbara, CA 93106

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R. Gunawan, National University of Singapore

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<sup>¶</sup>Author to whom correspondence should be addressed: frank.doyle@icb.ucsb.edu

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*Henry Mirsky<sup>\*</sup>, Rudiyanto Gunawan<sup>†</sup>, Stephanie Taylor<sup>‡</sup> and Francis J. Doyle III<sup>\*§¶</sup>*

*<sup>\*</sup>Program in Biomolecular Science and Engineering  
University of California, Santa Barbara  
Santa Barbara, CA 93106*

*<sup>†</sup>Department of Chemical and Biomolecular Engineering  
National University of Singapore  
Singapore 119077*

*<sup>‡</sup>Department of Computer Science  
University of California, Santa Barbara  
Santa Barbara, CA 93106*

*<sup>§</sup>Department of Chemical Engineering  
University of California, Santa Barbara  
Santa Barbara, CA 93106*

*<sup>¶</sup>Author to whom correspondence should be addressed: [frank.doyle@icb.ucsb.edu](mailto:frank.doyle@icb.ucsb.edu)*

A circadian clock is a common feature among eukaryotes and an occasional feature among prokaryotes, imparting an ~24 hour period to many biological behaviors at the molecular, tissular, and organismal levels [1]. Some of the molecular aspects of the clock have been elucidated for organisms as diverse as *Neurospora* [2,3], *Arabidopsis* [4], *Drosophila* [5] and mouse [6-8]. In consequence, models of varying complexity, both deterministic [9-17] and stochastic [18,19], have been proposed.

In mammals, the “master clock” is located in the suprachiasmatic nucleus (SCN) of the anterior hypothalamus. This cluster of ~20,000 neurons receives light via the retinohypothalamic tract (RHT) to entrain the clock. The master clock then directs the timing of peripheral oscillators in other parts of the body [6].

Circadian rhythms are known to be robust, in that variations in kinetics (due to, for example, temperature fluctuations) do not appreciably affect performance. Applied to models of circadian rhythm, formal sensitivity analysis provides a tool to assess robustness. The sensitivity of a system to perturbations in parameters is given, for each state and each parameter in the system, as the infinitesimal change in each state with respect to each parameter. The smaller this value, the greater the robustness of this state to perturbations in this parameter. When these values are taken over all states and all parameters, one can determine the relative effect of perturbations in each parameter on the whole system. Sensitivities can then be used to generate the Fisher Information Matrix (FIM) and a rank-ordering of parameter effects on the circadian system.

BioSens software [20], part of BioSPICE, has been used to determine the rank-order of parameter sensitivities on two models of the mouse circadian clock – the 19-state model proposed by Leloup and Goldbeter [12] and the 74-state model proposed by Forger and Peskin [9].

For each model, rank-ordered sensitivities were grouped by parameter type. Parameters were separated into “global” and “local” sets [21]. Global parameters are those that affect many processes in addition to the circadian clock (e.g. transcription rates, translation rates), and local parameters are those that influence only the circadian clock (e.g. phosphorylation of clock proteins). A third category, “mixed” was provided for those parameters that are neither truly global nor local. It was found that for both the Leloup and Goldbeter model and the Forger and Peskin model the circadian system is statistically more sensitive to perturbations in “global” parameters than to perturbations in “mixed” or “local” parameters. This was previously found to be the case for the Leloup and Goldbeter *Drosophila* model [21]. Therefore, the current work extends the previous hypothesis into another phylum.

To further test this conclusion we introduced extraneous noise into selected reactions in the stochastic version of the Forger and Peskin model [18]. It was found that noise introduced into reaction propensities governed by circadian system-sensitive parameters propagates through the system and appears as a change in the distribution of level in miscellaneous clock components. However, noise introduced into reaction propensities governed by circadian system-insensitive parameters does not lead to this effect.

To assess the possibility that noise propagation is due to proximity between the points of noise introduction and the measured states, rather than arising when noise is introduced at highly sensitivity points, we determined the correlation between noise propagation and node distance. We found that no such correlation exists.

An examination of the timecourses of various states indicates that noise functions in the clock by damping oscillations. To achieve an unaltered mean propensity for any given reaction with multiplied lognormal noise, the propensity will either cluster close to zero or be very high. When the propensity is driven high, that reaction channel becomes very likely to “fire.” And if, for example, the reaction is a degradation, then the level of the degraded species will be driven to very low levels.

In addition to explorations of system sensitivities with the nominal parameter set, we performed Monte Carlo assessments of system sensitivities in a sweep of parameter space. It was found that for both the Leloup and Goldbeter model and the Forger and Peskin model, the mean rank-ordered sensitivities to parameter perturbations observed throughout the explored portions of parameter space are well correlated with the rank-ordered sensitivities found in the nominal models. Therefore, the separation of robustnesses and fragilities in the circadian systems is the same for all parameter sets investigated. And thus, the separation of robustnesses and fragilities in the circadian systems are inherent in the hierarchical network structure, not in the choice of parameter sets.

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