Strengthening the continuous time formulation of a mixed plant cyclic scheduling problem

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Extended abstract

The problem addressed is the optimization of the cyclic schedule of a mixed plant in order to maximize its productivity. A mixed plant is composed of batch and continuous tasks. A batch task has a fixed processing time and outputs at the end a fixed amount of product. A continuous task is processed continuously and its decision variable is the processing rate at which this continuous task is performed. Both tasks consume resources whose capacity or availability is limited.

Two types of formulations are typically proposed in the literature to model such scheduling problems as mixed integer programs : the discrete time formulation and the continuous time formulation. Discrete time formulations are based on time intervals of fixed duration and require a large number of small time intervals to model the real problem accurately and obtain realistic solutions. The formulation is typically quite strong (i.e. the duality gap is small) but of very large size. Continuous time formulations have time intervals of variable duration. The number of time intervals is smaller and close to the number of events that really occur and so the formulation is of reduced size, but is usually weak (i.e. the duality gap is large).

The problem studied here has relatively stable product demand which implies that we are interested by an optimal cyclic schedule where the objective is to maximize the long term productivity. In the literature, this cyclic scheduling problem has been modeled by discrete time formulations (see for example Shah et al. [10]) and continuous time formulations (see for example Wu and Ierapetritou [12], Schilling and Pantelides [9]). The paper by Castro et al. [2] models a case study problem with both types of formulations and conclude that the discrete time formulation gives, in a reasonable amount of time, a solution

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of better quality than the continuous time formulation.

Here, we use a continuous time formulation to model the productivity optimization problem for the cyclic scheduling of a mixed plant. In our model, some tasks have to start after some others without any waiting time. In such a case the exact timing of events is crucial and it is not possible to remove the big M constraints as proposed in the paper by Sundaramoorthy and Karimi [11]. Therefore, the continuous time mixed integer linear programming (MILP) formulation becomes weak.

One way to improve the model formulation is to get rid of this big M type of constraints by decomposing the problem into an assignment master problem and a sequencing subproblem. In the paper by Maravelias and Grossmann [6], for example, the assignment of production units to task is modeled by a MILP formulation and the sequencing subproblem is modeled and solved by constraint programming (CP). Another related decomposition approach is proposed by Maravelias in [5]. The assignment problem is also modeled as a mixed integer linear program, but the feasibility check and the deduction of feasible schedules are performed by combinatorial sequencing algorithms.

Another way to improve model formulations with big M type of constraints is to tighten the model formulation. To tighten a formulation in general, we can use strong or facet defining valid inequalities for the problem studied (see Nemhauser and Wolsey [7]). We also can tighten formulations by using an extended space of variables (see Pochet and Wolsey [8]) or by strengthening techniques (see Andersen and Pochet [1]).

Our first contribution in this paper is to show that by using some tightened continuous time formulation, we solve problem instances quicker and with less branch and bound nodes than if we use the initial continuous time formulation. The tightened formulation was obtained by using a combination of strengthening techniques and the analysis of small polytopes, see Christof and Loebel [3], related to the formulation of the scheduling of the batch tasks.

We also tried to improve the model formulation of the continuous part of the problem. The results obtained so far suggest that only one of the valid inequalities found can help to solve the problem instances more rapidly. This deserves further research.

Using the tightened formulation obtained, we detect that for many instances, the CPU solution time and the total number of nodes in the branch and bound tree are drastically reduced. Nevertheless the time needed in some cases to find the optimal solution remains very long because of the difficulty of finding good feasible solutions. Therefore, we pay attention to some MIP heuristic techniques in order to obtain good feasible solutions quickly, while trying to take advantage of the improved formulation obtained. The MIP heuristic methods can be subdivided into two groups. The fist type are the construction heuristic methods that construct feasible solutions from scratch (LP-and-Fix or Cut-and-Fix , Relax-and-Fix, . . .) and the second type are the improvement heuristic methods that try to improve some initial feasible solutions (Relaxation Induced

Neighborhood Search (RINS) , Local Branching (LB) , Exchange (EXCH)). For more details about these MIP heuristic methods, see for instance Pochet and Wolsey [8].

In Kelly and Mann [4], the use of a Relax-and-Fix heuristic is proposed in order to speed up the resolution of production scheduling problems. They showed that with this heuristic, it is possible to find a good feasible solution quickly.

Our second contribution in this paper is to show for some large instances that the heuristic method used, which is a combination of various well-known MIP heuristic methods such as Relax-and-Fix and Local Branching, allows us to find better feasible solutions in the same computing time than exact solution methods.

Keywords : Cyclic scheduling, Tighter continuous time formulation , MIP heuristics

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