Determining Sensor Locations for Stable Nonlinear Systems: The Multiple Sensor Case

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In this work, the sensor location procedure introduced in Singh and Hahn (2005b) for placing a single sensor is extended to the multiple sensor case. The presented technique for sensor location is based on the total response energy of the outputs. In this approach sensors are placed in the system so that a trade-off between the total output energy of the system, sensor cost, and measurement redundancy is achieved. The presented technique results in a mixed-integer nonlinear programming problem when multiple sensors are placed, however, the optimization problem can be reformulated to significantly reduce the computational burden required for its solution. The resulting optimization problem is ideally suited for solution by genetic algorithms (GA) as only binary variables need to be used for the optimization after the problem has been reformulated. The technique has been illustrated by applying it to a distillation column where up to 6 temperature sensors are placed along the height of the column.

Keywords: Sensor placement, Nonlinear system, State estimation.

1.0 Introduction

Various techniques have been proposed in the past for sensor placement that take into account different performance criteria for sensor placement (Bagajewicz, 2001; Muske and Georgakis, 2003; Muslin et al., 2005), like system observability or redundancy of measurements. However, most of these techniques are based on linear or linearized systems and only a few contributions for sensor location for nonlinear systems exist. These include the work by Wouwer et al. (2000) and Alonso et al. (2004) for distributed systems and Georges' work (1995) involving nonlinear observability functions. However, the application of the latter approach is limited to low-dimensional systems due to the high computational burden. An alternative is presented in the work by Singh and Hahn (2005b) for placing a single sensor on a system.

This work extends the technique shown in Singh and Hahn (2005b) to the case where multiple sensors are placed. The concept of output energy plays a fundamental role for the present approach and will be explained in detail. The output energy of a linear system corresponds to the singular values of the observability gramian and, accordingly, the total output energy is given by the trace of the observability gramian (Fairman, 1998). Similarly, the output energy of nonlinear systems can be computed by using empirical observability gramian. These empirical observability gramians can be viewed as extension of linear observability gramians to nonlinear systems and can, therefore, be used for the observability analysis of nonlinear system over an operating region (Singh and Hahn, 2005a). The total output energy of the system is equal to the sum of the diagonal elements of the empirical observability gramian. Therefore the key idea behind achieving a high degree of observability is to maximize the trace of the empirical observability gramian matrix.

Measurement redundancy is also a desirable goal for sensor placement in addition to increasing observability of a system. Redundancy in a measurement refers to the ability to be able to make statements about a state variable even if a sensor fails. The presented sensor location technique incorporates the concept of redundancy and sensors can be placed in the system so that a desirable amount of redundancy for state variables is present in the sensor network.

A mixed integer nonlinear optimization problem has been formulated to take into account the total sensor cost, process output energy, and measurement redundancy. Sensors are placed in the system where a trade-off between process information, sensor cost, and information redundancy is taken into account. The optimization problem is reformulated such that it decomposes into a problem which is significantly less computationally demanding. This is achieved by performing the main steps of the observability and redundancy analysis outside of the optimization loop. The resulting problem is in a form which makes it very suitable for solution by a genetic algorithm as optimization variables correspond to possible locations of the sensors and each vector entry contains information in a binary form. As such a GA can be directly applied without having to discretize the solution space (Goldberg, 1989).

The presented technique has been applied to a nonlinear binary distillation column. The locations of temperature sensors have been computed for monitoring the distillation column subject to an upper bound on the number of sensors. Sensor location of up to six temperature sensors has been investigated in the presented work.

2.0 Placing multiple sensors on a nonlinear system

In this work a new technique for sensor location is presented that is based on maximizing an objective function representing a trade-off between process information obtained from the outputs, measurement cost, and information redundancy. The optimization problem for determining optimal sensor locations is given by:

$$\min_{x_i, \quad i=1, \quad \ldots, \quad n} J$$
$$\text{s.t.} \quad \sum_{i=1}^{n} x_i \le a, \quad x_i = 0,1 \quad \forall i \in \{1, \quad \ldots, \quad n\} \tag{1}$$

where a is the largest possible number of sensors to be placed, and $x_i = 1$ if state i is measured and $x_i = 0$ if no sensor is placed for measuring state i. The objective function for this work is given as follows:

$$J = \begin{pmatrix} Measurement \\ Cost \end{pmatrix} - \begin{pmatrix} Process \\ Information \end{pmatrix} - \left(Redundancy \right) \tag{2}$$

The objective is to determine an optimal trade-off between minimizing the installation cost of the sensors, trying to maximize the information extracted from the system, and maximizing information redundancy. It is also possible to include the inequality constraint from equation (1) into the objective function of the system via a penalty term or by weighting the measurement cost. The resulting optimization problem from equations (1) and (2) is then given by:

$$\min_{x_i,\ i=1,\ldots,n}\left[\alpha\sum_{i=1}^{n}C_i x_i-(1-\beta)\sum_{j=1}^{n}\max_{i=1,..,n}\left(x_i\left(W_{O,jj}\right)_i\right)-\beta\sum_{i=1}^{n}x_i\left(\sum_{j=1}^{n}W_{O,jj}\right)_i\right] \quad (3)$$

$$\text{s.t.}\quad \sum_{i=1}^{n}x_i\le a,\ \ x_i=0,1\quad \forall i\in\{1,\ \ldots,\ n\}$$

The optimization problem give by equation (3) is a mixed integer nonlinear programming (MINLP) problem that can be computationally expensive to solve if it is not properly formulated. In order to solve the optimal sensor location problem, the optimization problem given by equation (3) has been reformulated. In the reformulated version, most of the observability computation is carried out outside of the optimization loop such that only an explicit linear expression has to be repeatedly solved during the optimization to determine the empirical observability gramians for any combination of sensors.

The resulting problem is in a form that makes it well suited for genetic algorithms. In the presented optimization problem, the decision variables correspond to possible locations of the sensors and each decision variable is in binary form. Each decision variable is represented by a single binary bit, called gene, and the number of genes in each chromosome is equal to the total number of available sensor locations or decision variables. Due to the binary structure of the presented optimization problem, a GA can be directly applied without having to discretize the solution space.

3.0 Results

The presented technique is illustrated by applying it to a nonlinear distillation column model (Singh and Hahn, 2005b) where up to six sensors are placed along the height of the column. The column model is described by a set of 32 nonlinear ordinary differential equations with temperatures as state variables and 33 explicit algebraic equations.

Temperature sensors have been placed such that a trade-off between process information and sensor cost can be determined. Redundancy, has not been considered for this example to make the results more illustrative ($\beta=0$ for equation (3)). Neglecting the condenser and the reboiler for this analysis, there are thirty possible locations for sensor placement in the column, as the temperature on each tray can be measured. A series of problems has been formulated in the following with an increasing number of sensors. As only temperature sensors have been considered in the presented cases each sensor is assumed to have the same cost.

If only one temperature sensor is to be placed on the column, then the sensor placement problem reduces to maximizing the trace of the empirical observability gramian. The values of the trace for varying measurement locations are shown in Figure 1. The best location corresponds to the largest value of the measure, which can be found for measuring the temperature on the sixth tray. In addition to providing information about the best sensor location, it is also possible to determine approximately how much each individual state contributed to the measure. This is illustrated in Figure 2 where the values of the diagonal elements of the empirical observability gramian are plotted over the tray number. It should be noted that the dark area in Figure 2 corresponds to the value of the measure shown in Figure 1 for placing a sensor at the 6^{th} tray.

If two sensors are to be placed then the objective function given by equation (3) is minimized using a GA. The best locations are determined to be the 6^{th} and the 25^{th} tray (Figure 3). Figure 3 can also illustrate the concepts of new state information vs. information redundancy. The sum of the dark and lightly shaded area represents the information that can be extracted from the two measurements. The lightly shaded area corresponds to the redundant information as it represents the area where the contribution of the states to the two individual measurements overlap. However, this area is only counted once for the optimization problem formulated in equation (5). For the case of six sensors, the optimal locations are at the 2^{nd}, 4^{th}, 6^{th}, 8^{th}, 24^{th}, 26^{th} tray (Figure 4). In addition to solution by GA, the global optimum has also been determined by techniques searching through the entire solution space. The presented results were indeed found to be the global solution to the problem..

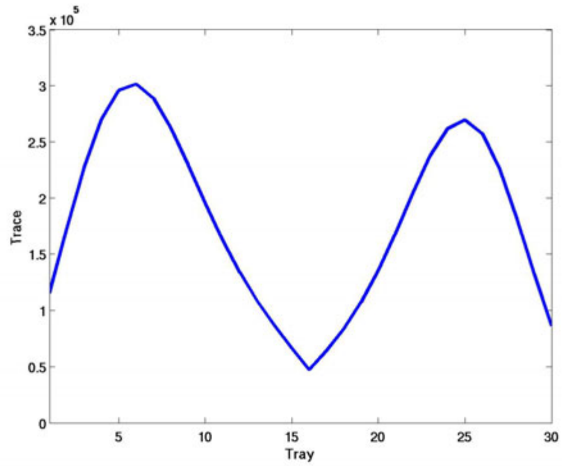

Figure 1: Measure for placing one sensor on the distillation column

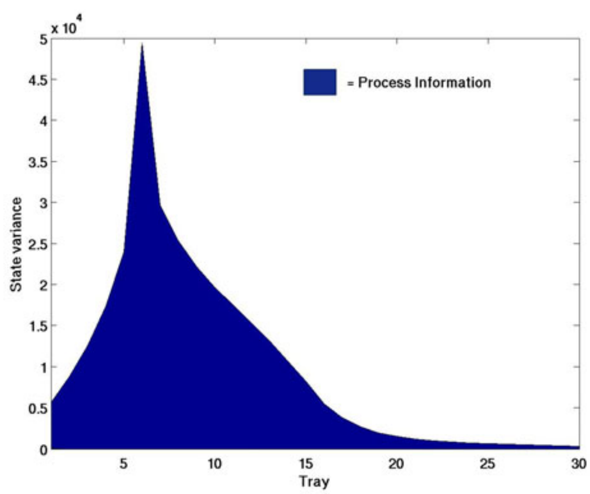

Figure 2: Contribution of individual states to observability for a sensor placed at the 6^{th} tray

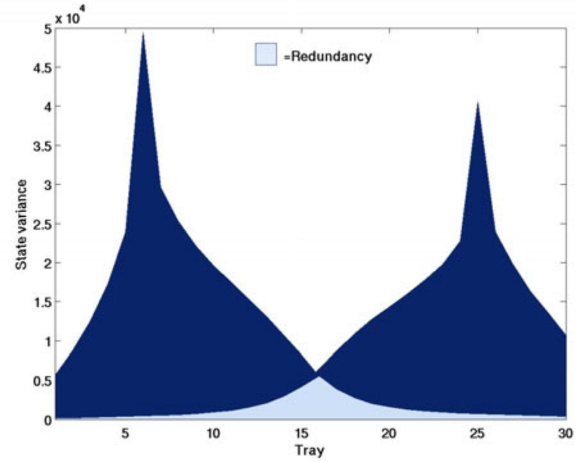

Figure 3: The area presents the value of the objective function for measurements at the 6th tray and the 25th tray

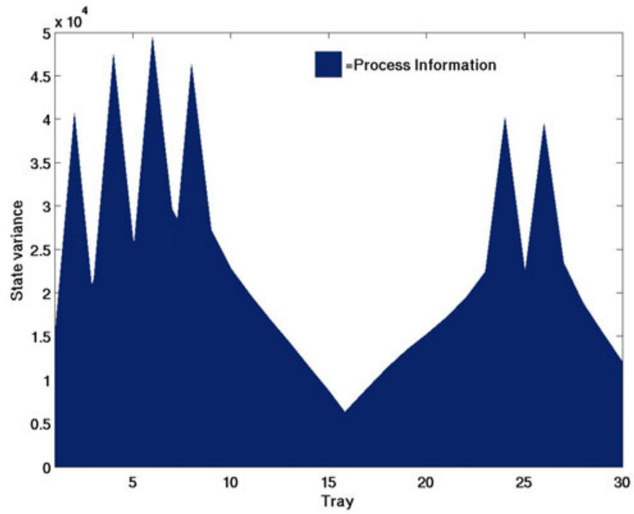

Figure 4: Six sensors located at the 2nd,4th, 6th,8th, 24th, and 26th tray

4.0 Conclusion

This paper presents a new technique for sensor location for stable nonlinear systems. Special emphasis is placed on the aspect that the system can be nonlinear and on properties arising from placing multiple sensors, e.g. sensor cost and measurement redundancy. The nonlinearity is taken into account by using empirical gramians, rather than gramians of a linearized system, while the issues of sensor cost and information redundancy are addressed via the formulation of an optimization problem.

The presented technique can easily handle sensor networks of a size of interest to realistic processes. This has been illustrated by applying the technique to 30-tray nonlinear distillation column where up to six sensors were placed.

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