89f Interval Analysis of Ode Systems with Parametric Uncertainty

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Standard numerical methods for initial value problems (IVP's) for ordinary differential equations (ODE's) compute an approximate solution that satisfies a user-specified tolerance. Unfortunately, the solutions are not validated. There are situations when guaranteed bounds or enclosure of the true solution are desired or needed from safety or reliability point of view. In this presentation we discuss a collection of methods (under the umbrella of VNODE) which provide validated solutions for ODE/IVP's. The methods produce a guaranteed bound for the true solution and also verify that a unique solution to the problem exists. Many ODEs arising in practical applications contain parameters estimated from experimental data. Mindful of this, we have employed VNODE to study systems for which there are parameter uncertainties. We have also studied cases where the initial conditions are uncertain. Specifically, we have considered the Bergman minimal model for glucose and insulin time course in the body. The model consists of three ODEs for glucose concentration, active insulin concentration and plasma insulin concentration. Overall, the model has 5 estimated parameters (3 parameters for glucose kinetics and 2 parameters for insulin kinetics). Different subsets of the parameters were regarded as interval-valued, while the remainder was treated as being crisp intervals. The results show that p1 (insulin independent rate constant of glucose in muscles and liver) is the most important parameter in glucose kinetics and Gamma (rate of insulin release after the glucose injection) is the important parameter in insulin kinetics. Some of the parameter combinations above led to runaway kinetics. We are currently trying to understand the biological significance of these results assuming the kinetic model accurately reflects the true kinetics.