A Homemade Sliderule as a Manipulative Study Aid for Chemical Reactor Design

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Abstract

In this paper, we demonstrate the construction and use of a simple slide rule, and demonstrate its application as a manipulative aid intended for studying chemical reactor design. The slide rule is simple enough that it can easily be built by a student or a teacher, and designed to represent any form of chemical kinetics. In this paper, we demonstrate the construction and use of a slide rule for first order irreversible decomposition reaction in a batch, CSTR and PFR reactor.

Introduction

The study of engineering is strongly coupled to the study of mathematics. To the extent that we can facilitate learning of mathematical concepts, we can facilitate the learning of engineering concepts. A large body of education literature has established a substantial relationship between the use of manipulative materials and student's achievements in the classroom, particularly at the primary through high school level.[1] The major rationale for the use of manipulative or concrete operations can be attributed to Piaget,[2] Bruner,[3-4] and Dienes.[5-7] The main idea is that at early stages of elementary learning, concrete operations facilitate the development of abstract thinking. The concept goes beyond the use of practical examples that we typically use in the engineering classroom, and refers specifically to devices that can be manipulated with the hands. As children progress into adulthood, their need for concrete examples is somewhat reduced due to the development of more advanced learning schemas, but this dependence is not eliminated.[1] The kinds of thought possesses that characterize the use of concrete operations are thought to be utilized at all developmental levels.[2]

Chemical engineering educational methods contain many examples of learning devices that can be categorized as manipulative in nature. Many graphical solution methods were developed early in the last century, more out of necessity that any desire to improve learning strategies. The McCabe-Thiele graphical solution of the binary distillation problem is perhaps the most familiar and widely used method.[8] Another widely used technique is the Hunter-Nash-Kinney method for solving liquid-liquid extraction problems.[9-10] While reliable CAD software has been available for at least twenty years, it is still very common for students to be using these methods in the classroom. The reason for this is that simultaneous equilibrium and mass balance equations can be visualized in graphical solutions. Another reason is that the students learn the concepts better or more efficiently.

Another example of a manipulative device is the slide rule. For almost 350 years, the slide rule was perhaps the most important calculating tool available to the scientist and engineer.[11] There is a natural curiosity on the part of many engineering students to learn about how they work. Between 1965 and 1975, the slide rule as a computing tool was almost entirely replaced in the US by the electronic calculator.[12] There were many reasons for this, including cost, ease of use, and precision. The modern scientific calculator is inexpensive, and has facilities for solving differential equations, integrals, plotting, and spreadsheet operations. Certainly computing horsepower continues to become more portable, and many students have access to palm-top or laptop computers that are routinely brought to the classroom. However, given the increase in computing power that is now available, construction of manipulative devices such as slide rules is fairly straightforward, and this is what we shall show here. The end product, at the very least, is another graphical form for presentation of kinetic data. In the extreme, the working slide rule can be easily transformed into a concrete, handheld device that can be used as a working manipulative.

How a Slide Rule Works

Before getting into the details of our specific application, we first outline the basic operating principles. There are essentially two reasons that conventional slide rules work. First, adding any two numbers is equivalent to adding distances. Second, multiplying numbers is equivalent to adding logarithms. We illustrate each of these two points with examples.

In the first example, we demonstrate a simple addition problem using two linear scales, shown in Figure 1a. The top scale in the figure is the mirror image of the bottom scale. Each scale is numbered from 0 to 11. Figure 1b shows how the scales are used to solve the simple addition problem three plus seven. The arrow on the lower left side of Figure 1b points to the three on the lower scale. The left edge of the upper scale is aligned with the three on the lower scale. The arrow on the upper right side of the figure points to the 7 on the upper scale. These are the two numbers we are trying to add. The answer is then read from the lower scale directly below 7, namely, 10 (also indicated with an arrow on the lower right in the figure).

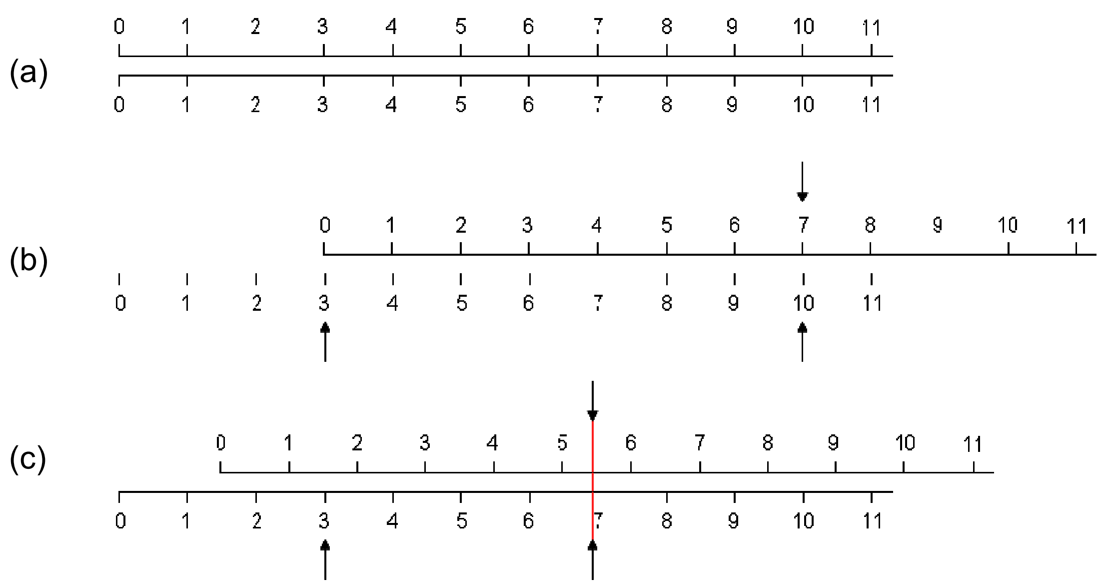


Figure 1. Addition Scales. (a) Two mirror image linear scales. (b) Scales used to solve “3+7=10.” (c) Scales used to solve “1.4+5.5=6.9.”

Larger problems can be accommodated by using multiples of ten. The scales in Figure 1 might also represent 0 to 110, in which case Figure 1b shows 30+70=100. Also note that fractions can be added by estimating between the tick marks. Figure 1c shows the addition of 1.4 + 5.5 to obtain 6.9. The right arrows in this case are emphasized with a dashed line “crosshair” to facilitate estimating the answer. Fractional precision is limited by our ability to visualize the distances between the tick marks. Clearly 14.01 is inappropriate estimate, but 14.1 might be achievable with a good eye and some extra tick marks. Subtractions are just the inverse of additions, and we run the problem backwards. For example, in Figure 2b, moving in sequence from the lower right arrow to the upper right arrow to the lower left arrow would represent 10-7=3.

Multiplications are performed by adding logarithms. Figure 2a shows two logarithmic scales. The positions of the tick marks are given by the log of the numbers above the marks. As before, the top scale is just the mirror image of the bottom scale. Figure 2b shows a simple multiplication. The arrow at the lower left of Figure 2b points to 3. The arrow on the upper right points to 7. The dotted line cross hair crosses at 21, which is the product of 3 and 7, also indicated by the arrow at the lower right. The inverse operation, division, is accomplished by working in reverse. As another example, Figure 2c shows a sample problem involving fractions, namely 2.1x5.5=11.6. As before, larger numbers are handled by scaling.

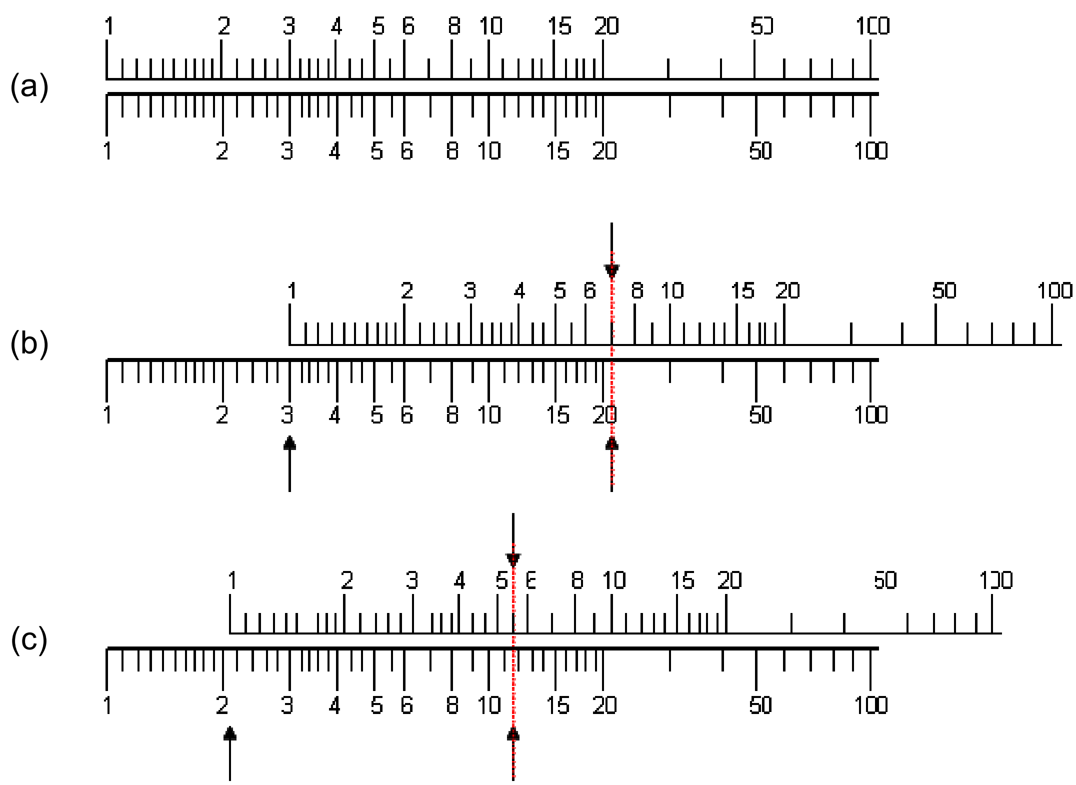


Figure 2. Multiplication Scales. (a) Two mirror image log scales. (b) Scales used to solve “3×7=21.” (c) Scales used to solve “2.1×5.5=11.6.”

Construction Methods

To construct a slide rule scale, the positions of the tick marks must be carefully laid out on the page. There are some choices here, depending upon available resources. The simplest addition scales can be made with a piece of paper, a ruler, and a pencil. The log scales can be drawn by hand using a calculator or a log table to determine the logs, although this work is greatly simplified with a computer and a spreadsheet. In this work, PowerPoint was used to draw the scales and Excel was used to calculate the logs. PowerPoint has the advantage of allowing us to set the size and position of the tick mark to within 0.01 inches. Similar freeware such as OpenOffice can also be used by Microsoft or Linux users. In PowerPoint, start by drawing a short line, double click on the line, and then click the position tab. You can then set the horizontal and vertical positions. For a complete scale, the vertical positions of all of the tick marks are held constant, and the horizontal position is scaled to the log of the number that the tick mark represents.

Start by deciding the numerical values of the tick marks, and take the logarithm of these numbers. The logs are then placed at the appropriate position on the paper. In our case, we would like the left-most tick mark to appear at 1.64 inches from the left edge of the paper, and the right most tick mark appears at 6.62 inches, where the dimensions were chosen arbitrarily. Using the scales in Figure 2 as an example, the smallest number is 1 and the largest is 100. The scale factor for converting logarithms to inches is then just (6.22-1.64)/($\log_{10}$(100)-$\log_{10}$(1))=2.29. The distance to the edge of the paper needs to be added as a constant. To facilitate the procedure, we have written the mapping in equation form as Equation 1 below:

$$\text{tick position} = f(x) = c + x \cdot \frac{d - c}{\log_{10}(b) - \log_{10}(a)} = 1.64 + 2.29x \qquad (1)$$

In this equation, x is the $\log_{10}$ of the tick mark value, d is the right-most tick position on the paper, c is the left most position, a is the numerical value of the left-most tick, and b is the numerical value of the right-most tick.

Construction of Kinetics Slide Rules

In the typical undergraduate chemical engineering reactor design course, the approach that is widely is to make a plot of the inverse of the reaction rate against the fractional conversion.[13] This results in a plot of f(x) versus x, where f(x) = 1/r(x), and r(x) is the rate of the chemical reaction as a function of fractional conversion x. This reciprocal-rate or inverse-rate graph is then used to scale the volume of the chemical reactor. The form of this plot is a completely natural consequence of the mass balance equation that describes the reactor. The space-time of a CSTR, for example, is just the product of x and f(x) at some specified x. The space-time of a PFR is just the integral of f(x) with respect to x from 0 to x. To build a slide rule, one only needs to recognize that the kinetic data can also be graphed in one dimension, as linear distances on a piece of paper.

We will consider as an example a first-order irreversible chemical reaction of the form A→ products. We will also assume that the reaction is occurring in a constant density medium, such as an aqueous solution, in a mixed flow (CSTR). For the special case of constant density systems, the space-time is given by the mole balance equation:

$$\tau = \frac{1}{k}\frac{X_A}{1-X_A} \qquad (2)$$

The space-time τ is the volume of the reactor divided by the volumetric flow rate of fluid moving through the reactor. The fractional conversion of A is X_A, and is the moles of A reacted divided by the moles of A fed. The first-order rate constant is k. Taking the log of both sides produces

$$\log(\tau) = \log\left(\frac{1}{k}\right) + \log\left(\frac{X_A}{1-X_A}\right) = \log(a) + \log(b) \qquad (3)$$

Thus, finding the space-time is equivalent to multiplying two numbers, which is equivalent to adding the logarithms. To construct a slide rule, we then just construct a table of 1/k and x/(1-x) values, and taking the log of both sets of data, and plotting them in the manner shown above.

The resulting scales for this example are shown in Figure 3. The top scale is set to line up with .10 on the bottom scale, which corresponds to 1/k. For a conversion of say .90, we can see that the resulting space-time is 90. Since the upper scale begins with 0.01, the answer must be multiplied by this quantity, resulting in a final answer of .90. We could have just as easily started the scale with 1 and used percent conversion. The choice is arbitrary and is only dependent on the desired scaling. The volume of the reactor is readily obtained by multiplying the answer by the volumetric flow rate.

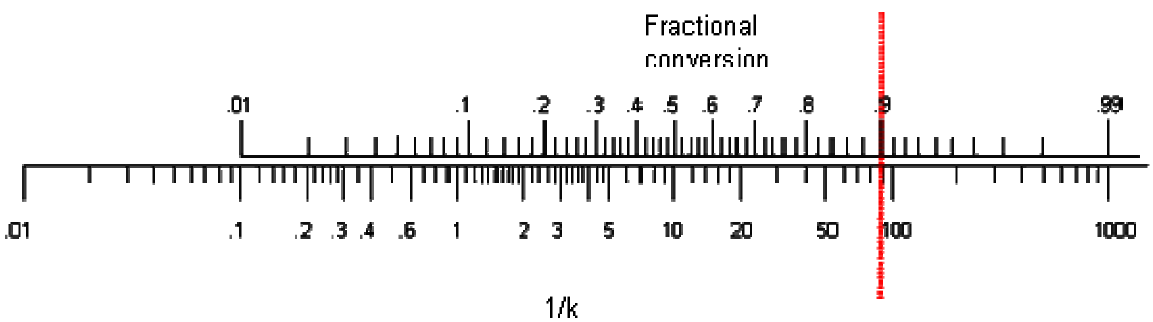
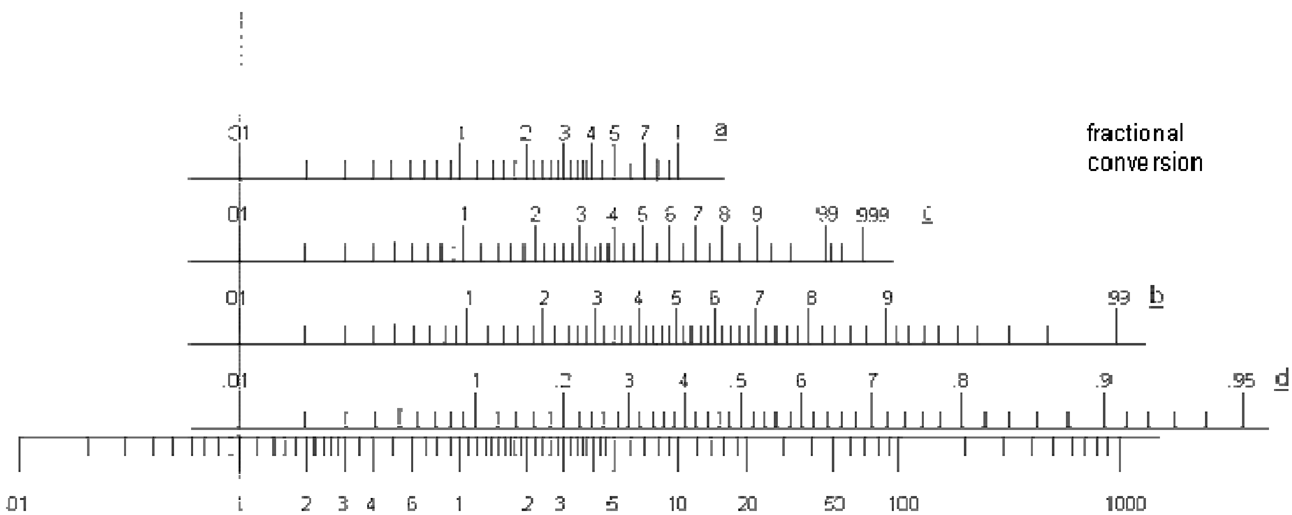


Figure 3. First-Order CSTR slide rule, used to solve for the space time at 90% conversion when 1/k=.1.

Figure 4 shows a slide rule constructed from four different reactor design equations. Each of the four upper scales corresponds to one of the three basic chemical reactor design equations, batch, CSTR, or PFR. Note that some of the design equations are identical in form but differ in terms of whether time or space-time is used as the design variable. We have chosen zero, first, and second-order irreversible kinetics, although in principle, any kinetic form can be represented. Thus, nine different cases are covered by this combined slide rule, which would be equivalent to making nine reciprocal rate plots. Which scale to use is indicated in the upper table in the figure, and the corresponding characteristic reaction time is selected from the lower table. Reading from the top scale, as an example, with 1/k=.1, at a fractional conversion of 0.73, a space time of ~.79 results in a CSTR or PFR. In the batch reactor, this would be the reaction time.

$A \rightarrow$ products

	batch	CSTR	PFR
0th order	a	a	a
1st order	c	b	c
2nd order	c	d	c



	batch	CSTR	PFR
0th order	C_{A0}/k	C_{A0}/k	C_{A0}/k
1st order	$1/k$	$1/k$	$1/k$
2nd order	$1/kC_{A0}$	k/C_{A0}	$1/kC_{A0}$

Figure 4. Basic Reactor Slide Rule Scales. Scales are lettered a-d, and the letter corresponds to the kinetics and reactor configuration, as indicated in the upper table. The lower scale represents the characteristic time scale, as indicated in the lower table.

The utility of the device in educational terms is in the representation of the kinetics/reactor system as a concrete manipulative device. That is, the scales can be mounted on a physical device which is held in the hands and manipulated like a traditional slide rule, as shown in Figure 5 below. Alternatively, the scales can be embedded into digital media to construct a “virtual manipulative.” As for whether this methodology will enhance learning of reactor design theory, we have no data at this time, and this will be the subject of a future study. However, initial student reaction has been very positive, and we have used this device as an exercise for outside of the classroom in student projects.

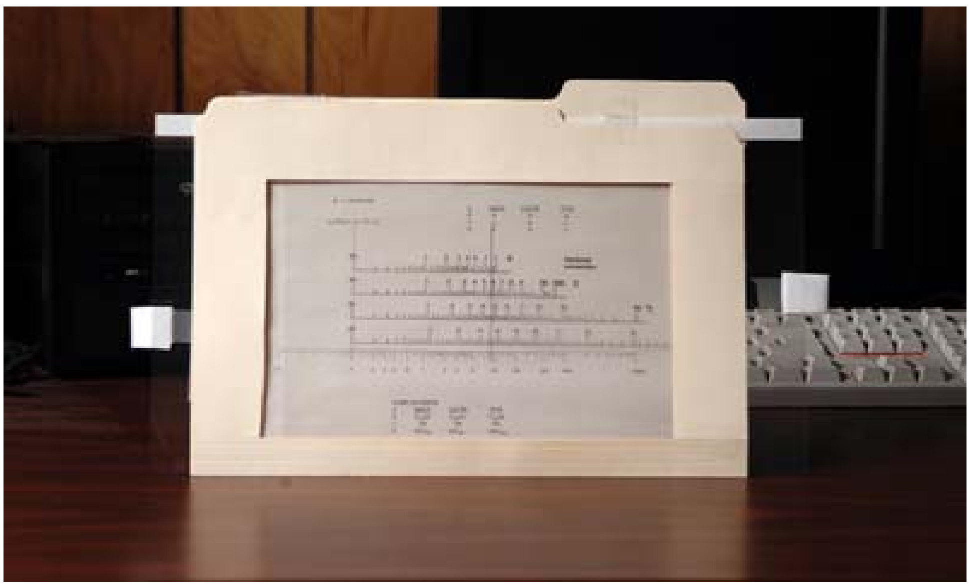


Figure 5. A prototype slide rule based on Figure 4. Movable scales are made with transparency film, and the slides are held together in a file folder.

Use of Multimedia

The use of drawing programs such as PowerPoint to lay out the scales provides a great deal of flexibility for designing multimedia or web applications, or classroom presentations. Simple “grouping” of the tick marks allows one to slide a scale back and forth using the mouse or arrow keys. This allows for very rapid scale manipulations in front of the class. When doing this, the arrow keys are excellent since they allow motion in only one dimension. The scales can also be saved as a bitmap or jpeg image and used to construct virtual slide rules for web applications. Also, plotting functions in Excel can also be used to make plots of tick marks directly, without the need to manipulate tick marks one at a time.

Customized Use for Industrial Reaction Kinetics

A small hand-held tool could be useful for a plant engineer who wants to know an instantaneous estimate of reactor performance, as a “rule of thumb” measurement. A device such as that proposed here can easily be customized for a wide range of chemical kinetics and reactor configurations. We have constructed rules for reversible kinetics, variable volume, recycle, and variable temperature systems. All that is required is that one knows the specific kinetic parameters for the reaction of interest. The specific kinetic parameters for the system of interest can then be plotted directly onto a slide rule scale in the manner shown above.

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