Planning under Correlated and Truncated Price and Demand Uncertainties

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ABSTRACT

Until now, most research work on uncertainty assumes that the demand and price are independent because of the difficulty in computing the bivariate integral originated from the correlated demand and price. This can cause significant discrepancies in revenue calculation and hence yield sub-optimal planning strategies. This paper presents a novel approach to handle correlated and truncated demand and price uncertainties. To compute the expectation of plant revenue, which is the main difficulty for a planning problem under uncertainty, we use a bivariate normal distribution to describe demand and price. The double integral for revenue calculation is reduced to several single integrals after detailed derivation. The unintegrable standard normal cumulative distribution function in the single integrals is approximated by polynomial function. Case studies show that, for a large enough CV of a product, assuming independent price and demand may underestimate the revenue by up to 20%. Since the real world demands or prices vary in limited ranges, integrating over the whole range of a normal distribution, which some research has done, may give incorrect results. This paper thus approximates a bivariate double-truncated normal distribution for demand and price. The influence of degree of truncation on plant revenue is studied.

To handle possible unmet customer demands, the hard-to-specify penalty functions of the two-stage programming are avoided and replaced by two of the decision maker's service objectives, namely the confidence level and fill rate objective. Confidence level or the type I service level, which is the probability of satisfying customer demands, is commonly used in chance-constrained programming. However, fill rate or the type II service level, which is the proportion of demands that are met from a plant, is a greater concern of most managers. In this paper, fill rate is efficiently calculated using the derived formulae and the maximal plant profit that satisfies certain fill rate objectives can thus be obtained. Case studies show that a planning strategy that satisfies certain confidence level objectives might be too generous compared to a strategy that satisfies a fill rate objective. Case studies including refinery planning problems were used to illustrate the proposed approach. The proposed approach can be generally applied for modeling other chemical plants under uncertainty.

1. INTRODUCTION

Due to the fluctuating product demands, volatile raw material prices, and other changing market conditions, many parameters in a planning/scheduling model are uncertain. Upon realization of uncertainty, a schedule built on a deterministic approach will be non-robust or in some cases even infeasible[1,2]. Since Dantzig's seminal work on uncertainty appeared[3], research on uncertainty has been attracting the attention of numerous researchers.

Most research work[4,5] on uncertainty assumes that the demand and price are independent

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because of the difficulty in computing the bivariate integral originated from the correlated demand and price. However, by regressing real world demand and price data from EIA (the U.S. Energy Information Administration) [6], the correlation coefficient between gasoline (New York Harbor Gasoline Regular) price and its demand is 0.44 for the year 2003 to 2004. For world crude oil in 2003 and 2004, the correlation coefficient is 0.30. These data show that the demand and price are far from independent. Considering the correlation between the price and demand and studying its influences on plant revenue is the main concern to be addressed in this paper.

2. Revenue Calculation Methods

The computation of the revenue of a plant involves uncertain variables such as market demand and product price. How to compute the expectations of uncertain functions introduced by these uncertain variables generates the main difficulty in stochastic rogramming[7]. Several approaches have been used in the literature to compute these expectations[8]. Li et al[5] categorized different revenue calculation approaches into three types which include: A) Minimizing cost. The objective is to minimize the total costs and the computation of plant revenue is avoided. B). Maximizing profit I. The revenue is calculated by the product of the market price and the amount of the product produced by the plant. In this approach, it is assumed that a product can always be absorbed by the market. C). Maximizing profit II. The revenue is calculated by the product of the market price and the market demand. In this approach, it is assumed that the amount of a product is always greater than the market demand. However, the assumptions in B) and C) are not always true in the real world. As pointed out by Petkov et al[4] and Li et al[5], in many cases, if the market demand is less than the product amount, only part of the product can be sold; otherwise if the market demand is higher than the product amount, then only part of the demand can be satisfied. The revenue should then be calculated by:

$$\text{Revenue} = E[\sum_{c}\sum_{x} C * \min(P, x)] \qquad (1)$$

where C is the price, P is the production rate of the product and x is the demand.

3. Derivation of Revenue Calculation for Correlated price and demand

The derivations in most of the works in the literature[4,5,9] are based on the assumption that the demand and price are independent and the price is assumed to be a constant. This assumption may introduce significant discrepancy in revenue computation due to correlated price and demand for a real world plant. In this section, the formulae for plant revenue computation considering the correlation between price and demand are derived.

In stead of a constant, the price is assumed to conform a normal distribution. The price, c, and demand, x, is further assumed to conform a two-dimensional normal distribution whose probability density function is represented by:

$$\varphi(c,x)=\frac{1}{2\pi\sigma_c\sigma_x\sqrt{1-\rho^2}}e^{\frac{-1}{2(1-\rho^2)}[\frac{(c-\bar{c})^2}{\sigma_c^2}-\frac{2\rho(c-\bar{c})(x-\theta)}{\sigma_c\sigma_x}+\frac{(x-\theta)^2}{\sigma_x^2}]} \qquad (2)$$

where, $\bar{c}$ is the mean of price, σ_c is the standard deviation of price. θ is the mean of demand and σ_x is the standard deviation of demand and ρ is the correlation coefficient. The normally distributed price and demand are independent if $\rho=0$.

Combining eqs (1) and (2), the revenue is $\int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{P} x*c*\varphi(c,x)dxdc$ if $x \le P$ and $\int_{c=-\infty}^{+\infty}\int_{x=P}^{+\infty} P*c*\varphi(c,x)dxdc$ if $x > P$. Then, eq (1) becomes

$$\begin{aligned}\text{Revenue} &= \int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{P} x*c*\varphi(c,x)dxdc+\int_{c=-\infty}^{+\infty}\int_{x=P}^{+\infty} P*c*\varphi(c,x)dxdc \\ &= \int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{P} x*c*\varphi(c,x)dxdc+\int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{+\infty} P*c*\varphi(c,x)dxdc-\int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{P} P*c*\varphi(c,x)dxdc \\ &= A + B - C\end{aligned}$$

Where,

$$A=\int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{P} x*c*\varphi(c,x)dxdc\,,$$

$$B=\int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{+\infty} P*c*\varphi(c,x)dxdc$$

$$C=\int_{c=-\infty}^{+\infty}\int_{x=-\infty}^{P} P*c*\varphi(c,x)dxdc\,.$$

After detailed derivation, A can be reduced to A1 to A5 and C can be reduced to C1 and C2. The revenue is computed by

$$\text{Revenue} = A1 + A2 + A3 + A4 + A5 + B - C1 - C2 \qquad (3)$$

Where,

$$A1=\frac{-\sqrt{1-\rho^2}\sigma_x\sigma_c}{2\pi}\int_{m=-\infty}^{\infty} me^{-\frac{m^2}{2}}e^{-\frac{U^2}{2}}dm$$

$$A2=\frac{-\sqrt{1-\rho^2}\sigma_x\bar{c}}{2\pi}\int_{m=-\infty}^{\infty} e^{-\frac{m^2}{2}}e^{-\frac{U^2}{2}}dm$$

$$A3 = \frac{\rho\sigma_x\sigma_c}{\sqrt{2\pi}} \int_{m=-\infty}^{\infty} m^2 e^{-\frac{m^2}{2}} \Phi(U)dm$$

$$A4 = \frac{\sigma_c\theta + \rho\bar{c}\sigma_x}{\sqrt{2\pi}} \int_{m=-\infty}^{\infty} me^{-\frac{m^2}{2}} \Phi(U)dm$$

$$A5 = \frac{\bar{c}\theta}{\sqrt{2\pi}} \int_{m=-\infty}^{\infty} e^{-\frac{m^2}{2}} \Phi(U)dm$$

$$(B) = P\bar{c}$$

$$C1 = \frac{P\bar{c}}{\sqrt{2\pi}} \int_{m=-\infty}^{+\infty} e^{-\frac{m^2}{2}} \Phi(U)dm$$

$$C2 = \frac{P\sigma_c}{\sqrt{2\pi}} \int_{m=-\infty}^{+\infty} me^{-\frac{m^2}{2}} \Phi(U)dm$$

$$m = \frac{c - \bar{c}}{\sigma_c}$$

$$U = \frac{\frac{P-\theta}{\sigma_x} - \rho m}{\sqrt{1-\rho^2}}$$

In the above equations,$\Phi(.)$ is the standard normal cumulative function:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-t^2}{2}} dt$$

4. Derivation of Revenue Calculation for Correlated and Truncated price and demand

Besides the independence assumption, in the derivation of the works in the literature[4,5], the integration ranges of price and demand are assumed to be (-∞, +∞), which is not the case in the real world. This may bring further discrepancy in revenue computation. To handle this, the formulae for truncated price and demand are derived in this paper and the influence of degree of truncation on revenue computation is studied using some case studies.

Now suppose that the range of demand is $[X_L, X_U]$, where $-\infty < X_L < X_U < +\infty$ and the range of price is $[C_L, C_U]$, where $-\infty < C_L < C_U < +\infty$. Then the pdf (probability density function) of BBTN (Bivariate Bi-Truncated Normal distribution) is:

$$f_{BBTN}(c,x) = \begin{cases} \frac{\varphi(c,x)}{F_{LU}}, & x_L \le x \le x_U \text{ and } c_L \le c \le c_U \\ 0, & \text{otherwise} \end{cases} \qquad (4)$$

where, $\varphi(c,x)$ is the pdf of the two-dimensional normal distribution function defined by eq (2), $F_{LU} = F(x_U, c_U) - F(x_L, c_L)$ and $F(x,c) = \int_{-\infty}^{c}\int_{-\infty}^{x} \varphi(c,x)dxdc$.

The plant revenue is then

$$\text{Revenue} = \int_{c=c_L}^{c_U}\int_{x=x_L}^{P} x*c*f_{BBTN}(c,x)dxdc + \int_{c=c_L}^{c_U}\int_{x=P}^{x_U} P*c*f_{BBTN}(c,x)dxdc$$
$$= A_T + C_T$$

where,

$$A_T = \int_{c=c_L}^{c_U}\int_{x=x_L}^{P} x*c*f_{BBTN}(c,x)dxdc$$

$$C_T = \int_{c=c_L}^{c_U}\int_{x=P}^{x_U} P*c*f_{BBTN}(c,x)dxdc$$

Here, the value of P should locates in [$\mathbf{X_L}$, $\mathbf{X_U}$].
After detailed derivation for each term, the revenue can be computed by

$$\text{Revenue} = A_T1 + A_T2 + A_T3 + A_T4 + A_T5 + C_T1 + C_T2 \qquad (5)$$

Where,

$$A_T1 = \frac{-\sqrt{1-\rho^2}\sigma_x\sigma_c}{2\pi F_{LU}} \int_{\frac{c_L-\bar{c}}{\sigma_c}}^{\frac{c_U-\bar{c}}{\sigma_c}} me^{-\frac{m^2}{2}}[e^{-\frac{U^2}{2}} - e^{-\frac{L^2}{2}}]dm$$

$$A_T2 = \frac{-\sqrt{1-\rho^2}\sigma_x\bar{c}}{2\pi F_{LU}} \int_{\frac{c_L-\bar{c}}{\sigma_c}}^{\frac{c_U-\bar{c}}{\sigma_c}} e^{-\frac{m^2}{2}}[e^{-\frac{U^2}{2}} - e^{-\frac{L^2}{2}}]dm$$

$$A_T3 = \frac{\rho\sigma_x\sigma_c}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L-\bar{c}}{\sigma_c}}^{\frac{c_U-\bar{c}}{\sigma_c}} m^2e^{-\frac{m^2}{2}}[\Phi(U) - \Phi(L)]dm$$

$$A_T4 = \frac{\sigma_c\theta + \rho\bar{c}\sigma_x}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L-\bar{c}}{\sigma_c}}^{\frac{c_U-\bar{c}}{\sigma_c}} me^{-\frac{m^2}{2}}[\Phi(U) - \Phi(L)]dm$$

$$A_T5 = \frac{\bar{c}\theta}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L-\bar{c}}{\sigma_c}}^{\frac{c_U-\bar{c}}{\sigma_c}} e^{-\frac{m^2}{2}}[\Phi(U) - \Phi(L)]dm$$

$$C_T 1 = \frac{P\bar{c}}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L - c}{\sigma_c}}^{\frac{c_U - c}{\sigma_c}} e^{-\frac{m^2}{2}} [\Phi(U_U) - \Phi(U)]dm$$

$$C_T 2 = \frac{P\sigma_c}{\sqrt{2\pi}F_{LU}} \int_{\frac{c_L - c}{\sigma_c}}^{\frac{c_U - c}{\sigma_c}} me^{-\frac{m^2}{2}} [\Phi(U_U) - \Phi(U)]dm$$

$$L = \frac{\frac{x_L - \theta}{\sigma_x} - \rho m}{\sqrt{1-\rho^2}}$$

$$U_U = \frac{\frac{x_U - \theta}{\sigma_x} - \rho m}{\sqrt{1-\rho^2}}$$

5. Approximation of the standard normal cumulative function

In the equations derived in the above sections, there is an unintegrable term, $\Phi(.)$. To facilitate the revenue computation, we use some simpler functions to approximate $\Phi(.)$. There exist some accurate approximations to the standard normal cumulative function in the literature[10]. However, those approximations are still too complicated to integrate. In this paper, a seventh-order polynomial function is used to approximate $\Phi(.)$.

$$\Phi(x) = a + b*x + c*x^3 + d*x^5 + e*x^7 \qquad (6)$$

When x is in the range of [-3, 3], the coefficients in eq (6) are: a, 0.5; b, 0.3942473009; c, -0.058125270; d, 0.0056884266 and e, -0.000228133.

6. Type I and II service levels

Two types of service levels (or customer satisfaction levels) are commonly used in the industry. The Type 1 service level (usually also called the confidence level) is the probability of not stocking out in all scenarios or horizons[11]. The Type 2 service level (also often called the fill rate) is the proportion of demands that are met from a plant[11]. The Type 1 service level is widely applied in chance constrained programming up to now. However, the Type 1 service level is not how service is interpreted in most applications[11]. The type II service level is a greater concern of most managers in industry[5,11]. The difference of Type I and II service level can be found in Nahmias' work[11].

To apply Type I service level, the following constraint should be added in the model[5,9]:

$$\Pr\left(D \geq d\right) = \alpha,$$

where D is the production rate (no inventory) or the deliverable amount (with inventory), d is the market demand. α is the Type 1 service level or confidence level. The above constraint is transformed to the following by applying chance constrained programming[5,9]:

$$D \geq \Phi^{-1}(\alpha). \tag{7}$$

In the above constraint, Φ^{-1} is the known reverse cumulative distribution function of the product demand.

7. Derivation of truncated loss function

Loss function, defined as $LF(P) = \int_P^{\infty} (x-P)\rho(x)dx$ (for non-truncated and continuous distribution), represents the amount of unmet demand (the backorder level) of a plant facing uncertain demand[12]. It is essential to compute value of loss function to apply the Type II service level in a model.

For non-truncated and normally distributed demand, loss function has been effectively applied and approximated to compute the actual fill rate in the literature[5]. In this paper, we extend the research to cases when demand is truncated and normally distributed.

Again, we assume that the range of demand is $[X_L, X_U]$. Then the bi-truncated density function of demand x is:

$$\rho_{BTN}(x) = \frac{\phi(\frac{x-\theta}{\sigma_x})}{\sigma_x[\Phi(\frac{x_U-\theta}{\sigma_x}) - \Phi(\frac{x_L-\theta}{\sigma_x})]} = \frac{\phi(\frac{x-\theta}{\sigma_x})}{\sigma_{BTN}}.$$

where, $\phi(x)$ is the standard normal density function and $\Phi(x)$ is the standard normal cumulative function and $\sigma_{BTN} = \sigma_x[\Phi(\frac{x_U-\theta}{\sigma_x}) - \Phi(\frac{x_L-\theta}{\sigma_x})]$.

Thus, the bi-truncated loss function, $LF_{BTN}(P)$, is:

$$LF_{BTN}(P) = \int_P^{X_U} (x-P)\rho_{BTN}(x)dx = \frac{1}{\sigma_{BTN}}\int_P^{X_U} x\phi(\frac{x-\theta}{\sigma_x})dx - \frac{P}{\sigma_{BTN}}\int_P^{X_U} \phi(\frac{x-\theta}{\sigma_x})dx$$

let $t = \frac{x-\theta}{\sigma_x}$, then

$$LF_{BTN}(P) = \frac{1}{\sigma_{BTN}}\int_{\frac{P-\theta}{\sigma_x}}^{\frac{X_U-\theta}{\sigma_x}} (t\sigma_x + \theta)\frac{\sigma_x}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}dt - \frac{P}{\sigma_{BTN}}\int_{\frac{P-\theta}{\sigma_x}}^{\frac{X_U-\theta}{\sigma_x}} \frac{\sigma_x}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}dt$$

let $Z_{XU} = \frac{X_U - \theta}{\sigma_X}$ and $Z_{XP} = \frac{P - \theta}{\sigma_X}$, then

$$LF_{BTN}(P) = \frac{\sigma_X^2}{\sigma_{BTN}\sqrt{2\pi}} \int_{Z_{XP}}^{Z_{XU}} te^{\frac{-t^2}{2}} dt + \frac{\theta\sigma_x}{\sigma_{BTN}\sqrt{2\pi}} \int_{Z_{XP}}^{Z_{XU}} e^{\frac{-t^2}{2}} dt - \frac{P\sigma_x}{\sigma_{BTN}\sqrt{2\pi}} \int_{Z_{XP}}^{Z_{XU}} e^{\frac{-t^2}{2}} dt$$

$$= \sigma_x \{ \frac{-\sigma_x}{\sigma_{BTN}} [\phi(Z_{XU}) - \phi(Z_{XP})] - \frac{(P-\theta)}{\sigma_{BTN}} [\Phi(Z_{XU}) - \Phi(Z_{XP})] \} \quad (8)$$

When the demand conforms to Left Truncated Normal Distribution**,** We have $X_U \to +\infty$ and $Z_{XU} \to +\infty$. Then

$$\Phi(Z_{XU}) = 1,\ \phi(Z_{XU}) = 0,$$
$$\sigma_{BTN} \to \sigma_{LTN} = \sigma_x[1 - \Phi(Z_{XL})]$$

Thus the left truncated loss function, $LF_{LTN}(P)$, becomes,

$$LF_{LTN}(P) = \sigma_x \{ \frac{\sigma_x \phi(Z_{XP})}{\sigma_{LTN}} - \frac{(P-\theta)}{\sigma_{LTN}} [1 - \Phi(Z_{XP})] \}$$

When the demand conforms to Right Truncated Normal Distribution**,** We have $X_L \to -\infty$ and $Z_{XL} \to -\infty$. Then

$$\Phi(Z_{XL}) = 0,\ \phi(Z_{XL}) = 0,$$
$$\sigma_{BTN} \to \sigma_{RTN} = \sigma_x \Phi(Z_{XU})$$

Thus the right truncated loss function, $LF_{RTN}(P)$, becomes,

$$LF_{RTN}(P) = \frac{-\sigma_x}{\Phi(Z_{XU})} [\phi(Z_{XU}) - \phi(Z_{XP})] - \frac{(P-\theta)}{\Phi(Z_{XU})} [\Phi(Z_{XU}) - \Phi(Z_{XP})]$$

For Non-Truncated normal distribution, that is, $X_U \to +\infty$ and $X_L \to -\infty$, we have $Z_{XU} \to +\infty$ and $Z_{XL} \to -\infty$ and $\sigma_{BTN} \to \sigma_x$. Thus the eq (8) is reduced to

$$LF(P) = \sigma_x \{ \phi(Z_{XP}) - Z_{XP}[1 - \Phi(Z_{XP})] \}$$

The above formula can also be found in the literature[4, 5].

8. Case Study

Case studies are used to illustrate the influences of correlation and truncation of price and demand on plant revenue. The problem is taken from the case 1 of Li et al.[5]. Figure 1[5] shows the configuration of the problem. MTBE and GASO (gasoline) enter the gasoline blending (GB) unit to produce two products: 90# (GASO'90) and 93# (GASO'93) gasoline. The price of GASO and MTBE are 1400 and 3500 Yuan/tom, respectively. The price of 90# and 93# gasoline are 3215 and 3387 Yuan/ton, respectively. The octane number of GASO and MTBE are 70 and 101, respectively. The octane number of 90# and 93# gasoline are 90 and 93, respectively. The blending requirement is that the octane number of each product should equal or be greater than the required octane number of that product. No inventory is considered and the overproduced products are assumed to be valueless.

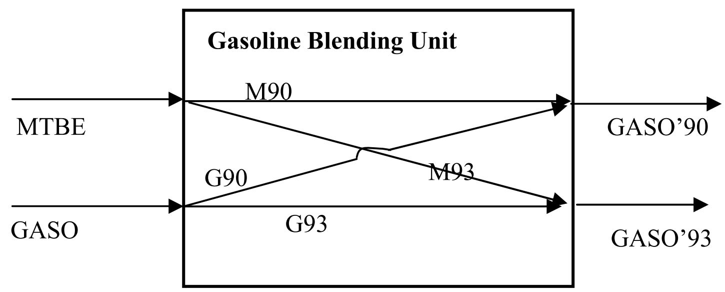

Figure 1. The configuration of the problem

8.1. Non-truncated and correlated case

In this case, we consider the influence of correlation coefficient on plant revenue at different CVs (Coefficient of Variation, $CV = \frac{\text{Standard Deviation}}{\text{mean}}$). The means of demand for 90# and 93# gasoline are 50 and 70 tons, respectively. The standard deviation of 90# and 93# gasoline at different CVs are listed in Table 1. The standard deviation of 90# and 93# gasoline prices at different CVs are assumed to be fixed at 600 and 620 Yuan/ton, respectively.

	CV		
Products	0.2	0.3	0.5
90# gasoline	10	15	25
93# gasoline	10	20	35

Table 1 Standard deviation of products at different CVs

The revenues at different correlation coefficients at CV=0.5 are listed in Table 2. It can be seen that, the revenue at correlation coefficient of 0.4 (near the real world data) is 21.1% higher than the revenue calculated by assuming independent demand and price (correlation coefficient=0.0). That means, if for a large enough CV of a product, assuming independent price and demand may underestimate the revenue by up to 20%.

Correlation coefficient	Revenue, Yuan	Difference,%
0	10097.5	0.0
0.1	10589.83	4.9
0.2	11096.87	9.9
0.3	11638.74	15.3
0.4	12225.64	21.1
0.5	12853.69	27.3

Table 2 Revenue at different correlation coefficients (CV=0.5).

From other cases, we also found that, as the standard deviation of price increases, the revenue increases slightly. However, as the standard deviation of demand increases, the revenue decreases significantly. The revenue difference between the independent and correlated cases depends on the CV of products. In the problem studied here, if the CV of a product takes value of 0.2 and correlation coefficient is 0.4 to 0.5, the revenue difference is about 2%. This difference is about 5% for CV of 0.3 (According to the regressed data from EIA, the correlation coefficient between demand and price is around 0.4).

8.2. Truncated and correlated case

Integrating over the whole range of a normal distribution may give incorrect results to revenue calculation. The formulae derived for bivariate double-truncated normal distribution are applied in the model. Some results are shown in Tables 3 and 4 (the product production rates of truncated cases are fixed to those of the non-truncated case for comparison). In Table 3 (CV=0.2), it can be seen that, if the price and demand vary inside two standard deviations of their mean values, integrating over the whole range will underestimate the revenue by 2 to 3%. If the price and demand vary inside one standard deviation of their mean values, integrating over the whole range will underestimate the revenue by about 12%. The revenue difference between truncated and non-truncated becomes much more significant for large enough CV. In Table 4 (CV=0.5), integrating over the whole range will underestimate the revenue from 20% up to 130%.

	Integration Range of Non-truncated case	Integration Ranges of Truncated Cases	
Correlation coefficient	$(-\infty, +\infty)$	(mean+/-2*standard deviation)	(mean+/-standard deviation)
0	38685.9	39847.5837	43513.6285
0.1	38851.1635	39948.7	43772.7047
0.2	39021.0171	40107.0	43958.1995
0.3	39202.2534	40418.5	44166.1383
0.4	39398.5284	40792.3	44369.2797
0.5	39608.5912	39418.8	44445.1523

Table 3. Revenues of truncated and non-truncated cases (CV=0.2)

	Integration Range of Non-truncated case	Integration Ranges of Truncated Cases	
Correlation coefficient	(-∞, +∞)	(mean+/-2*standard deviation)	(mean+/-standard deviation)
0	10097.5	14986.6021	23935.1191
0.1	10589.8324	15234.4	24545.507
0.2	11096.8739	15556.8	25029.963
0.3	11638.7364	16067.2	25457.6564
0.4	12225.6447	16693.6	25907.5828
0.5	12853.6936	15504.5	26286.4896

Table 4. Revenues of truncated and non-truncated cases (CV=0.5)

9. Conclusion

In this paper, the correlation between price and demand as well as their integration ranges are studied. Theoretical derivations are performed and several case studies are developed to study the influences of correlation and truncation on plant revenue.

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