57f Robust Control of Inhomogeneous Patterns in Reaction-Diffusion Systems Using Reduced Order Models

Carlos Vilas, Míriam R. García, Julio R. Banga, and Antonio A. Alonso

Reaction-diffusion (RD) systems are object of intensive research as their underlying mechanism plays a central role in many chemical and biological processes. The dynamics of these systems usually presents spatially inhomogeneous patterns, in the form of travelling waves (waves that move without change of shape) or even spirals. This class of solutions which is typical of chemical systems such as the oxidation of CO on Pt(110) surface, but also explains phenomena taking place in biological instances such as neural communication, or cardiac rhythms. A well known model which qualitatively captures the key dynamic aspects of diffusion-reaction systems, namely the development of chemical waves is the so-called FitzHugh-Nagumo (FHN) model, which is probably the most extended simplified version of the Hodgkin-Huxley model. This system is very sensitive to changes in the parameters as slight changes on them could result in completely different behaviours.

Standard approaches to robust control of distributed process systems, and in particular RD systems, are based on the spatial discretization of the original set of partial differential equations (PDEs) to obtain a set of, usually large scale, ordinary differential equations (ODEs), this allows to employ standard finite-dimensional methods to construct the controller. However, and as pointed out by Christofides and co-workers, this approach presents some disadvantages such as the computational cost of the resolution of the resulting set of ODEs which limits real time applications or the fact that controllability and observability properties might depend on the number of discretization points as well as their locations.

The use of reduced order models has been proposed recently as an alternative to overcome these disadvantages. The underlying idea is to take into account the spatially distributed nature of the system and make use of the Galerkin method to approximate the system by a low-dimensional set of ODEs. Christofides and co-workers (see [1], [2]) employed this approach to derive robust stabilizing controllers based on feed-back linearization. In [3] the authors applied a Gershgorin theorem based method to construct a finite dimensional regulator which is able to stabilize front solutions of RD systems. From another point of view, Alonso and co-workers used the second law of thermodynamics and passivity to develop passive stabilizing controls for reaction-diffusion systems [4], and made use of the connections between these ideas and the theory of robust nonlinear control to obtain a proportional controller that is able to stabilize arbitrary modes in distributed process systems [5].

In this contribution we consider the problem of stabilizing, in a robust way (taking into account that some uncertainties on the nonlinear terms or in the system parameters are present), a given solution of the FHN system when the systems operates under a different and uncertain set of parameter values. To that purpose, we make use of a Lyapunov redesign technique to develop a class of robust nonlinear controllers for diffusion-reaction systems able to stabilize fronts. In order to adapt this technique to infinite dimensional systems we make use of the physics of dissipative processes and the spectral decomposition method for model reduction.

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