

57a Actuator/Sensor Scheduling for Distributed Processes with Quantized Control Systems

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The study of distributed parameter systems has been a subject of considerable interest in process control research as evidenced by the large and growing number of research studies dealing with various aspects of the analysis and control of distributed process systems (e.g., see [1], [2], [4], [5], [9]). The motivation for these studies stems in part from the distributed nature of the control problem arising in many prominent chemical processes, such as transport-reaction systems. The fusion of analytical and computational tools from several domains, such as the dynamics of infinite-dimensional systems, model reduction and control, has enabled researchers to address a variety of control problems for distributed processes. This significant progress notwithstanding, an observation common to most of the existing work on control of distributed systems is the fact that the control problem is often formulated and solved within the classical feedback control setting where the output of the process is assumed to be passed directly to the controller, which generates the control input and in turn passes it directly back to the process.

In practice, however, this paradigm often needs to be re-examined, in part because the interface between the controller and the process features some additional information-processing devices that should be accounted for in the design of the controller as well as in the analysis of its stability and performance properties. One important aspect to take into account in such situations is signal quantization. Generally speaking, quantization refers to the phenomenon of converting a real-valued signal into a piecewise constant one taking on a finite set of values. Quantization effects are common in most practical control systems and are usually tied to inherent physical or technological constraints on the sensing and actuating devices. For example, many signals cannot be measured quantitatively such as the biomass concentration in bio-reactors, substance concentrations in the liquid or the gaseous phase, the temperature in cement kilns or blast furnaces. Even the pH-sensors that are often used in the process industry have substantial measurement errors and give, in principle, only an interval of possible current pH values. These limitations give rise to quantized sensors whose measurements provide only a limited (i.e., discrete) information on the state of the system. Quantization can also affect the control input as many inputs can only be switched between discrete values rather than be varied continuously. Examples include on/off valves, pumps with discrete settings, stepping motors and a variety of other event-driven actuators. In addition to actuator and sensor discrete settings and device limitations, communication constraints are another important source of quantization. With the advent of networked control systems and emerging applications involving large numbers of distributed sensors and actuators, the example of digitally interconnected systems controlled through finite communication channels capable of transmitting only discrete information between the plant and the controller is becoming commonplace. In such systems, limits on the information (bit) rate requires that the measured output be quantized before being sent through the channel.

Over the past two decades, the problem of dynamic systems analysis and control synthesis in the presence of quantization has received considerable attention. In the earlier control literature, quantization of inputs has been mostly regarded as a disturbance to be rejected (e.g., [3]). Typical results in this spirit are those showing that, with a finite quantized control set, stability can only be achieved in a weak sense (e.g., practical stability [7]) and that stability bounds can be made arbitrarily small by refining quantization sufficiently. More recently, some attention has been focused on quantized control systems as specific models of hierarchically organized systems with interaction between continuous dynamics and logic (e.g., [6], [8]). In these works, quantization is regarded as a fundamental characteristic of systems where the resources for implementing the control scheme are limited. The focal point of this research has been to understand how to quantize the control system best (in some suitable sense) to meet a pre-specified control objective, rather than assessing the robustness of a given design

with respect to quantization. In [6], for example, the optimal (coarsest) quantization for asymptotically stabilizing a linear system was derived. Also, in [8] a hybrid quantized feedback control strategy that varies the quantization levels at certain times was devised to achieve asymptotic stability.

In many practical settings, however, quantization levels are fixed by the quality and precision of the actuating and sensing devices used, as well as by the bandwidth of the communication medium, and cannot be varied. In these instances, an important problem is to analyze the robustness of a given controller to the errors introduced through quantization. This kind of analysis is helpful because it allows the designer to identify the performance limits of a given controller and to decide a priori whether a desired control objective can be achieved with a certain kind of actuator or sensor. Examination of the literature on quantized control systems also reveals that the available results have focused exclusively on lumped systems modeled by linear or nonlinear ordinary differential equations. There are many examples in the process industries, however, where the process dynamics are characterized by spatial variations owing to the underlying physical phenomena, such as diffusion, convection, and phase-dispersion. Unlike lumped processes, the quantization problem for spatially-distributed control systems not only impacts the performance and stability of the controller but also imposes limitations on where the control actuators and measurement sensors should be placed to attain the desired control objectives. The abundance of spatially-distributed systems in process control applications as well as the frequent presence of quantization effects provide a strong motivation for (1) the analysis of the fundamental limitations imposed by quantization on the performance and stability of a distributed control system and (2) the development of systematic control strategies that account explicitly for quantization effects.

Motivated by the above considerations, we focus in this work on the analysis and control of distributed processes, modeled by highly dissipative PDE systems, with both control and communication constraints. Control constraints are modeled using bounds on the magnitude of the control action, while communication constraints (between the plant and the control system) are represented using fixed actuator/sensor quantization levels. Using appropriate finite-dimensional approximations of the PDE system, we first characterize the inherent conflict that emerges in the control design objectives when both control and quantization constraints are considered, and the implications of this conflict for the actuator/sensor placement problem. At the heart of this conflict is the fact that control constraints limit the set of initial conditions starting from where stability can be achieved (stability region), while quantization constrains the set of terminal states that the system can be steered to (terminal region). Owing to the dependence of the input and measurement operators on spatial location, the conflict manifests itself through the dependence of both the stability and terminal sets on the spatial locations of the actuators and sensors. Using Lyapunov-based analysis and controller synthesis techniques, we obtain explicit characterizations of both the stability and terminal regions in terms of the control constraints, the quantization levels and the actuator/sensor locations. The analysis reveals the essence of the tradeoff, which is captured by the fact that the control configuration with the largest stability region also possess the largest terminal region. Stabilization from large initial conditions favors configurations with large stability regions, while close convergence to the steady-state favors configurations with small terminal sets. Therefore, steering the closed-loop state from large initial conditions to arbitrarily small neighborhoods of the desired steady-state cannot be achieved using a single control configuration (with a fixed actuator/sensor placement) and requires switching between multiple configurations. To resolve this conflict, we devise an actuator/sensor scheduling strategy that orchestrates the transitions between the different locations based on where the closed-loop state is with respect to the stability and terminal regions at any given time. We also characterize the minimum number of transitions needed to steer the closed-loop trajectory from a given initial condition to a given terminal set. Finally, the theoretical results are demonstrated using a benchmark diffusion-reaction process example.

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