**553e Lyapunov-Based Predictive Control of Hybrid Systems**

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The operation of chemical processes often involves controlled, discrete transitions between multiple, continuous modes of operation in order to handle, for example, changes in raw materials, energy sources, product specifications and market demands. The superposition of discrete events on the continuous process dynamics gives rise to a hybrid system behavior, i.e., intervals of piecewise continuous behavior interspersed by discrete transitions. In many processes, the hybrid nature of the process arises from the requirement to follow a prescribed switching schedule, where the switching times are prescribed via an operating schedule. The operation in each individual mode needs to account for practical issues such as constraints on the manipulated input (due to limited capacity of control actuators) and the state variables (which may arise due to performance or safety considerations), and additionally, guiding the system safely through a prescribed switching sequence imposes additional constraints that a control algorithm must incorporate. One control approach well suited to handling constraints within an optimal control framework is model predictive control (MPC) and has been studied extensively (see, for example the survey paper, [1]).

In a recent work [2] we proposed a predictive control framework for the constrained stabilization of switched nonlinear systems that transit between their constituent modes at prescribed switching times. While the work in [2] addressed the issue of characterizing the stability region for each individual mode subject to input constraints, it did not focus on the issue of incorporating and ensuring the satisfaction of state constraints during operation in the individual modes, and assumed precise knowledge of the switching times.

Motivated by these considerations, in this work we propose a predictive control framework for the constrained stabilization of switched nonlinear systems that transit between their constituent modes at prescribed switching intervals, subject to constraints on the manipulated inputs and process state variables. The main idea is to design a Lyapunov-based predictive controller for each constituent mode in which the switched system operates, and incorporate constraints in the predictive controller design which upon satisfaction ensure that the prescribed transitions between the modes occur in a way that guarantees stability of the switched closed-loop system. This is achieved as follows: for each constituent mode, a Lyapunov-based model predictive controller (MPC) is designed, and an analytic bounded controller, using the same Lyapunov function, is used to explicitly characterize a set of initial conditions for which the MPC, irrespective of the controller parameters, is guaranteed to be feasible, and hence stabilizing in the presence of both input and state constraints [3]. Then, constraints are incorporated in the MPC design which, upon satisfaction, ensure that: (1) the state of the closed-loop system, at the beginning of the transition interval, and during it, resides in the stability region of the mode that the system is switched into, and (2) the Lyapunov function for each mode is non-increasing wherever the mode is re-activated, thereby guaranteeing stability. The proposed control method is demonstrated through application to a chemical process example.

References:

[1] Mayne, D. Q., Rawlings, J. B, Rao, C. V and P. O. M. Scokaert, Constrained model predictive control: Stability and optimality, Automatica, 36:789-814, 2000.

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[3] Mhaskar, P., N. H. El-Farra and P. D. Christofides, "Stabilization of Nonlinear Systems with State and Control Constraints Using Lyapunov-Based Predictive Control," Syst. & Contr. Lett., in press.