553d An Optimization Based Approach for Stability Analysis of Nonlinear Model Predictive Controller

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Model predictive control (MPC) is based upon the solution of an optimization problem at regular time intervals – a parametric programming approach (Dua et al., 2002) that avoids the repetitive solution of the on-line optimization problem has recently been proposed by Pistikopoulos et al., (2002) and Bemporad et al., (2002). The optimization problem usually consists of a quadratic objective function, so as to minimize the weighted deviation of state and control variables from their corresponding target values. This optimization is carried out over a given future time horizon, also known as prediction horizon, for which the state variables are predicted for the given current state of the system and the control variables which are computed such that the objective function is minimized. The optimization problem also incorporates constraints on state and control variables. An increase in the prediction horizon can improve the stability of the system, but results in slower response and increase in the size of the problem. For linear time-invariant systems by choosing an infinite prediction horizon and a finite control horizon the system can be stabilized (Rawlings and Muske, 1993). Keerthi and Gilbert (1988) and Mayne and Michalska (1990) have presented a detailed study of stability analysis of model predictive controllers. Sistu and Bequette (1995) presented verifiable expressions that can be used to find a stabilising set of tuning parameters for nonlinear systems, similar to the results by Garcia and Morari (1982) for linear systems.

In this work we present a systematic method for computing prediction and control horizons that meet a given stability criterion for discrete time nonlinear model predictive controller by formulating and solving a mixed-integer nonlinear programming problem (MINLP). Corresponding to each state and control variable a set of 0-1 binary variables is introduced. A binary variable in a set represents a discrete time interval such that the summation of the binary variables of a set is the prediction or control horizon for the corresponding state or control variable. The objective is to minimize the summation of prediction and control horizons for all the state and control variables. The stability criterion is introduced by incorporating constraints so that the state and control variables reach the target values at the end of the corresponding horizons. This is achieved by using binary variables to model the logical conditions such that the square of the difference between the variables and the corresponding target values is positive before the end of the horizon and is zero at and after the end of the horizon. Another set of constraints is also included so that a binary variable in a set corresponding to a time interval is greater than or equal to the binary variable at the next time interval. This enforces the condition that the computed control law affects the solution only until the end of the horizon. The resulting MINLP is solved in GAMS (Brooke et al., 1998) by using the outer-approximation algorithm (Viswanathan and Grossmann, 1990). The solution provides minimum prediction and control horizons, for each of the state and control variables respectively, such that there exists a feasible control law. In general, more than one solution, i.e. prediction and control horizons, can be found which meet a given stability criterion. This issue is addressed by introducing integer cuts to identify all the possible solutions. The optimization based approach presented in this work is quite generic and can be used for systematic calculation of prediction and control horizons that meet given stability criterion. A number of process examples are presented to illustrate the concepts discussed above.

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