553c Control of Nonlinear Hybrid Systems Using Multiple Mld Models

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Chemical processes invariably exhibit a hybrid character where the system dynamics are determined by discrete decisions and continuous inputs. This hybrid nature is emphasized during startups/shutdowns, batch operations, scheduling, and grade transitions. During such operations, examples of discrete decisions include manipulation of on/off valves, gears switches, heater/cooler units, and speed selector switches among others. The process industry has typically integrated these discrete components through use of heuristic rules that encapsulate operator experience and process insights.

Several strategies that model the hybrid system by formally integrating the continuous and discrete variables have been presented in literature. Amongst these the mixed logical dynamical (MLD) formalism (Bemporad and Morari, 1999) has emerged as a global modeling framework, which is described by **linear** dynamic equations with continuous, discrete and mixed variables, and **linear inequality** constraints. Also, the availability of a compiler, namely HYSDEL (Torrisi et al., 2000) has popularized this approach. A key advantage of the MLD framework is that it allows design of model based control formulations such as MPC for hybrid systems. However, much of the literature targeting control applications is limited to linear hybrid systems (Van der Schaft et al., 1998; De Schutter, 2000; Bemporad et al., 2000; Potocnik et al., 2004;Thomas et al., 2004).

Chemical processes are inherently nonlinear and this nonlinearity is further pronounced for processes traversing a wide region of operation (such as during startups/shutdowns, batch operation and grade transition). In this work, we attempt to model and control a nonlinear hybrid system using multiple MLD models. The MPC formulation using such an approach will retain the MILP or MIQP optimization problem while addressing the nonlinearity of the hybrid system.

A model for a hybrid process may be represented by,

$$\dot{x} = f_{\ell}(x, u_c, u_d)$$

where u_c and u_d describe the continuous and discrete inputs, respectively and ℓ represents an index that enumerates the system dynamics for different logical inputs typically represented by jump rules. We develop a multiple MLD model approach by decomposing the operating regime into a set of separate local operating regions and describing each of these by local MLD models. The local operating regimes are generally characterized by values of the continuous variables. The linearization proceeds by first accommodating the jump rules using logical variables and subsequently performing a Taylor series expansion around the operating point for the continuous variables with fixed values of the discrete variables (Colmenares, 2001). Finally, the linearized model is discretized using an appropriate sampling time. Thus, for each of the m operating regions, the n binary (discrete) variables will result in 2^n linear models. These linear sampled models are then multiplied with suitably defined logical variables and summed up to obtain a single MLD model for a given operating region. The m multiple linear models may be used for control purposes as is accomplished in multiple model MPC. We demonstrate the above multiple MLD model approach by simulating the control of a three spherical tank system. This system is based on the COSY three-tank benchmark problem used by a number of researchers (Villa et al., 2004; Bemporad et al., 1999) with the main difference lying in the shape of each of the tanks. The nonlinearity results from the shape of the tanks as well as the constitutive relationship between the exit flows and the level in each tank. We consider two operating regions (m=2), namely high and low level. Six independent discrete (on/off) valves can be manipulated to interrupt the flows into or out of the three tanks. We demonstrate the transition between the high and low levels in the 3-tank system using an

MPC formulation. We also demonstrate the filling and emptying of the tanks with respect to the high and low level operating points. To quantify the benefits of employing a multiple MLD model, we compare our results with a single MLD model based control. We also document the increase in computational burden for MPC based on the multiple MLD models relative to a single MLD model. It is well known that the MILP or MIQP formulation encountered when using MPC based on MLD models is computationally expensive. In this regard, we explore the recently proposed state space polyhedral partition approach to reduce the size of the optimization problem (Thomas et al., 2004).

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