Application of Optimal Control Theory for Sustainable Ecosystem Management

Y. Shastri and U. Diwekar
Department of Bioengineering, University of Illinois at Chicago,
851 S. Morgan Street, Mail Code 063, Chicago, IL 60607

Abstract

The concept of sustainability, an abstract one by its nature, has been given a mathematical representation through the use of Fisher information as a measure. It is used to propose the sustainability hypotheses for dynamical systems, which has paved the way to achieve sustainable development through externally enforced control schemes. For natural systems, this refers to the task of ecosystem management, which is complicated due the lack of clear objectives. This work attempts to incorporate the idea of sustainability in ecosystem management. Natural regulation of ecosystems suggests two possible control options, the top-down control and the bottom-up control. A comparison of these two control philosophies is made on aquatic food chain models using objectives derived from sustainability hypotheses. Optimal control theory is used to derive the control profiles to handle the complex nature of models and objectives. Results indicate a strong relationship between the hypotheses and dynamic behavior of the models, supporting the use of Fisher information as a measure. As regards to the aquatic ecosystem management, it has been observed that top-down control is more aggressive but can result in instability while the bottom-up control option is guaranteed to give stable and improved dynamic response.

1 Introduction

Sustainability is a relatively new concept in the field of ecology. Sustainable development, a multifaceted approach to manage the environmental, economic, and social resources, calls for the consideration of long term effects in all the decisions relevant to the society as a whole. Being embodied in a multi-disciplinary environment, a suitable mathematical representation, or a measure, of sustainability is essential for successful communication amongst various fields encompassed by the concept. To this effect, Cabezas and Fath [1] have proposed Fisher information as a sustainability measure for dynamic systems and have formulated the sustainability hypotheses with particular focus on natural ecosystems. These hypotheses help rank various dynamic systems and decide the nature of evolution using sustainability as a criteria. As regards to natural evolution, there is not much that humans can do about. But when it comes to evolution caused by human interventions (e.g. ecosystem management), such a criteria can be helpful in deciding the right course of action.

Ecosystem management is much more complicated than management of engineering systems due to the complexities of natural systems. Not only are these systems poorly understood, but also the objectives to be achieved are quite obscure. This is where the multi-disciplinary concept of sustainability can make an invaluable contribution. Binding ecosystem properties of disparate temporal and spatial scales together, it becomes an effective tool for ecosystem management. Work by Cabezas and Fath [1] has made quantification of sustainability possible, paving the way to implement this abstract concept in management related decision making. This work, if broadly defined, concerns with the application of sustainability concept in management of aquatic ecosystems (such as lakes).

A key to effectively regulate any ecosystem is to first understand the natural regulation of the system, for which scientists propose two different control philosophies, the top-down control [2] and the bottom-up control [3]. These natural regulation paths can be used to advantage when trying to impose external control for ecosystem management. There has been an intense debate over the validity and relative importance of both philosophies and the general consensus to emerge is that both these regulations are dominant at different levels of the ecosystem represented as a food chain [4]. This work performs a relative assessment of these two management philosophies.

But given the complex nature of natural systems and the objective (sustainability), heuristics based derivation of control profiles is not justified. Advanced control theory has been developed to tackle such problems and optimal control theory has been at the forefront due to some of its obvious advantages. This work, therefore, uses this theory to formulate the control profiles. One of the necessities of the optimal control theory is a model of the system. This work models the aquatic ecosystem using two and three species predator-prey model, a class of general food web models. Uncertainty is inherent in any natural ecosystem due to our restricted understanding of them. It is therefore important to consider such uncertainties in the analysis which is done here through stochastic predator-prey model. In this model the mortality rate of a species is modelled as an uncertain parameter

To summarize, this work compares the top-down and bottom-up control philosophies, derived using optimal control theory, for an aquatic ecosystem, modelled as deterministic and stochastic systems, using sustainability, quantified by Fisher Information, as the objective. Fig. 1 gives the schematic explaining the relative contribution from each of the topics mentioned here.

The article is organized as follows. The next section reviews the theory behind the work. Section 3 gives the problem specific details and section 4 report and discuss the results for tri-trophic food chain ecosystem model. The article ends with comments on the computational aspects in section 5 and conclusions in section 6.

2 Theoretical Basics

2.1 Sustainability and Fisher Information

Sustainable development is defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" [5]. In its simplest terms, it calls for the consideration of long terms effects, benefits and drawbacks in all the decisions relevant to the society as a whole. Since its formalization, this new concept has become increasingly important and popular, attracting active research. Although the concept has been universally recognized to be of paramount importance, it is essential to have a quantifying measure for it to be used in the field of ecosystem management.

Cabezas and Fath [1] have proposed to use information theory in ecology to derive a

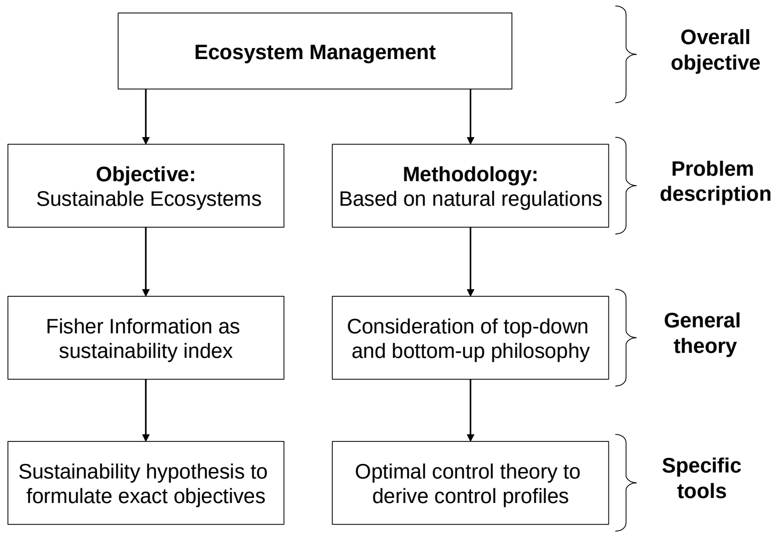

Figure 1: Integration of sustainability and ecosystem management: theory and tools

measure for the sustainability of a system, the hypothesis being based on the argument that information is a fundamental quantity of any system, irrespective of the discipline [6]. Various previous applications of information theory in ecology include using Shannon information [7] as an index of biodiversity; using entropy of information to investigate evolutionary processes [8]; measuring the distance of a system from the thermodynamic equilibrium (based on exergy) [9] and developing the concept of Ascendency [10]. Cabezas and Fath though use Fisher information as the quantity for their hypothesis.

Fisher information (FI), introduced by Ronald Fisher [11], is a statistical measure of indeterminacy. One of its interpretations, relevant for this work, is as a measure of the state of order or organization of a system or phenomenon [6]. The Fisher information, I, for one variable is given as [1]

$$I = \int \frac{1}{p(x)} \left(\frac{dp(x)}{dx} \right)^2 dx \tag{1}$$

where p is the probability density function (pdf) of variable x. This definition can be extended to a system of n variables. When the n variables constitute the state variable vector of a system, it gives the Fisher information of that system. Fisher information, being a local property, dependent on the derivative of the state vector density function, is sensitive to perturbations that affect the density function and therefore can be used as an indicator of the self-organization of the system. Here self-organization refers to the distribution of states in which the system exists. A chaotic system with many almost equally probable states is disorganized while a system with few preferred states is better organized. For a highly disorganized system, the lack of predictability due to a nearly uniform probability distribution of its states results in a low value of Fisher information. On the contrary, a highly organized system will have a high value of Fisher information. The central argument in the sustainability hypothesis [1] is that stability (static or dynamic) of a system is sufficient (but not necessary) for the sustainability of the state of the system. A statically or dynamically stable system, due to one or few preferred states, will have a high value of Fisher information. Thus Fisher information, being an indicator of system stability, is also an indicator of sustainability of the state of the system. For stability of an ecosystem (static or dynamic), it is important that the system is not losing or gaining species, affecting the system dimensionality and hence the value of Fisher information. The sustainability hypothesis therefore states that: the time-averaged Fisher information of a system in a persistent regime does not change with time. Any change in the regime will manifest itself through a corresponding change in Fisher information value [1].

Two additional corollaries stated in [1] to this hypothesis are: (1) if the Fisher information of a system is increasing with time, then the system is maintaining a state of self-organization and (2) if the Fisher information of a system is decreasing with time, then the system is losing its state of self-organization. These corollaries are based on the correlation between Fisher information and system order and give an idea of the quality of change, if the system is changing its state. An extensive review of Fisher information and sustainability hypotheses can be found in [1] and [12].

The sustainability hypotheses provide the theoretical basis for the presented work. From an ecosystem management perspective, two different objectives can be formulated based on these hypotheses:

- Maximization of the time averaged Fisher information: This objective is based on the idea that higher Fisher information is a consequence of a more organized system. Hence the objective attempts to push the system from its current state into a state that is more sustainable in the mathematical sense of Fisher information. It may demand rapid changes though.

- Minimization of the Fisher information variance over time: This objective is a direct consequence of the sustainability hypothesis. Minimizing Fisher information variance ensures constancy of the regime. The objective thus aims to maintain the system close to its current state

These different objectives are compared for different control philosophies. These control philosophies are explained in the following section.

2.2 Ecosystem management philosophies

This section gives a brief overview of various control philosophies proposed for a natural ecosystem, which will help decide the external control philosophies.

The earliest attempt to systematically understand the controlling effects of a natural ecosystem dates back to [13]. This has since encouraged more and more research in understanding the natural control of the various ecosystems and has led to the concept of trophic cascade hypothesis [2, 14]. For an aquatic ecosystem, it proposes that the predator-prey interactions are transmitted through food webs to cause variance in phytoplankton biomass and production at constant nutrient load [2] and that the responses are nonlinearly related to the strength of the interactions among adjacent trophic levels. Another possible regulatory effect in a food web is the effect of available resources on higher level species, e.g. nutrients supporting the phytoplankton in an aquatic food web, which in turn affect the top level species that feed on them.

From ecosystem management point of view this has led to the formulation of two control philosophies, the top-down control and the bottom-up control. Top-down control (also called as the consumer control) refers to controlling the ecosystem through top level predators. Bottom-up control (also called as the resource control) refers to controlling the ecosystem via available resources (e.g.

nutrients). Debate has been going on over the validity of individual control philosophies [15, 16, 17] and also on the relative importance of each of those in a food web [18, 19, 20]. The recent opinion, based on some of the published results, is that both the effects are apparent in a food chain and the relative importance of the two depends on the length of the food chain and the position of a particular species in the food chain [4, 3]. Thus, top-down control is more prominent in species at the top of the chain while the lower level species are more strongly under bottom-up control. Most of these results are based on experimental manipulations of lakes followed by observations over a long period of time.

The existence of these two natural regulation paths in ecosystems provides two different avenues to exercise external control of these systems. Thus, regulation by controlling the top predator and by controlling the lowest level resources are the two options explored and compared in this work for the given objectives.

2.3 Optimal control theory

Control theory aims to derive a time dependent profile of a particular system parameter, called the control variable, such that a specific objective is optimized over the considered time horizon. Development of control theory has primarily been motivated to solve performance problems of engineering systems such as mechanical, electrical and chemical, amongst others. But environmental problems such as ecosystem management also offer an exciting avenue for implementing some of the advanced control strategies. Some of these applications include fisheries management through fish harvesting, lake water quality management [21], fire management etc. Most of them aim to achieve immediate short term goals such as maximizing the harvesting income for a season, controlling periodic lake eutrophication etc. For such applications even time independent strategies may work. Sustainability though demands consideration of long term objectives and achievement of short term goals does not guarantee long term sustainability. For example, limiting nutrient or pollutant input into a lake at a constant level without giving proper credence to the environmental cycles and aquatic life cycles in the lake can affect species diversity and life expectancy. Such effects are not evident immediately and manifest themselves only over a long time period. Hence to achieve sustainability objectives, the time dependent nature of system parameters will need to be considered, which can only be achieved under a systematic mathematical framework. Moreover, there are multiple parameters in nature which are partially or completely under human control. In such cases, rigorous mathematical analysis needs to replace heuristics and logic in decision making. Use of advanced control strategies, therefore, might not just be an option but rather a necessity.

Optimal control is at the forefront of the advanced control strategies. Some of the advantages of optimal control over other advanced control strategies are: it does not make any assumption about the form of the control law, for the given objective function it theoretically gives the best control strategy and it can theoretically handle any type of system. Owing to the complexity of the natural systems and objective of sustainability, this work uses theory of optimal control to derive top-down and bottom-up control schemes for aquatic ecosystems.

The theory presents three possible methodologies to derive the optimal control law: dynamic programming (Hamilton-Jacobi-Bellman equation), calculus of variation (Euler-Lagrange equation) and Pontryagin's maximum principle [22]. In this work, Pontryagin's maximum principle has been used for the deterministic systems. For stochastic system, the stochastic maximum principle, an extension of the deterministic maximum principle, is used [23]. Both the methods results in a set of algebraic equations (optimality condition) and ordinary differential equations (state and adjoint equations) to be solved as a boundary value problem. The boundary values of the state and adjoint variables depend on the problem specification [24]. The control trajectory thus obtained using the optimality condition is optimal for the considered objective function and starting conditions. A detailed explanation of the theory is beyond the scope of this article and interested reader is referred to [24, 23].

Application of optimal control theory necessitates the knowledge of three aspects, the objective, the control variable and the model of the given system. The objectives are given by the sustainability hypothesis while the control variable is decided by the control philosophies discussed. The model of the system, which in this case is the aquatic ecosystem, is explained in the next section.

2.4 Predator-prey model

The model used to represent the aquatic ecosystems is a predator-prey model, derived from the more general class of Lotka-Volterra-type models. These models give a simplistic mathematical representation of the observed dynamics in natural systems. In many applications, three level food chain models are often considered to be a good enough representation of the ecosystems [25]. This work uses the Rosenzweig-MacArthur model, which is frequently used in theoretical ecology [26, 27, 28]. The model is given by the following set of differential equations:

$$f_1 = \frac{dx_1}{dt} = x_1\left[r\left(1 - \frac{x_1}{K}\right) - \frac{a_2 x_2}{b_2 + x_1}\right] \tag{2}$$

$$f_2 = \frac{dx_2}{dt} = x_2\left[e_2\frac{a_2 x_1}{b_2 + x_1} - \frac{a_3 x_3}{b_3 + x_2} - d_2\right] \tag{3}$$

$$f_3 = \frac{dx_3}{dt} = x_3\left[e_3\frac{a_3 x_2}{b_3 + x_2} - d_3\right] \tag{4}$$

where x_1, x_2 and x_3 are population variables of the three species in the food chain, in the ascending order of position in the chain. These species are referred to as prey (x_1), predator (x_2) and super-predator (x_3) in the subsequent text. r and K are the growth rate and prey carrying capacity, respectively, and a_i, b_i, e_i and d_i, $i = 2, 3$, are maximum predation rate, half saturation constant, efficiency, and death rate of the predator ($i = 2$) and super-predator ($i = 3$). $x_i(0)$ is the population of specie i at starting time. The model parameters for the tri-trophic model are given in Table 1.

For uncertainty analysis, the mortality rate of the predator d_2 is considered to be uncertain and modelled by the Ito mean reverting process. Ito process has the ability model time dependent stochastic variables effectively. It has been extensively used in financial analysis (stock markets, real options theory) and also has been shown to model the human mortality rate [29]. The equation for the Ito mean reverting process is given as:

$$f_{ito} = \frac{dz}{dt} = \eta(\bar{d}_2 - z) + \frac{\sigma\ \epsilon}{\sqrt{\Delta t}} z \tag{5}$$

Here z is the Ito variable (mortality rate), $\bar{d}_2$ is the mean mortality rate, η is the speed of reversion, σ is the variance parameter, Δt is the time interval and ϵ is a normally distributed random variable with mean zero and standard deviation one [22].

3 Optimal control problem specifications

Based on the background theory explained in the last section, this sections gives the formulations used in this work.

The objectives, as mentioned in section 2.1 using sustainability hypotheses, are maximization of time averaged Fisher information and minimization of Fisher information variance over time. The definition of Fisher information given by equation 1 is in terms of state variable vector x. For dynamic systems, there is a one to one correspondence between system evolution (states) and time. Using this relationship and the chain rule of differentiation, the system pdf and Fisher information can be defined in terms of time. The time averaged Fisher information for a system with n species is thus given by:

$$I_t = \frac{1}{T_c} \int_0^{T_c} \left(\frac{a(t)^2}{v(t)^4} \right) dt \tag{6}$$

where T_c is the cycle time of the system and

$$v(t) = \sqrt{\sum_{i=1}^{n} \left(\frac{dx_i}{dt} \right)^2} \tag{7}$$

$$a(t) = \frac{1}{v(t)} \left[\sum_{i=1}^{n} \frac{dx_i}{dt} \frac{d^2 x_i}{dt^2} \right] \tag{8}$$

$v(t)$ and $a(t)$ are called as the velocity and acceleration terms of the ecosystem, respectively. Please refer to [12] for a formal derivation of equation 6.

The objectives can therefore be given as:

- Maximization of Fisher information:

$$J = \text{Max } \frac{1}{T} \int_0^{T} \left(\frac{a(t)^2}{v(t)^4} \right) dt \tag{9}$$

- Minimization of Fisher information variance

$$J = \text{Min } \int_0^{T} (I_t - I_{constant})^2 \, dt \tag{10}$$

Here T is the total time horizon under consideration. I_t given by Eq. (6) is the time averaged FI for one system cycle and $I_{constant}$ is the constant around which the Fisher information variation is to be minimized.

Table 1: Tri-trophic food chain model parameters

Prey	Predator	Super-predator
$x_1(0)$= 100	$x_2(0)$= 75	$x_3(0)$= 150
r = 1.2	a_2 = 2.0	a_3 = 0.1
K = 710	b_2 = 200	b_3 = 250
	e_2 = 1.12	e_3 = 1.12
	d_2 = 1.0	d_3 = 0.04

The top-down and bottom-up control philosophies are compared by performing separate analysis using x_3, the super-predator and x_1 the prey as the control variable, respectively. x_3 is controlled by manipulating the mortality rate d_3 of the super-predator and x_1 is controlled by manipulating the parameter K, which represents the prey carrying capacity of the system. Although control by manipulating K does not strictly represent the bottom-up approach, which proposes control by nutrient addition, nutrient addition affects the prey carrying capacity of the system [30]. The results are therefore expected to give the impact of manipulation in the lower food chain on the upper level in the chain.

The optimal control problem equations are formulated using the Pontryagin's maximum principle (for deterministic model) and the stochastic maximum principle (for stochastic model). Solution of the control equation and derivation of the control variable profile requires solution of the two point boundary value problem, which, for this highly complex differential-algebraic system of equations, is a cumbersome task. Numerical technique of the steepest ascent of Hamiltonian is therefore used to perform the boundary value problem solution [24, 31]. The technique solves the problem as an optimization problem by discretising the solution horizon, using the control variable at each time instant as a decision variable and trying to satisfy the optimality condition at each time point, given a tolerance limit.

The next section gives the simulations results for the deterministic as well as stochastic predator-prey model.

4 Results and discussion

In this analysis, both control options and both objectives are tested on the tri-trophic food chain model given by Eqs. (2), (3) and (4). Moreover, the uncontrolled model is considered to have unstable dynamics. It simulates situations when the ecosystem needs external intervention to avoid imbalance. The analysis assesses the ability of different sustainability based objectives and control philosophies to identify and manage such situations. Two such cases are considered:

- Super-predator extinction: The super-predator population x_3 is going extinct
- Super-predator explosion: The super-predator population x_3 is exploding

Apart from the predator half saturation constant b_2, all the parameters for the two cases are as reported in Table 1. Parameter b_2 is adjusted to change the dynamics of the food chain model. When K or d_3 is the control variable, reported value is the starting guess for the steepest ascent algorithm. The control variables are constrained to avoid numerical problems. Maximum principle can not take care of bounds on control variables. But, since the work uses the steepest ascent of Hamiltonian algorithm, which approximates the solution by maximum principle, it is possible to include constraints on control variables. The constraints are $0.03 \leq d_3 \leq 0.05$ for top-down control and $500 \leq K \leq 900$ for bottom-up control. The uncontrolled system is first simulated for the deterministic model and values of the average Fisher information and Fisher information standard deviation are noted. The model is then subjected to the two different control philosophies. For the stochastic model, uncontrolled system simulation is preceded by sampling of ϵ_t, the variable in the Ito process. The samples are stored and the same sample set is used in all the subsequent simulations of this system to allow a proper basis for comparison. The time dependent predator mortality rate is shown in Fig. 2. The results for the two cases follow.

4.1 Case 1: Super-predator extinction

In order to simulate super-predator extinction, the value of b_2 (predator half saturation constant) is modified to $1/0.0055$.

4.1.1 Deterministic System

Numerical values of the results obtained for this case are reported in Table 2. The super-predator dynamics for this case are shown in Fig. 3. The plots show that all the control options achieve the desired goal of arresting super-predator extinction. The objective of FI maximization, for both control options, exerts a stronger impact and for top-down control, the super-predator achieves a much higher population. The objective of FI variance minimization on the other hand shows a slower and weaker impact but achieves the goal in the long run. The predator-prey dynamics are shown in Fig. 4. It can be noticed that the effect of bottom-up control on the predator-prey dynamics is stronger than the top-down control.

4.1.2 Stochastic System

The numerical results obtained for this case are reported in Table 3 which indicate that the desired objective is achieve in terms of numerical values. The super-predator population dynamics for this case are plotted in Fig. 5. As for the deterministic case, the objective of super-predator extinction is achieved in all cases. Moreover, the relative results are also qualitatively similar to those for the deterministic model leading to the same conclusions that the objective of FI maximization, particularly in combination with top-down control option, has a stronger impact that FI variance minimization. The predator-prey dynamics for this case are quite mixed up. But careful consideration indicates that bottom-up control impacts the predator-prey dynamics more than the top-down control (plots not shown).

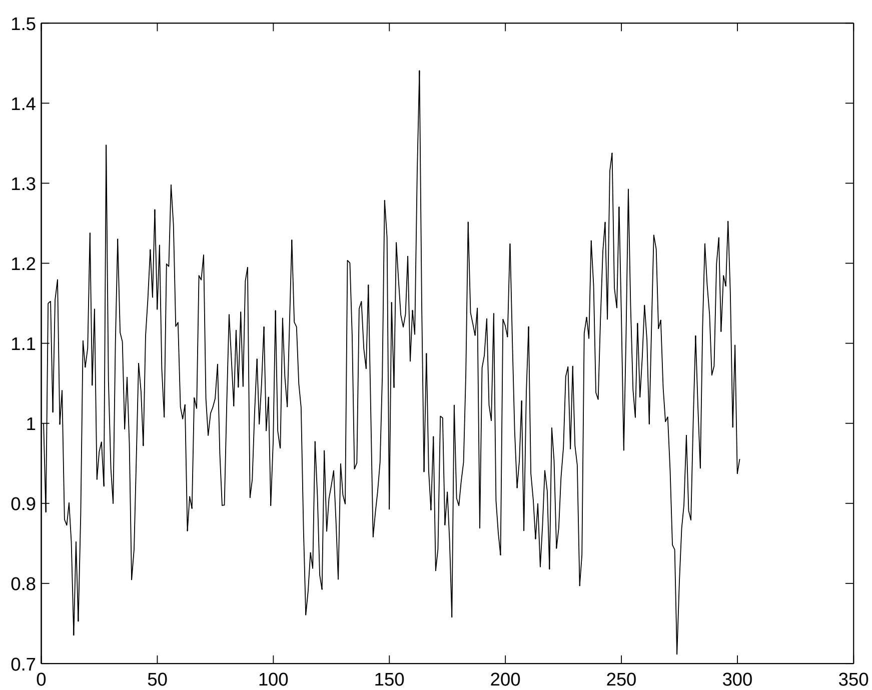

Figure 2: Time dependent random predator mortality rate for stochastic model

Table 2: Deterministic model: Case 1 results

Type of analysis	Top-down control		Bottom-up control	
	FI	FI Standard deviation	FI	FI Standard deviation
Uncontrolled model	4.71×10^{-5}	1.30×10^{-5}	4.71×10^{-5}	1.30×10^{-5}
F.I. Maximization	4.72×10^{-5}	1.50×10^{-5}	8.30×10^{-5}	6.27×10^{-5}
F.I. Variance Minimization	4.72×10^{-5}	1.29×10^{-5}	3.44×10^{-5}	8.71×10^{-6}

Table 3: Stochastic model: Case 1 results

Type of analysis	Top-down control		Bottom-up control	
	FI	FI Standard deviation	FI	FI Standard deviation
Uncontrolled model	1.39×10^{-4}	1.08×10^{-4}	1.39×10^{-4}	1.08×10^{-4}
F.I. Maximization	1.85×10^{-4}	2.25×10^{-4}	1.13×10^{-2}	3.98×10^{-2}
F.I. Variance Minimization	1.36×10^{-4}	1.05×10^{-4}	6.28×10^{-5}	3.09×10^{-5}

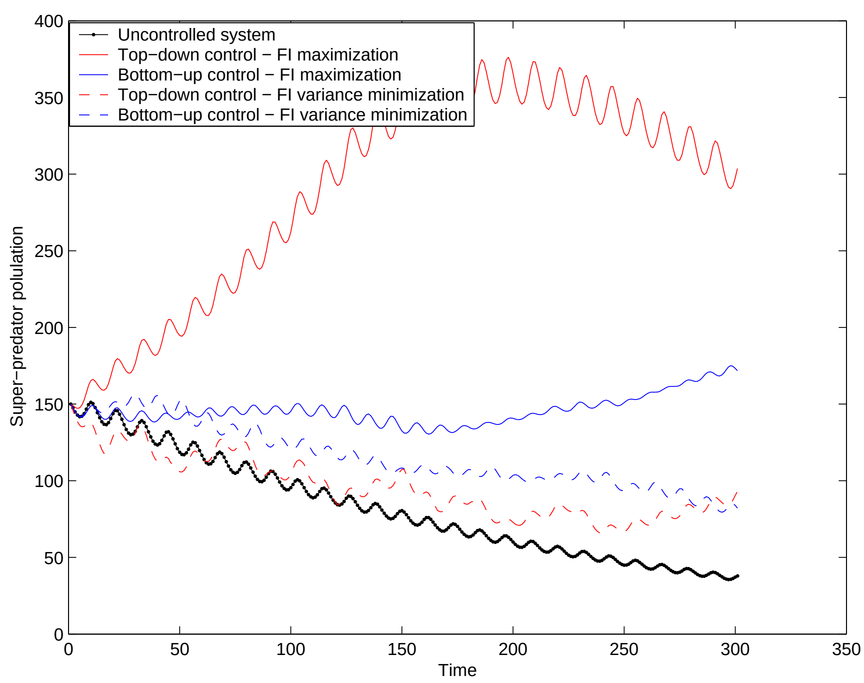

Figure 3: Deterministic model: Super-predator population dynamics for case 1

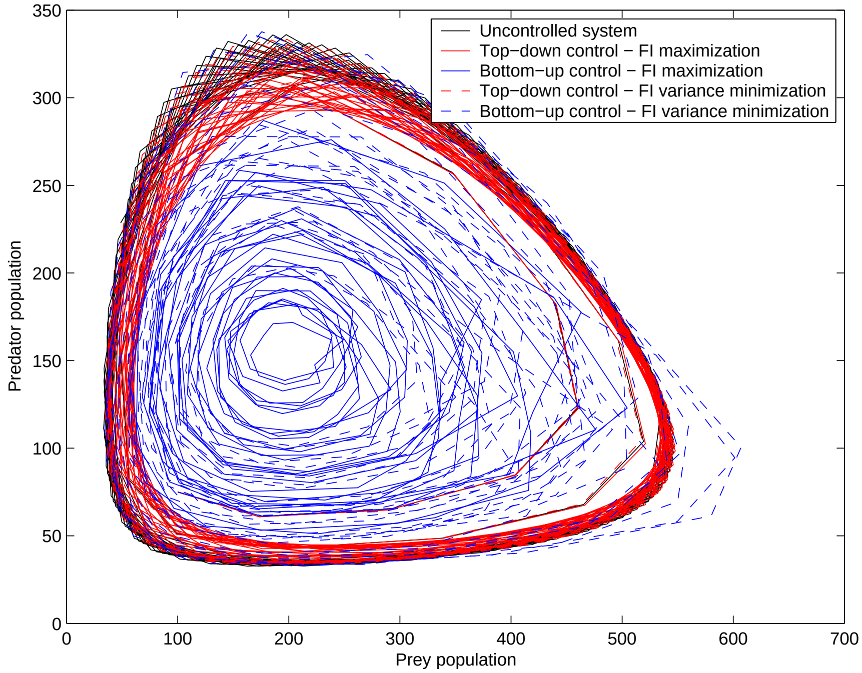

Figure 4: Deterministic model: Predator-prey population dynamics for bottom-up control of case 1

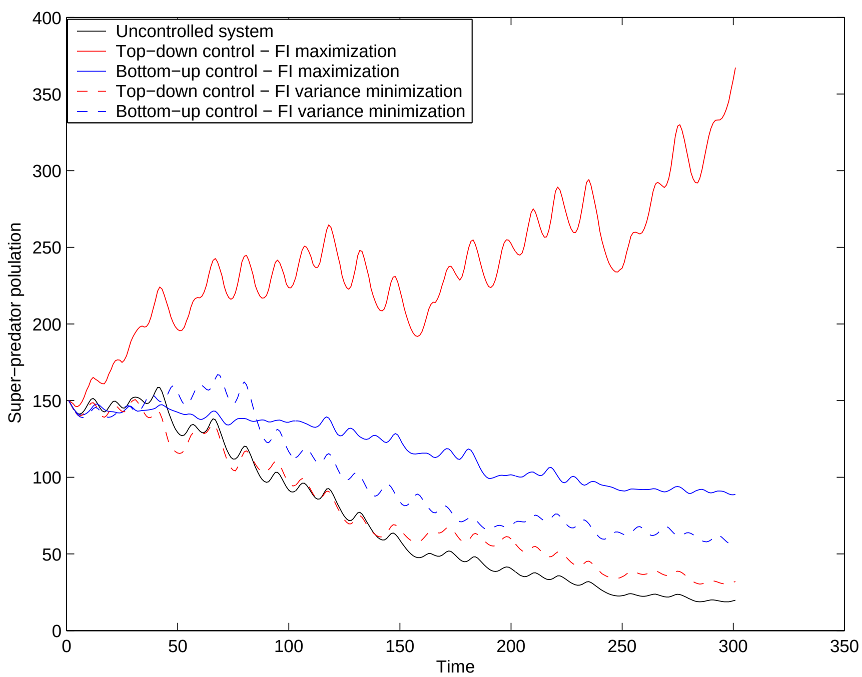

Figure 5: Stochastic model: Super-predator population dynamics for case 1

4.2 Case 2: Super-predator explosion

4.2.1 Deterministic System

To simulate super-predator explosion, b_2 (predator half saturation constant) is modified to $1/0.0049$. Numerical values obtained for this case are reported in Table 4. The super-predator dynamics are plotted in Fig. 6. It can be observed that the objective of FI maximization further supports the super-predator population explosion. The objective of FI variance minimization manages to restrict the super-predator population explosion. The predator-prey dynamics for this system are shown in Fig. 7. The plots reveal that the predator-prey dynamics are more significantly affected by the bottom-up control and this effect is more pronounced for the objective of FI maximization.

4.2.2 Stochastic System

For this case, $b_2 = 1/0.0048$ to simulate super-predator explosion. The numerical values obtained in this case are reported in Table 5 while the super-predator population dynamics are plotted in Fig. 8. The plots indicate that the objective of FI maximization with top-down control increases the super-predator population further. For the other cases, desired goal of controlling the super-predator extinction is achieved. As in case 1, the predator-prey dynamics are mixed up but indicate that the bottom-up control affects the predator-prey dynamics more than the top-down control (plot not shown). A comparison with the deterministic model again suggest a qualitatively similar behavior, apart from the case of FI maximization using bottom-up control, a case for which the deterministic system showed super-predator explosion. This suggests a stabilizing effect of uncertainty on the dynamics.

Table 4: Deterministic model: Case 2 results

Type of analysis	Top-down control		Bottom-up control	
	FI	FI Standard deviation	FI	FI Standard deviation
Uncontrolled model	3.14×10^{-5}	7.76×10^{-6}	3.14×10^{-5}	7.76×10^{-6}
F.I. Maximization	3.31×10^{-5}	8.89×10^{-6}	3.50×10^{-5}	1.09×10^{-5}
F.I. Variance Minimization	3.07×10^{-5}	6.21×10^{-6}	3.00×10^{-5}	3.92×10^{-6}

Table 5: Stochastic model: Case 2 results

Type of analysis	Top-down control		Bottom-up control	
	FI	FI Standard deviation	FI	FI Standard deviation
Uncontrolled model	1.80×10^{-3}	5.80×10^{-3}	1.80×10^{-3}	5.80×10^{-3}
F.I. Maximization	2.64×10^{-1}	1.18	2.60×10^{-3}	8.10×10^{-3}
F.I. Variance Minimization	8.72×10^{-5}	6.29×10^{-5}	6.18×10^{-5}	3.58×10^{-5}

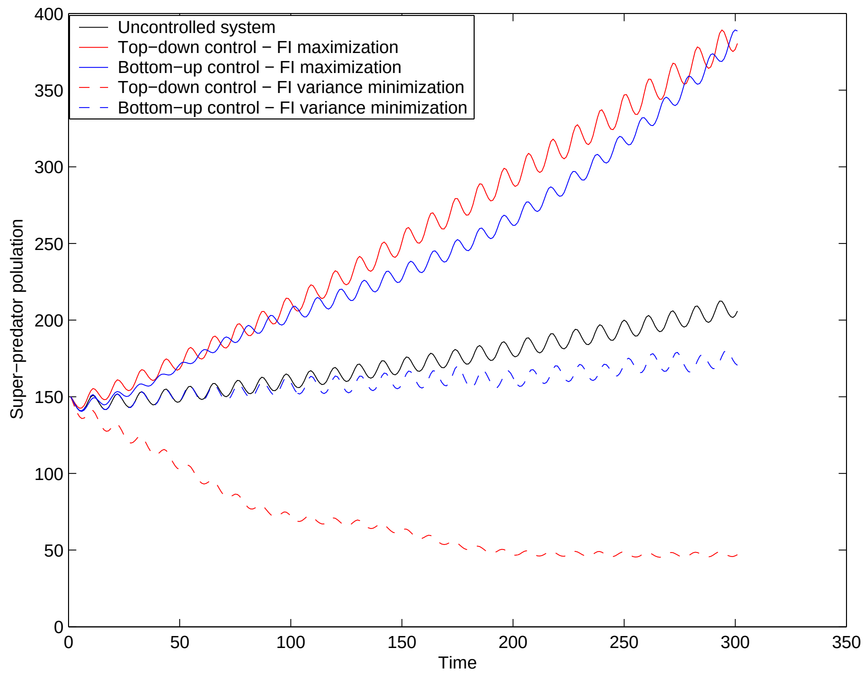

Figure 6: Deterministic model: Super-predator population dynamics for case 2; top-down control and FI maximization objective

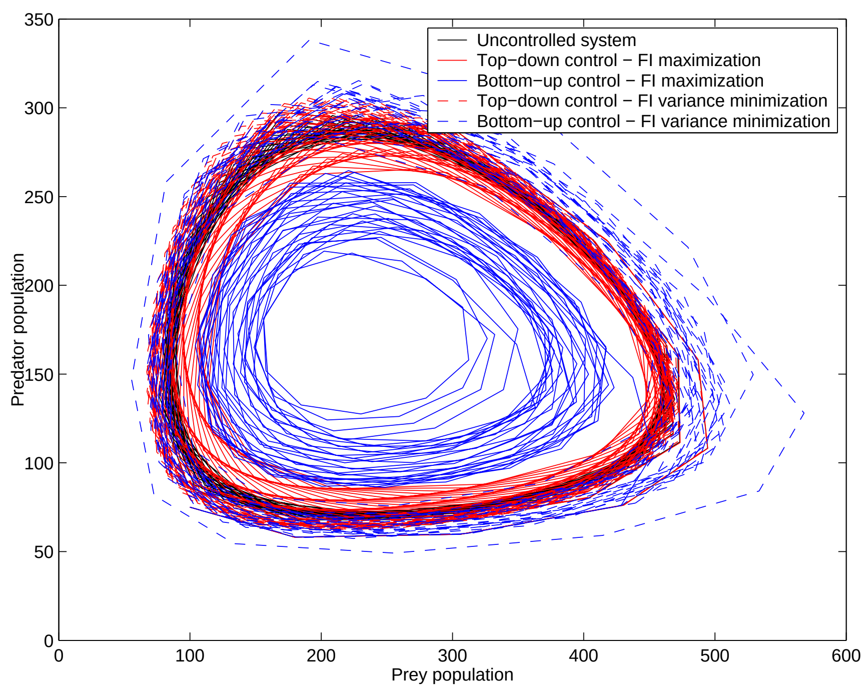

Figure 7: Deterministic model: Predator-prey population dynamics for case 2 with FI maximization objective

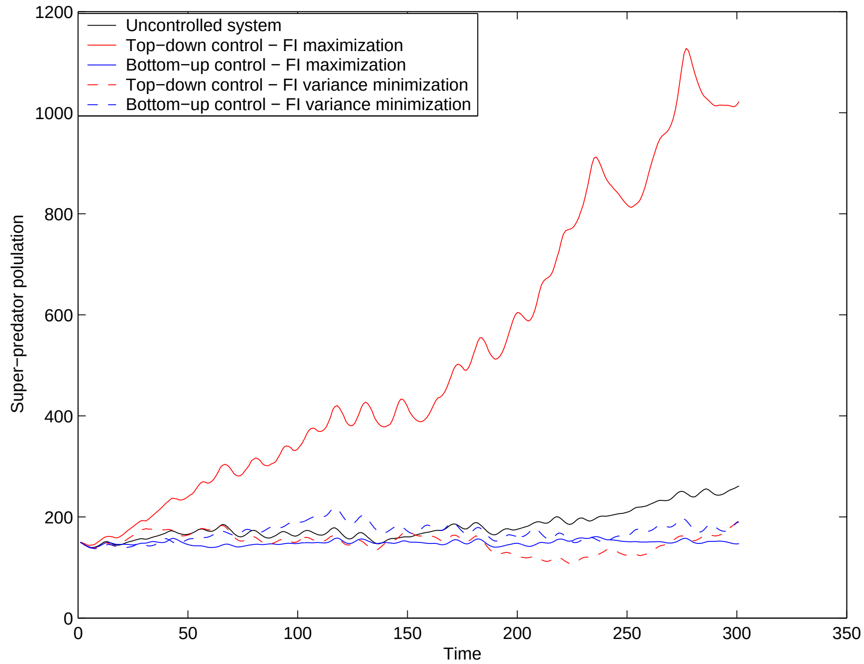

Figure 8: Stochastic model: Super-predator population dynamics for case 2

5 Computational considerations

Since the system of equations being solved is quite complex, computational problems need to be carefully avoided. Depending on the absolute value of the Fisher information, the objective function may need to be linearly scaled up or down using a constant to avoid numerical errors during the solution of the equations. Experience shows that this greatly reduces the convergence time. The termination constant and the step size for the steepest ascent method need to be carefully chosen for converging results. Most often, it is a compromise between faster convergence and running the risk of making the solution divergent. It was also observed that a good initial guess is important to have convergence.

6 Conclusion

The objective of the work was to incorporate ideas of sustainable development in the field of ecosystem management. For this, Fisher information along with sustainability hypotheses was chosen as the indicator of sustainability and aquatic ecosystems, represented by predator-prey models, were considered for application. Section 4 reported the results for the deterministic and stochastic tri-trophic food chain model. The objective was to compare two different control philosophies and two different objectives under different scenarios. An important conclusion to draw from those results is the manifestation of a strong correlation between Fisher information based sustainability hypotheses and ecosystem dynamics. Favorable changes in Fisher information, as suggested by the hypotheses, reflect, in general, in favorable ecosystem dynamics. This should give enough incentives for further investigations into the application of Fisher information for ecosystem management. Based on the results, following conclusions can be drawn:

- The results for the deterministic and stochastic models are qualitatively similar.

- The objective of FI maximization results in an elevated super-predator population in all but one cases. For case 2 when the uncontrolled system shows super-predator explosion, this response is highly undesirable and suggests that the system might become unstable.

- The objective of FI variance minimization achieves the desired objective of super-predator population control in both cases.

- Regarding the predator-prey dynamics, bottom-up control is observed to have a greater impact on it, evident by the population dynamics considerably different from the uncontrolled case, and this impact is more significant for FI maximization objective. For top-down control, the impact on predator-prey dynamics is not significant.

- In terms of the absolute values of the objective functions, the bottom-up controlled systems obtain better values (i.e. higher time averaged FI and lower FI standard deviations) than the top-down controlled systems.

- The objective of FI maximization resulted in significantly worse values of the second objective (standard deviation), while the objective of FI variation minimization did not alter the value of the second objective (average FI) by much.

- For the deterministic model results, the control variable profiles for the FI maximization objective fluctuate more than those for the FI variance minimization objective. The typical profiles are shown in figure 9. For the stochastic model results, control profiles for the FI maximization objective fluctuate much more than than those for the FI variance minimization objective. The typical variations are shown in figure 10. These results suggest that the results from variance minimization objective will be easier to implement on a physical system.

To summarize the results, one can argue that the objective of FI variance minimization will give a stable response without much disturbance while the FI maximization objective may result in significant disturbances and might increase the abundance of the super-predator. It was also observed that the FI variance minimization objective is not able to recover an uncontrolled system from significant disturbances (such as fast specie extinction). It is likely that some of these trends are system dependent. Based on the presented results though, FI variance minimization objective should be preferred in natural systems where maintenance of the system might be preferred over risking significant disturbance in an attempt to improve the state. On the contrary, for systems such as fisheries, where maximization of the fish harvesting is desirable, FI maximization objective should be preferred, particularly since the impacts can be kept localized and system is under better human control than completely natural systems.

The results obtained here are based on simulations of simplified models. It is well accepted that models can never truly represent the complex natural ecosystems. One might therefore doubt the validity of these findings. But the experimental results come with their own set of deficiencies. The results are quite often system specific (i.e. valid only for the particular lake or river), non-reproducible and the observations could be affected by many unknown factors not in direct human control. In such a situation, a strategic combination of theoretical and experimental approaches is needed. Theory helps import novel and proven ideas from other fields into the field of ecosystem management while experiments help estimate the goodness of these findings. The results presented here are therefore to be viewed with this perspective. Application of control theory to achieve sustainable ecosystems should guide an experimentalist to try different management options. It is expected that such an approach, if replaces logic and heuristics, will simplify the task of experimental biologists.

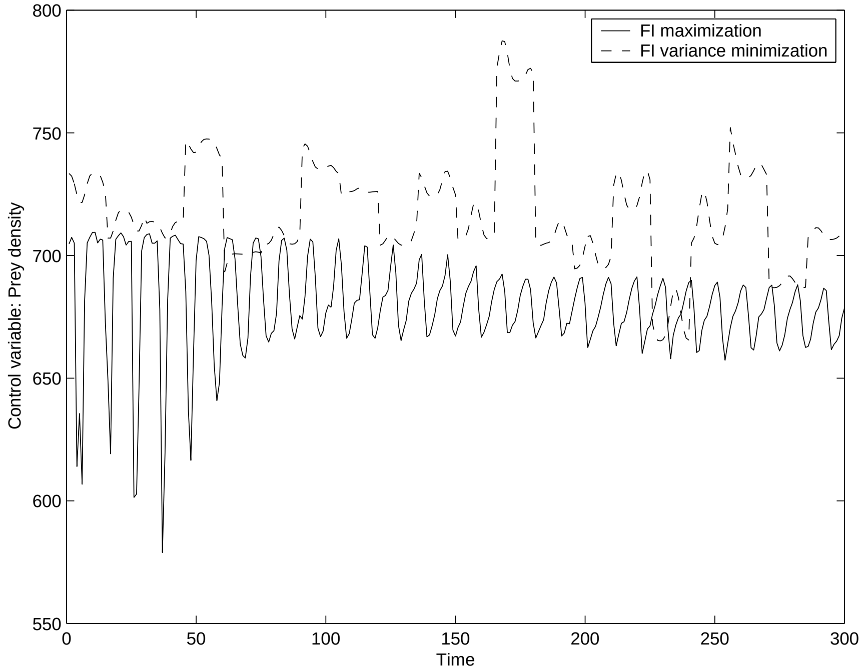

Figure 9: Deterministic model: Bottom-up control variable profile of case 2

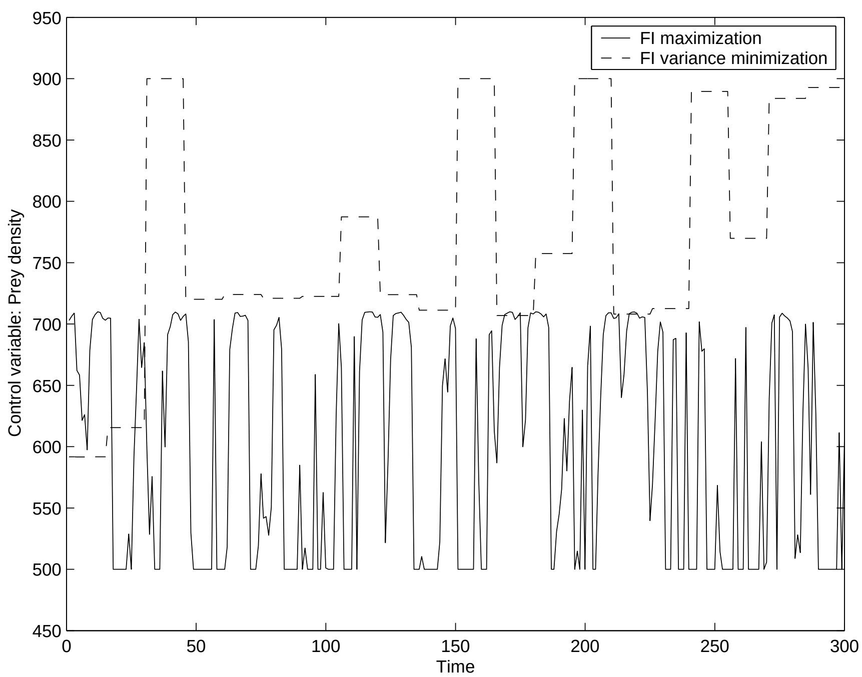

Figure 10: Deterministic model: Bottom-up control variable profile for case 2

Acknowledgement

This work is funded by the National Science Foundation under the grant CTS-0406154.

References

[1] F. Cabezas and B. Fath. Towards a theory of sustainable systems. *Fluid Phase Equilibria*, 2:194–197, 2002.

[2] S.R. Carpenter, J.F. Kitchell, and J.R. Hodgson. Cascading trophic interactions and lake productivity. *BioScience*, 37(10):634–639, 1985.

[3] D.J. McQueen, J.R. Post, and E.L. Mills. Trophic relationships in freshwater pelagic ecosystems. *Canadian Journal of Fisheries and Aquatic Sciences*, 43:1571–1581, 1986.

[4] M.T. Brett and C.R. Goldman. Consumer versus resource control in freshwater pelagic food webs. *Science*, 275(5298):384–386, 1997.

[5] C. Tomlinson. *Our Common Future*. World Commision on Environment and Development, Oxford University Press, Oxford, 1987.

[6] B.R. Frieden. *Physics for Fisher Information: A Unification*. Cambridge University Press, Cambridge, 1998.

[7] C. Shannon and W. Weaver. *The mathematical theory of communication*. University of Illinois Press, Champaign IL, 1949.

[8] J.L. Brooks and S.I. Dodson. Predation, body size, and the composition of plankton. *Science*, 150(3692):28–35, 1965.

[9] H.F. Mejer and S.E. Jørgensen. Exergy and ecological buffer capacity. In S.E. Jørgensen, editor, *Environmental Science and Applications*, volume 7:State-of-the-Art in Ecological Modeling, pages 829–846, Copenhagen, Denmark, 1979. International Society for Ecological Modelling, Pergamon Press.

[10] R.E. Ulanowicz. *Growth and Development: Ecosystems Phenomenology*. Springer, NY, 1986.

[11] Ronald Fisher. On the mathematical foundations of theoretical statistics. In *Philosophical Transactions of the Royal Society of London*, A 222, pages 309–368, 1922.

[12] B.D. Fath, H. Cabezas, and C. Pawlowski. Regime changes in ecological systems: an information theory approach. *Journal of Theoretical Biology*, 222:517–530, 2003. doi:10.1016/S0022-5193(03)00067-5.

[13] N. G. Hairston, F.E. Smith, and L.S. Slobodkin. Community structure, population control, and competition. *American Naturalist*, 94(879):421–425, 1960.

[14] R.T. Paine. Food webs: Linkage interaction strength, and community infrastructure. *Journal of Animal Ecology*, 49:667–685, 1980.

[15] S.R. Carpenter and J.F. Kitchell. Consumer control of lake productivity. *BioScience*, 38(11):764–769, 1988.

[16] C. Townsend. Fish, fleas and phytoplankton. *New Scientist*, 118(1617):67–70, 1988.

[17] M.R.S. McQueen, D.J. Johannes, J.R. Post, T.J. Stewart, and D.S.R. Lean. Bottom-up and top-down impacts on freshwater pelagic community structure. *Ecological Monographs*, 59(3):289–309, 1989.

[18] M. Lynch and J. Shapiro. Predation, enrichment and phytoplankton community structure. *Limnology and Oceanography*, 26(1):86–102, 1981.

[19] M.J. Vanni and J. Temte. Seasonal pattern of grazing and nutrient limitation of phytoplankton in a eutrophic lake. *Limnology and Oceanography*, 35(3):697–709, 1990.

[20] A.D. Rosemond, P.J. Mulholland, and J.W. Elwood. Top-down and bottom-up control of stream periphyton: Effects of nutrients and herbivores. *Ecology*, 74(4):1264–1280, 1993.

[21] D. Ludwig, S. Carpenter, and W. Brock. Optimal phosphorous loading for a potentially eutrophic lake. *Ecological Applications*, 13(4):1135–1152, 2003.

[22] U.M. Diwekar. *Introduction to Applied Optimization*. Kluwer Academic Publishers, Dordrecht, 2003.

[23] V. Rico-Ramirez and U. Diwekar. Stochastic maximum principle for optimal control under uncertainty. *Computers and Chemical Engineering*, 28:2845–2845, 2004. doi:10.1016/j.compchemeng.2004.08.001.

[24] D. Kirk. *Optimal Control Theory: an introduction*. Prentice-Hall, Englewood Cliffs, NJ, 1970.

[25] C. Ryan Gwaltney, M.P. Styczynski, and M.A. Stadtherr. Reliable computation of equilibrium states and bifurcations in food chain models. Technical report, Departement of Chemical and Biomolecular engineering, University of Notre Dame, 182 Fitzpatrick Hall, Notre Dame, IN 46556, USA, 2003.

[26] P.A. Abrams and J.D. Roth. The effects of enrichment of three-species food chains with non-linear functional responses. *Ecology*, 75(4):1118–1130, 1994.

[27] O. De Feo and S. Rinaldi. Yield and dynamics of tritrophic food chains. *American Naturalist*, 150(3):328–345, 1997.

[28] A. Gragnani, O. De Feo, and S. Rinaldi. Food chains in the chemostat: Relationships between mean yield and complex dynamics. *Bulletin of Mathematical Biology*, 60(4):703–719, 1998. doi:0092-8240/98/040703 + 17.

[29] J. Duggempudi and U. Diwekar. Sustainability, options theory, and quality control. In *Proceedings of AIChE National Meeting, Sustainability and Life Cycle Topical Conference*, San Francisco, 2003.

[30] P.A. Abrams. Effect of increased productivity on the abundances of trophic levels. *American Naturalist*, 141(3):351–371, 1993.

[31] U.M. Diwekar. *Batch Distillation: Simulation, Optimal Design and Control*. Taylor and Francis, Washington, DC, 1996.