539a Distribution Dynamics of Complex Systems

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Analyzing and designing complex systems, defined as having many interacting and interdependent parts with emergent self-organization, represents a challenge in engineering science. Many natural and engineered systems are composed of different interdependent parts that may be described by a distribution of properties and may be linked together in a network configuration. We have examined the distribution kinetics and dynamics of such systems by population balance methods. Power law distributions of properties (e.g., size) are often found in fabricated and natural systems. For example, the size distributions of particles, aerosols, corporations, and cities are often power laws. Each system is an ensemble of clusters, comprising units that combine with or dissociate from the cluster. Constructing models and investigating their properties are needed to understand how such systems evolve. To describe the growth of clusters, we hypothesize that a distribution obeys a governing population dynamics equation based on a reaction-like reversible process. The rate coefficients are considered generally to depend on the cluster size as power expressions, thus providing an explanation for the evolution of power law distributions. Complex systems can frequently be described as networks composed of many nodes and links, for example, metabolic pathways, ecosystems, the Internet, the World-Wide-Web, highways, the US power grid, the propagation of infections, and regulatory gene networks. How networks come into existence and how they change with time are fundamental issues in such physical and social systems. Based on the concept of a nodal-linkage distribution, we propose a unified population dynamics approach for the evolution of networks to exponential (single-scale) or power law (scale-free) conformations. The functional form of the rate coefficients for addition or removal of links governs the asymptotic forms, which are independent of initial states. The population balance equation, cast either as an integrodifferential equation, a difference-differential equation, or a partial differential equation, can be solved by standard methods, including a moment method. For non-linearly growing networks, when the total number of connections increases faster than the total number of nodes, the network is said to accelerate. We propose a systematic model for the dynamics of growing networks in the context of their kinetics represented by distribution dynamics equations. We define the nodal-linkage distribution, construct a population dynamics equation based on the association-dissociation process with the proposed rate coefficients, $k_g(\xi) = \gamma\xi^\lambda$ and $k_d(\xi) = \kappa\xi^\nu$, and perform the moment calculations to describe the network dynamics. For nondirectional networks, the moments are the total number of nodes, the total number of connections, and the degree distribution (the average number of connections per node), represented by the average moment. Size independent rate coefficients ($\lambda = 0$) yield an exponential network describing the network without preferential attachment; size dependent rate coefficients produce a power law network with preferential attachment.