A General Resource-constrained Short-term Scheduling Model for Multipurpose Batch Plants using Synchronous Slots

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Abstract

Short-term scheduling of multipurpose batch plants is a challenging problem for which several formulations exist in the literature. In these plants, there are many kinds of resource constraints which often lead these problems even tougher. Based on the work of Sundaramoorthy & Karimi (2005), we extend the mixed integer linear programming (MILP) formulation to the problems with different kinds of resource constraints. Due to the same continuous-time representation with synchronous slots with the original formulation, our extended formulation with fewer binary variables, constraints, and nonzeros seems to have good efficiency for these tough problems. Finally, several examples will be illustrated to demonstrate the superiority of our model.

INTRODUCTION

Short-term scheduling of multipurpose batch plants has received considerable attention in the last decade. Early attempts (Kondili et al., 1993, Shah et al., 1993) used mixed integer linear programming (MILP) formulations based on the uniform discrete-time representation. However, as the advantages of alternate representations such as non-uniform discrete-time (Mockus and Reklaitis, 1994; Lee et al., 2001) and continuous-time became clear, the recent trend (Ierapetritou and Floudas, 1998; Castro et al., 2001; Giannelos and Georgiadis, 2002; Maravelias and Grossmann, 2003) has favored continuous-time representations.

The research efforts using continuous-time representation in batch process scheduling have opted to tag themselves with two flavors. The so called slot-based formulations (Karimi & McDonald, 1997) represent time in terms of ordered blocks of unknown variable lengths. The so called event-based formulations (Ierapetritou and Floudas, 1998; Giannelos and Georgiadis, 2002) use unknown points in time at which events such as starts of tasks may occur. Maravelias and Grossmann (2003) recently attempted to rationalize the different types of time representation.

Sundaramoorthy & Karimi (2005) presented a slot-based continuous-time formulation for short-term scheduling in multipurpose batch plants. Their formulation required no big-M constraints and a few binary variables. In their evaluations on a variety of test problems, they proved that their formulation performed best compared to other formulations (Maravelias and Grossmann, 2003; Giannelos and Georgiadis, 2002) in the literature. In their work, they considered only materials and units as resources. In this paper, we extend their basic formulation to account for utilities besides materials and units. We present an example to illustrate performance of our model.

We begin with a problem statement followed by a brief discussion on the basic formulation. We then present the extension to it, and finally illustrate its performance with an example.

PROBLEM STATEMENT

A multipurpose batch plant or production facility (F) produces multiple products using a number of shared production units that constrain the plant operation. We use recipe diagram (RD) to describe the productions in F. The facility houses J (j = 1, 2, …, J) units and performs I (i = 1, 2, …, I) tasks. Each unit j can perform a set $\boldsymbol{I}_j$ of tasks in the RD. Similarly, a set $\boldsymbol{J}_i$ of units can perform a task i. We use index m to represent materials in the RD. Let $\boldsymbol{M}_i$ denote the set of materials ($m \in \boldsymbol{M}_i$) that a task i consumes or produces. $\boldsymbol{M}_i$ includes all the different states of raw materials, intermediates, and final products associated with task i. For each task i, we propose a general mass balance as,

$$\sum_{m \in \boldsymbol{M}_i} \sigma_{mi} (\text{Material } m) = 0$$

where, σ_{mi} is analogous to the stoichiometric coefficient of a species in a chemical reaction except that it can be in kg/kg units instead of mol/mol. Thus, $\sigma_{mi} < 0$, if task i consumes material $m \in \boldsymbol{M}_i$ and $\sigma_{mi} > 0$, if it produces $m \in \boldsymbol{M}_i$. Furthermore, for each task i, we designate a primary material μ_i, with respect to which we define the extent of task i. The batch size of a task i is defined as the amount of the primary material μ_i that task i consumes or produces in a batch.

For the short-term scheduling of such a multipurpose batch plant with resource constraints, we need to determine:

(i) the optimal sequence and schedule of different tasks on each unit
(ii) the batch size of each batch of each task on each unit at various times

using:

(a) RD for the plant with material and unit requirements
(b) Suitability of units (processing and storage) and utilities for tasks, their capacity limits, and batch processing time information
(c) Time horizon H for profit maximization or fixed product demands D_m for makespan minimization
(d) Final product revenues, net or otherwise
(e) Time and schedule of resource consumption

We consider only two scheduling objectives (maximizing the profit/revenue from the sales of finished products and minimizing the makespan) in this work. However, other objectives such as minimizing inventory, production, or setup costs and even minimizing the tardiness or earliness can be readily accommodated in the proposed formulation with minor modifications. We assume the following in our formulation.

1. Transfer and setup times are lumped into batch processing times of tasks.
2. The batch processing time of task i on unit j is either a constant (τ_{ij}) or varies linearly with its batch size as $\alpha_{ij} + \beta_{ij}$(Batch size), where α_{ij} and β_{ij} respectively are known.
3. Product revenues have accounted for various production costs.

MILP FORMULATION

Sundaramoorthy & Karimi (2005) proposed a novel slot-based continuous-time MILP formulation for short-term scheduling in multipurpose batch plants. Their formulation uses no big-M constraints, fewer binary variables, constraints and nonzeroes. They believed that all these features contributed to the computational superiority of their formulation. Since we will extend their work for general resource constraints, we discuss their formulation briefly in the following section. We use the same notations for variables and parameters. Further details can be seen in Sundaramoorthy & Karimi (2005).

$$T_k = T_{k-1} + SL_k \tag{1}$$

$$\sum_{k=1}^{K} SL_k \le H \tag{2}$$

Eqs. (1) and (2) define the slot arrangement in the time representation used by this formulation.

$$Z_{jk} = \sum_{i \in \boldsymbol{I}_j} Y_{ijk} \qquad 0 \le k < K \tag{3}$$

$$B_{ij}^{L}\, Y_{ijk} \le B_{ijk} \le B_{ij}^{U}\, Y_{ijk} \qquad i > 0 \tag{4}$$

Eq. (3) makes sure that at most one task can start on a unit j at any T_k. Eq. (4) enforces that when task i does not start at T_k, then $B_{ijk} = 0$, and vice versa.

$$y_{ijk} = y_{ij(k-1)} + Y_{ij(k-1)} - YE_{ijk} \qquad 0 < k < K \tag{5}$$

$$Z_{jk} = \sum_{i \in \boldsymbol{I}_j} YE_{ijk} \qquad 0 < k < K \tag{6}$$

$$\sum_{i \in \boldsymbol{I}_j} y_{ijk} + Z_{jk} = 1 \qquad 0 < k < K \tag{7}$$

$$y_{ijk} + Y_{ijk} \le 1 \qquad 0 < k < K \tag{8a}$$

$$y_{ijk} + YE_{ijk} \le 1 \qquad 0 < k < K \tag{8b}$$

About five equations define status of a processing unit. Eqs. 3, 5, 6, 7, and 8 force y_{ijk} and YE_{ijk} to be 0 or 1 only (even though they are continuous 0-1 variables), as long as Y_{ijk} are binary. In fact, it is easy to see that eqs. 3, 6, and 7 make eqs. 8a-b redundant. Also, with eq. 5 in effect, one of eqs. 3, 6, and 7 is redundant.

$$t_{j(k+1)} \ge t_{jk} + \sum_{i \in \boldsymbol{I}_j} (\alpha_{ij} Y_{ijk} + \beta_{ij} B_{ijk}) - SL_{(k+1)} \qquad k < K \tag{9}$$

$$b_{ijk} = b_{ij(k-1)} + B_{ij(k-1)} - BE_{ijk} \qquad i > 0,\ k > 0 \tag{10}$$

$$B_{ij}^{L}\, y_{ijk} \le b_{ijk} \le B_{ij}^{U}\, y_{ijk} \qquad i > 0,\ 0 < k < K \tag{11}$$

$$B_{ij}^{L}\, YE_{ijk} \le BE_{ijk} \le B_{ij}^{U}\, YE_{ijk} \qquad i > 0,\ 0 < k < K \tag{12}$$

$$t_{jk} \le \sum_{i \in \boldsymbol{I}_j} (\alpha_{ij} y_{ijk} + \beta_{ij} b_{ijk}) \qquad 0 < k < K \tag{13}$$

Eq. (9) gives a time balance at T_{k+1} in unit j. Similar to the status of task assignment on a unit, Eqs. (10)-(13) define batch amount balance in unit j

$$I_{mk} = I_{m(k-1)} + \sum_{i \in \boldsymbol{OI}_m, i \ne 0} \sum_{j \in \boldsymbol{J}_i} \frac{\sigma_{mi}}{\sigma_{\mu_i}} BE_{ijk} + \sum_{i \in \boldsymbol{II}_m, i \ne 0} \sum_{j \in \boldsymbol{J}_i} \frac{\sigma_{mi}}{\sigma_{\mu_i}} B_{ijk} \tag{14}$$

Eq. (14) involves with the inventory balance for a material m at T_k.

$$SL_k \le \max_j \left[\max_{i \in \boldsymbol{I}_j} \left(\alpha_{ij} + \beta_{ij} B_{ij}^{U} \right) \right] \tag{15}$$

$$t_{jk} \le \max_{i \in I_j}\left(\alpha_{ij} + \beta_{ij} B_{ij}^U\right) \qquad (16)$$

$$I_{mk} \le I_m^U \qquad (17)$$

$$B_{ijk},\ b_{ijk},\ BE_{ijk} \le B_{ij}^U \qquad (18)$$

$$Z_{jk},\ y_{ijk},\ YE_{ijk},\ SL_k,\ t_{jk},\ B_{ijk},\ b_{ijk},\ BE_{ijk},\ I_{mk} \ge 0 \qquad (19)$$

Finally, the following equations impose upper and lower bounds on all variables in the formulation.

$$P = \sum_m g_m I_{mK} \qquad (20)$$

$$I_{mK} \ge D_m \qquad (21)$$

$$MS = \sum_{k=1}^{NK} SL_k \qquad (22)$$

With Eq. (20) as objective function, the formulation for maximizing sales or revenue comprises eqs. 2-6, 9-14, 20, and the bounds (eqs. 15-19). While using Eqs. (21) and (22), the complete formulation for minimizing the makespan comprises eqs. 3-6, 9-14, 21, 22, and the bounds (eqs. 15-19).

EXTENSION TO BASIC FORMULATION

The above basic formulation has not considered resources other than materials and units (processing and storage). However, there are several other resources such as utilities and manpower that can significantly impact the production amounts. In the following, we mainly consider the availability of utilities such as steam, cooling water and electricity for a smooth production. These utilities are available only in limited quantities, which each production task has to share. Thus, scheduling of production tasks must monitor the availability of utilities throughout the horizon so that they are consumed without any violation in the limits.

We assume that whenever a task *i* begins on unit *j*, a fixed amount (σ_{uij}) of utility *u* is consumed. Based on the batch size B_{ijk}, an additional amount (τ_{uij}) of utility *u* is also required. Since we synchronize the slots on all units at each time point, we are able to monitor the availability of utilities easily. Let U_{uk} represent the total consumption of utility *u* at T_k. Then,

$$\sum_{i \in I_j} \sum_j \sigma_{uij}(Y_{ijk} + y_{ijk}) + \tau_{uij}(B_{ijk} + b_{ijk}) = U_{uk} \qquad (23)$$

Obviously, U_{uk} should be within available limits at each time point. Thus,

$$0 \le U_{uk} \le U_u^{\max} \qquad (24)$$

Where $U_u^{\max}$ is the maximum available limit of utility *u*. Eqs. 23 and 24 along with the basic formulation accounts for constraints on resources such as materials, production units, storage units and utilities.

EXAMPLE

We considered Example 2 from Maravelias and Grossmann (2003) to test the performance of our model. All the necessary data and description of the example can be seen in the above paper. Given the price and demand details, the objective is to maximize the sales of products. We solved the above problems in GAMS 21.7 (Brooke et al., 1998) using CPLEX 9.0 on IBM P4 machine with 512 MB RAM. The model required 50 binary variables, 407 continuous variables, 424 constraints and 1469 nonzeroes. Our model took 8.2 CPU s to obtain the optimal value of $6499.31. The optimal schedule gave a detailed plan of production activities for the given horizon of 8h without any violation of the available resources.

CONCLUSION

The novel continuous-time formulation presented in this paper uses synchronous slots and does not decouple tasks from units (i.e. uses 3-index binary assignment variables), but it still has fewer binary variables, constraints, and nonzeros. At the same time it is simpler, more efficient, and potentially tighter than the best models (event-based or otherwise) in the literature on short-term scheduling in multipurpose batch plants. In contrast to the existing models, it is equally efficient for both sales maximization and makespan minimization even with variable batch processing times, and has no big-M constraints. We believe that the latter may be a major contributor to our model's better efficiency. Lastly, this paper presents a novel idea of balances (time, mass, resource, etc.) in developing scheduling formulations, and considers resource constraints on materials, units and utilities.

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