520f Optimal Input Design for Identification of Nonlinear Systems

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Certain classes of nonlinear systems can be described by a finite order Volterra series expansion, which can be viewed as a generalization of the convolution integral for a Linear Time Invariant System. The familiar property of superposition that is valid for stable LTI systems is not valid for nonlinear systems. Hence, the steady state output of a general nonlinear system (under certain general conditions) when subjected to a sum of harmonically related sines or multi-sine input is a multi-harmonic signal containing in general, the input frequencies (the contribution due to the linear kernel) and additional harmonics (due to the nonlinear kernels).

Parameterization of nonlinear systems by Volterra series requires a large number of parameters. Hence, if the goal is to identify both the linear and the nonlinear contributions, the number of parameters to be identified and consequently identification time can be particularly large. Several runs with varying amplitudes or other characteristics of the inputs may need to be performed. Hence, the input should be carefully chosen so that as many parameters can be independently estimated as possible to reduce identification time and improve quality of estimated parameters.

Some of these ideas are more intuitive when considering identification in the frequency domain. Consider a nonlinear system that has a linear component and a second order contribution. The ultimate output of such a system to a multi-sine input is composed of the input frequencies (arising from the linear component) and additional frequencies. These additional frequencies are exactly those that are the sums and differences of the original input frequencies taken two at a time. In general, several nonlinear combinations can contribute to a measured output frequency and hence, independent estimates of the respective combinations is not possible. The problem is to choose the input frequencies so that the contributions due to the nonlinear effects fall at distinct frequencies that are both different from the input frequencies and different from each other, whenever possible. This will permit adequate use of the signal energy, since each measured frequency will have a distinct contribution. This property is sometimes referred to as “frequency separation” in literature.

Apart from frequency separation, it is often necessary to ensure that the smallest harmonic and the largest harmonic in the input are small and that the grid is evenly spaced out. There are several objectives that need to be satisfied and hence, choosing a suitable set of input harmonics or “input grid” is a multi-objective optimization problem. Choosing such an input set for a higher number of harmonics is not trivial and there have been some suggestions in literature (Boyd et al. 1983; Chua and Liao, 1989; Rees et al 1996, 2002, Schoukens et al. 2002). However, the solutions described are either not necessarily optimal with respect to the above criteria or do not satisfy the frequency separation requirement exactly.

A multi-objective optimization approach for phase selection has been presented previously by the authors (Narasimhan and Rengaswamy, 2004). We present a novel Integer Linear Programming (ILP) formulation to determine the optimal harmonics of the input signal. The frequency separation requirements form an exclusion disjunction and are coded in a novel manner using binary variables and linear constraints. The multiple objectives are handled in a lexicographic manner to arrive at a an optimal choice for the input harmonics. The ILP formulation can be extended to designing input signals for identification of higher order kernels. The above described approach is validated by simulated identification experiments of nonlinear systems.

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