Paper 520b

An Integrated Methodology for Plant-Friendly Input Signal Design and Control-Relevant Estimation of Highly Interactive Processes

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Abstract

A novel methodology that integrates a directional multisine input design procedure and control-relevant parameter estimation is developed in this paper, leading to desirable models for the control of highly interactive multivariable process systems. *A priori* information on system directionality is utilized in the input design procedure and recognized in the subsequent parameter estimation step, which consists of control-relevant curvefitting of frequency responses obtained from identification data. With input-output data based on a directionally adjusted input signal, the control-relevant parameter can accurately estimate the dynamic singular values of a system. As a result, a systematic procedure for generating a control-relevant model with balanced gain directionality is developed, appropriate for highly interactive processes. A case study involving a binary distillation column and Model Predictive Control is presented in this paper to demonstrate the effectiveness of the proposed approach.

1 Introduction

Effective system identification of highly interactive processes for multivariable control purposes has been viewed as a challenging problem by many investigators (Andersen and Kümmel, 1992; Chien and Ogunnaike, 1992; Jacobsen and Skogestad, 1994; Koung and MacGregor, 1993; Li and Lee, 1996). Such systems respond largely in the high gain direction by virtue of strong interaction, which makes it non-trivial to precisely capture low gain directionality in the data (Andersen and Kümmel, 1992; Morari and Zafiriou, 1988; Varga and Jørgensen, 1994; Zhu, 2001; Chou *et al.*, 2000). Conventional multivariable input signal designs are usually inadequate for estimating accurate gain directionality, particularly under

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noisy conditions (Koung and MacGregor, 1993; Koung and MacGregor, 1994; Jacobsen, 1994). Therefore, multivariable identification techniques that can address strong ill-conditioning and interaction are valuable for advanced control applications.

With this challenge in mind, we consider the problem of developing an input design procedure that takes advantage of *a priori* knowledge of gain directionality to obtain an informative input signal, and examine the effective use of data generated from this signal in the subsequent step of control-relevant parameter estimation. The resulting model serves as a useful nominal model for a high performance advanced control system, such as Model Predictive Control.

Recently, a multisine signal design with modified zippered spectrum meaningful for highly interactive systems was proposed by the authors (Lee *et al.*, 2003*b*). In this paper, the design procedure is extended to systematically enable the user to emphasize any particular direction of interest with a desirable level of power; in most cases, the user-specified direction will correspond to the weak gain direction. Constrained optimization techniques can be further applied to these signals to enable plant-friendly implementation (Lee *et al.*, 2003*b*; Lee *et al.*, 2003*a*).

In the control-relevant parameter estimation step, directionality is systematically considered through the specification of weights which emphasize control performance requirements; these weights are consistent with preserving the low gain output direction which is demanded by high performance advanced control systems (de Callafon *et al.*, 1996; Bayard, 1994; Gaikwad and Rivera, 1997; Lee and Rivera, 2005). Curvefitting of frequency responses obtained from input-output data to discrete-time models corresponding to a Matrix Fraction Description (MFD) model representation (de Callafon *et al.*, 1996) is accomplished using a computationally fast numerical procedure that recognizes the presence of orthogonal ("zippered") frequency grids (Lee and Rivera, 2005).

The principal purpose of this work is to present a comprehensive procedure involving directional multisine input design with control-relevant curvefitting, meaningful to the control of demanding multivariable systems, such as highly interactive processes. The analysis of the paper is demonstrated using a case study based on the high-purity distillation column by Jacobsen and Skogestad (1994) that illustrates the effectiveness of this integrated methodology. This paper is organized as follows: Section 2 introduces a brief overview of multisine input signals. Section 3 describes a design procedure for directional multisine input signals, suitable for estimating highly interactive systems. Section 4 summarizes the control-relevant parameter estimation problem. Section 5 describes the distillation column case study and Section 6 presents Summary and Conclusions.

2 Multisine Input Signal Designs

Multisine signals are deterministic, periodic signals whose power spectrum can be directly specified by the user (Guillaume *et al.*, 1991; Schroeder, 1970). A multisine input $x_j(k)$ for the *j*-th channel of a multivariable system with *m* inputs can be defined as,

$$\begin{aligned} x_j(k) &= \sum_{i=1}^{m\delta} \hat{\delta}_{ji} \cos(\omega_i kT + \phi_{ji}^{\delta}) + \sum_{i=m\delta+1}^{m(\delta+n_s)} \alpha_{ji} \cos(\omega_i kT + \phi_{ji}) \\ &+ \sum_{i=m(\delta+n_s)+1}^{m(\delta+n_s+n_a)} \hat{a}_{ji} \cos(\omega_i kT + \phi_{ji}^{a}),\ j = 1,\ldots,m \end{aligned} \quad (1)$$

where T is sampling time, N_s is the sequence length, m is the number of channels, δ, n_s, n_a are the number of sinusoids per channel ($m(\delta+n_s+n_a) = N_s/2$), $\phi_{ji}^{\delta}, \phi_{ji}, \phi_{ji}^{a}$ are the phase angles, α_{ji} represents the Fourier coefficients defined by the user, $\hat{\delta}_{ji}, \hat{a}_{ji}$ are the "snow effect" Fourier coefficients (Guillaume *et al.*, 1991), and $\omega_i = 2\pi i/N_sT$ is the frequency grid. Here, users should provide Fourier coefficients in terms of an input power spectrum and phases for the multisine inputs that determine the properties of input signals.

In designing an input signal, the primary frequency band of interest for excitation is determined by the dominant time constants of the system to be identified and the desired closed-loop speed-of-response,

$$\omega_* = \frac{1}{\beta_s \tau_{dom}^H} \leq \omega \leq \omega^* = \frac{\alpha_s}{\tau_{dom}^L} \tag{2}$$

α_s and β_s that specify the high and low frequency ranges of interest in the signal, respectively for a given range of low and high dominant time constants (defined by τ_{dom}^L and τ_{dom}^H). The bandwidth per (2) is bounded by the following inequality based on the choice of design parameters,

$$\frac{2\pi m(1+\delta)}{N_sT} \leq \omega_* \leq \omega \leq \omega^* \leq \frac{2\pi m(n_s+\delta)}{N_sT} \leq \frac{\pi}{T} \tag{3}$$

which in turn translates into the following inequalities for number of sinusoids, sampling time, and sequence length (n_s, T, and N_s, respectively):

$$(1+\delta)\frac{\omega^*}{\omega_*} \leq n_s+\delta \leq \frac{N_s}{2m} \tag{4}$$

$$T \leq \min\left(\frac{\pi}{\omega^*}, \frac{\pi}{\omega^*-\omega_*}(\frac{n_s-1}{n_s+\delta})\right) \tag{5}$$

$$\max\left(2m(n_s+\delta), \frac{2\pi m(1+\delta)}{\omega_* T}\right) \leq N_s \leq \frac{2\pi m(n_s+\delta)}{\omega^* T} \tag{6}$$

For more details on guidelines for choosing parameter variables in the input design the reader is referred to (Lee *et al.*, 2003*b*).

2.1 Zippered Multisine Input Signals

A "zippered" power spectrum uses orthogonal frequency grids for each input channel that makes a signal length longer than that of a shifted signal design (see Figure 1). A zippered power spectrum gives independence between channels and provide greater flexibility to the design interface, i.e., the Fourier coefficients and phases of each input can be determined independently (Lee *et al.*, 2003*b*). To achieve a zippered spectrum we define the Fourier coefficients α_{ji} as:

$$\alpha_{ji} = \begin{cases} \neq 0, & i = m\delta + j, m(\delta+1)+j, \ldots, m(\delta+n_s-1)+j \\ = 0, & \text{for all other } i \text{ up to } m(\delta+n_s) \end{cases} \tag{7}$$

Theoretical system requirements such as persistence of excitation, harmonic suppression (a key consideration in the identification of nonlinear systems), and control-relevance can be satisfied without loss of generality through the specification of Fourier coefficients.

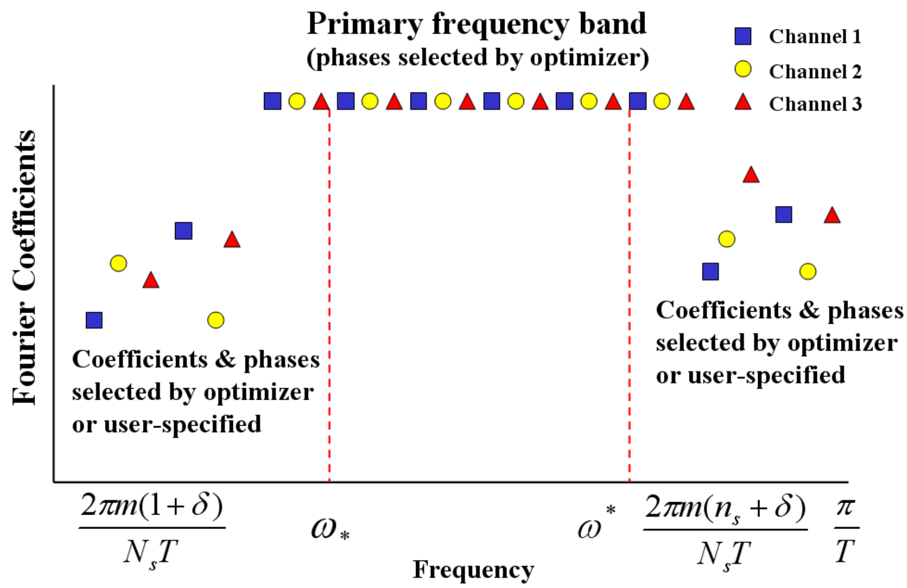


Figure 1: Conceptual design of a standard zippered power spectrum for 3-channel signal

2.2 Modified Zippered Multisine Signals

Stec and Zhu (2001) utilize sequential cycles of high-magnitude correlated and low-magnitude uncorrelated signals that promotes balanced directional content in the data. Their philosophy is adopted in our design procedure to define a *modified* zippered spectrum for a multisine input design, suitable for identifying highly interactive systems. A conceptual representation of the modified zippered spectrum is illustrated in Figure 2.

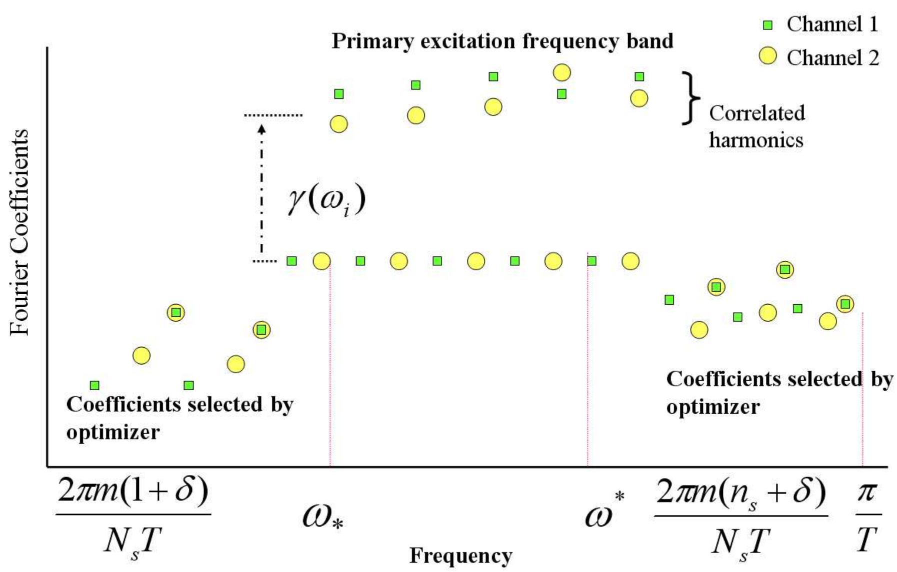


Figure 2: Conceptual design of a modified zippered power spectrum for 2-channel signal

To achieve the above modified zippered input power spectrum (Figure 2) we define the Fourier coefficients α_{ji} as:

$$\alpha_{ji} = \begin{cases} \neq 0, & i = (m+1)\delta + j, (m+1)(\delta+1) + j, \ldots, \ (m+1)(\delta + n'_s - 1) + j \text{ (uncorrelated)} \\ \neq 0, & i = (m+1)(\delta+1), (m+1)(\delta+2), \ldots, \ (m+1)(\delta + n'_s) \text{ (correlated)} \\ = 0, & \text{for all other } i \text{ up to } (m+1)(\delta + n'_s) \end{cases} \quad (8)$$

For efficient gain-directional estimation, the amplitudes $\gamma(\omega_i)$ and phases of the correlated harmonics need to be scaled and adjusted based on *a priori* knowledge of a system to be identified. This is explained in the ensuing sections.

3 Directional Multisine Input Design

Koung and MacGregor (1993) take advantage of knowledge of the condition number to increase the information contents of the low gain direction, comparable to that of the high gain direction. Similarly, the correlated multisine harmonics in a modified zippered spectrum can be designed to be collinear in a user-specified direction, usually the low gain direction, with a corresponding amplitude adjustment in the frequency domain.

In general, a $n \times m$ gain matrix (K) is represented in a Singular Value Decomposition (SVD) as follows:

$$SVD(K) \ = \ U \, \Sigma \, V^H$$
$$U \ = \ [u_1, u_2, \ldots u_n] \qquad V^H \ = \ [v_1, v_2, \ldots v_m]$$

where Σ contains a diagonal nonnegative definite matrix Σ_1 of singular values arranged in descending order as in

$$\Sigma = \begin{pmatrix} \Sigma_1 \\ 0 \end{pmatrix}, \qquad n \geq m \quad (9)$$

$$\Sigma = (\Sigma_1 \quad 0), \qquad n \leq m \quad (10)$$

and

$$\Sigma_1 = diag\{\sigma_1, \sigma_2, \ldots, \sigma_k\}, \qquad k = \min\{m, n\} \quad (11)$$

where $\bar{\sigma} = \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_k = \underline{\sigma}$ (Morari and Zafiriou, 1988). The output (U) and input (V^H) directional vectors are unitary and orthogonal, i.e., $[v_k] \times [v_j] = 0$ for $k \neq j$. If an input signal sequence $[x]$ is collinear to the j_{th} input directional vector in V^H such that $[x] = \alpha [v_j]^T, \ \alpha > 0$, then, $[v_j] \times [x]$ becomes

$$[v_j] \times [x] = [v_j] \times \alpha [v_j]^T = \alpha (\sum_{i=1}^{m} v_{ij} v_{ij}) = \alpha \quad (12)$$

This enables the direction and power amplitude adjustments using correlated harmonics in the multisine signal.

3.1 Directional Adjustment of Multisine Inputs

A multisine input signal in the time-domain is transformed into the frequency-domain, consisting of a series of power amplitudes and phases. A multisine equation given as

$$x(k) = \sum_{i=1}^{n_s} \alpha_i \cos(\omega_i kT + \phi_i) \tag{13}$$

can be transformed into at a specific frequency ω_i:

$$X(\omega_i) = \tilde{\alpha}_i e^{-j\tilde{\phi}_i} \tag{14}$$

where $X(\omega_i) = FFT([x])$, $\tilde{\alpha}_i$=$\sqrt{2\alpha_i} N_s$, $\tilde{\phi}_i$=$\omega_i T + \phi_i$. A multisine signal for multiple channels becomes such that

$$X(\omega_i) = \begin{bmatrix} \tilde{\alpha}_{1i} e^{j\tilde{\phi}_{1i}} \\ \tilde{\alpha}_{2i} e^{j\tilde{\phi}_{2i}} \\ \vdots \\ \tilde{\alpha}_{mi} e^{j\tilde{\phi}_{mi}} \end{bmatrix} \tag{15}$$

Furthermore, the above multisine input also represents the correlated harmonics in a modified zippered spectrum where the amplitude and phases are identical. The correlated harmonics $X_c(\omega_i)$ are

$$X_c(\omega_i) = \begin{bmatrix} \tilde{\alpha}_i e^{j\tilde{\phi}_i} \\ \tilde{\alpha}_i e^{j\tilde{\phi}_i} \\ \vdots \\ \tilde{\alpha}_i e^{j\tilde{\phi}_i} \end{bmatrix} \tag{16}$$

and they are taken into the consideration of direction adjustment.

To achieve the directional signal design, $\tilde{\alpha}_i$ and $\tilde{\phi}_i$ of the correlated harmonics should be adjusted based on a selected input direction vector $[v_j]$. Since a input direction vector can be transformed into the spherical coordinate, the amplitudes and phases of the selected direction vector is obtained as

$${v_j}^T = \begin{bmatrix} v_{j1} \\ \vdots \\ v_{jm} \end{bmatrix} = \begin{bmatrix} \alpha_{j1}\, e^{j\phi_{j1}} \\ \vdots \\ \alpha_{jm}\, e^{j\phi_{jm}} \end{bmatrix} \tag{17}$$

For the first channel, the phases $\tilde{\phi}_i$ may be selected only by satisfying a plant-friendly criterion such as the crest factor of signal. As a result, the adjusted correlated $X_c'(\omega_i)$ is formulated such that

$$X_c'(\omega_i) = v_j^* \otimes X_c(\omega_i) = \begin{bmatrix} \alpha_{j1} \tilde{\alpha}_i e^{j(\tilde{\phi}_i + 0)} \\ \vdots \\ \alpha_{jm} \tilde{\alpha}_i e^{j(\tilde{\phi}_i + \Delta\phi_{jm})} \end{bmatrix}, \ \Delta\phi_{mi} = \begin{bmatrix} -\phi_{j1} + \phi_{j1} = 0 \\ \vdots \\ -\phi_{jm} + \phi_{j1} \end{bmatrix} \tag{18}$$

where $v_j^* = conj(v_j)$ and $\otimes$ is the *Shur* product and $\Delta\phi_{ji}$ indicates the whole correlated harmonics are rotated by $e^{j\phi_{j1}}$. The proof of directional adjustment is verified by a simple test by

$$\begin{aligned} v_j^* \times X_c'(\omega_i) &= \begin{bmatrix} \alpha_{j1}\, e^{j\phi_{j1}} \\ \vdots \\ \alpha_{jm}\, e^{j\phi_{jm}} \end{bmatrix}^T \times \begin{bmatrix} \alpha_{j1}\, \tilde{\alpha}_i\, e^{j(\tilde{\phi}_i\, -\, \phi_{j1} + \phi_{j1})} \\ \vdots \\ \alpha_{jm}\, \tilde{\alpha}_i\, e^{j(\tilde{\phi}_i\, -\, \phi_{jm} + \phi_{jm})} \end{bmatrix} \\ &= \left[\alpha_{j1}^2 e^{j(\phi_{j1} - \phi_{j1})} + \cdots + \alpha_{jm}^2 e^{j(\phi_{jm} - \phi_{jm})}\right] \times \tilde{\alpha}_i e^{j\tilde{\phi}_i} e^{j\phi_{j1}} \\ &= 1 \times \tilde{\alpha}_i e^{j\tilde{\phi}_i} e^{j\phi_{j1}} \end{aligned} \tag{19}$$

where $\alpha_{ji}^2 + \cdots + \alpha_{jm}^2 = 1$. Therefore, $v_j^* \times X_c'(\omega_i) \neq 0$, $\tilde{\alpha}_i > 0$. This directional adjustment is now utilized for selecting the low gain input direction (v_m) (or any other gain direction of user's interest).

3.2 Amplitude Adjustment for Correlated Harmonics

If the input signal $[x_1 \cdots x_m]^T$ is designed to be collinear to one input directional vector $[v_j]$, only its corresponding output direction $[u_j]$ is manifested through the system such that

$$Y(\omega_i) = \begin{bmatrix} u_{1j} \\ \vdots \\ u_{mj} \end{bmatrix} \sigma_j \, [v_{1j} \cdots v_{mj}] \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} (\omega_i) \tag{20}$$

Having designed how to adjust the input signal for any system input direction, we focus on the low or weak gain direction whose information contents we wish to increase in the output. If the input signal sequence is also collinear or close to the high gain direction (or not collinear to the low gain direction, the high gain direction (e.g., $j = 1$) is naturally dominant in the response because of the maximum singular value (Morari and Zafiriou, 1988).

The response in the low gain direction is relatively very small; therefore, the modified zippered spectrum is proposed to overcome this output gain deficiency by properly scaling the amplitudes of the correlated harmonics. This adjustment can be accomplished by applying a higher κ for the low gain directional responses such that

$$\left\| \begin{matrix} u_{11}\sigma_1 \\ \vdots \\ u_{m1}\sigma_1 \end{matrix} \right\|_2 \text{ by uncorrelated harmonics} \approx \kappa \left\| \begin{matrix} u_{1m}\sigma_m \\ \vdots \\ u_{mm}\sigma_m \end{matrix} \right\|_2 \text{ by correlated harmonics} \tag{21}$$

where the inputs are collinear to an input directional vector; $x = [v_j]^T$, $j = 1$ for the high gain direction using uncorrelated harmonics and $j = m$ for the low gain direction using correlated harmonics. The scaling factor κ for channels is bounded by

$$\min_{i=1\ldots m} \left\{ \left\| \frac{u_{i1}\sigma_1}{u_{im}\sigma_m} \right\|_2 (\omega_i) \right\} \leq \kappa(\omega_i) \leq \max_{i=1\ldots m} \left\{ \left\| \frac{u_{i1}\sigma_1}{u_{im}\sigma_m} \right\|_2 (\omega_i) \right\} \tag{22}$$

Since κ includes a contribution for all inputs, it should be distributed over m number of input channels such that

$$\gamma(\omega_i) = \frac{\kappa(\omega_i)}{m} \tag{23}$$

As a result, the correlated multisine harmonics with directional and power adjustments for promoting a selected gain direction is obtained by the following multisine input signal

$$X_c'(\omega_i) \;=\; \gamma(\omega_i) \begin{bmatrix} \alpha_{j1}(\omega_i)\ \tilde{\alpha}(\omega_i)\ \exp^{j(\tilde{\phi}_i\,+\,\Delta\phi_{j1,i})} \\ \vdots \\ \alpha_{jm}(\omega_i)\ \tilde{\alpha}(\omega_i)\ \exp^{j(\tilde{\phi}_i\,+\,\Delta\phi_{jm,i})} \end{bmatrix} \tag{24}$$

Moreover, the quality of output distribution should be also geometrically monitored for the balanced gain directional information content in the state-space.

4 Control-Relevant Curvefitting for Plant-Friendly System Identification

4.1 Control-Relevant Parameter Estimation

A key feature of control-relevant parameter estimation is that it emphasizes closed-loop control performance requirements during the estimation procedure. The goal is to obtain a MFD model $\tilde{P}$ representing a system P that is best suited for the end use of model, which is control system design. To this end, the work of Gaikwad and Rivera (1997) established that such a parameter estimation problem can be cast as a pre- and post-weighted 2-norm minimization using the linear fractional transformation (LFT) (Figure 3) such that

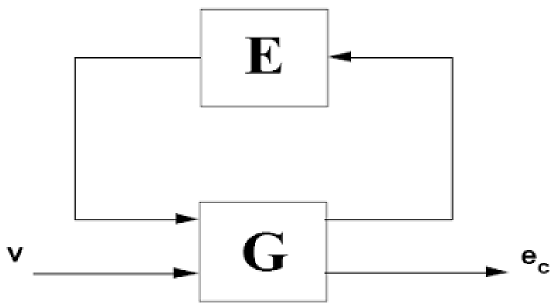


Figure 3: Linear fractional transform of closed-loop feedback system

$$\min_{E} \ \|W_y \tilde{S} E_m \tilde{H}(r-d)\|_2^2 \tag{25}$$

subject to the condition that

$$\sup_{\omega} \ \rho(E_m\tilde{H}) < 1, \quad -\pi \leq \omega \leq \pi \tag{26}$$

where the pre- and post-weights are functions of the closed-loop transfer functions as $\tilde{S} = (I + \tilde{P}C)^{-1}$, $\tilde{H} = \tilde{P}C(I + \tilde{P}C)^{-1}$ and $E_m = (P - \tilde{P})\tilde{P}^{-1}$. $\rho(E_m\tilde{H})$ arises from the Small Gain Theorem and can be used as a sufficient condition for nominal stability (Gaikwad and Rivera, 1997).

If zippered multisine signals are applied for simultaneously exciting all the input channels, a permutation matrix, T_m, should be applied to ensure only the relevant frequencies are captured in parameter estimation. The model estimation error that is then formulated as

$$\tilde{E}(\omega_i) \;=\; (G(\omega_i) - \theta\Phi(\omega_i))T_m(\omega_i) \tag{27}$$

where T_m is defined by

$$T_m(\omega_i) = diag(0, \,\ldots \underbrace{1}_{j_{th}} \ldots\, 0), \; T_m \in \Re^{m\times m} \tag{28}$$

θ and Φ are given as

$$\theta \;=\; [B_d \;\ldots\; B_{d+b-1}\, A_1 \;\ldots\; A_a] \in R^{p\times(mb+pa)} \tag{29}$$

$$\Phi(\omega_i) \;=\; \begin{bmatrix} I_{m\times m}\, \xi(\omega_i)^{-d} \\ \vdots \\ I_{m\times m}\, \xi(\omega_i)^{-(d+b-1)} \\ G(\omega_i)\; \xi(\omega_i)^{-1} \\ \vdots \\ G(\omega_i)\; \xi(\omega_i)^{-a} \end{bmatrix} \tag{30}$$

where $\xi(\omega_i) = j\omega_i$ in a continuous time model, whereas $\xi(\omega_i) = e^{j\omega_i T}$ represents the shift operator in a discrete-time model. In this paper, we utilize the left MFD parameterization for control-relevant parameter estimation purposes, i.e., $\tilde{P}$ is defined such that

$$\tilde{P}(\xi^{-1},\theta) = A(\xi^{-1},\theta)^{-1}B(\xi^{-1},\theta) \tag{31}$$

$$A(\xi^{-1},\theta) = I_{p\times p} \;+\; \xi \sum_{k=1}^{a} A_k\xi^{-k+1}, A_k \in \Re^{p\times p} \tag{32}$$

$$B(\xi^{-1},\theta) = \sum_{k=d}^{d+b-1} B_k\xi^{-k}, \;\; B_k \in R^{p\times m} \tag{33}$$

The parameter vector θ_t is estimated from an iterative minimization of the following objective,

$$\theta_t = \arg\min_{\theta\in\Re} \sum_{k=1}^{N} ||\tilde{W}_2(\omega_i,\theta_{t-1})\tilde{E}(\omega_i,\theta)W_1(\omega_i,\theta_{t-1})||_2^2\Delta\omega_i \tag{34}$$

where $\tilde{W}_2(\omega_i,\theta) = -\,W_y(\omega_i)\,\tilde{S}(\omega_i,\theta)A(\omega_i,\theta)^{-1}$ and $W_1(\omega_i,\theta) = \tilde{P}^{-1}(\omega_i,\theta)\,\tilde{H}(\omega_i,\theta)(r-d)$. $\Delta\omega_i$ represents the frequency interval for zippered or/and harmonic-suppressed input power spectra.

4.2 Control-Relevant Curvefitting Procedure

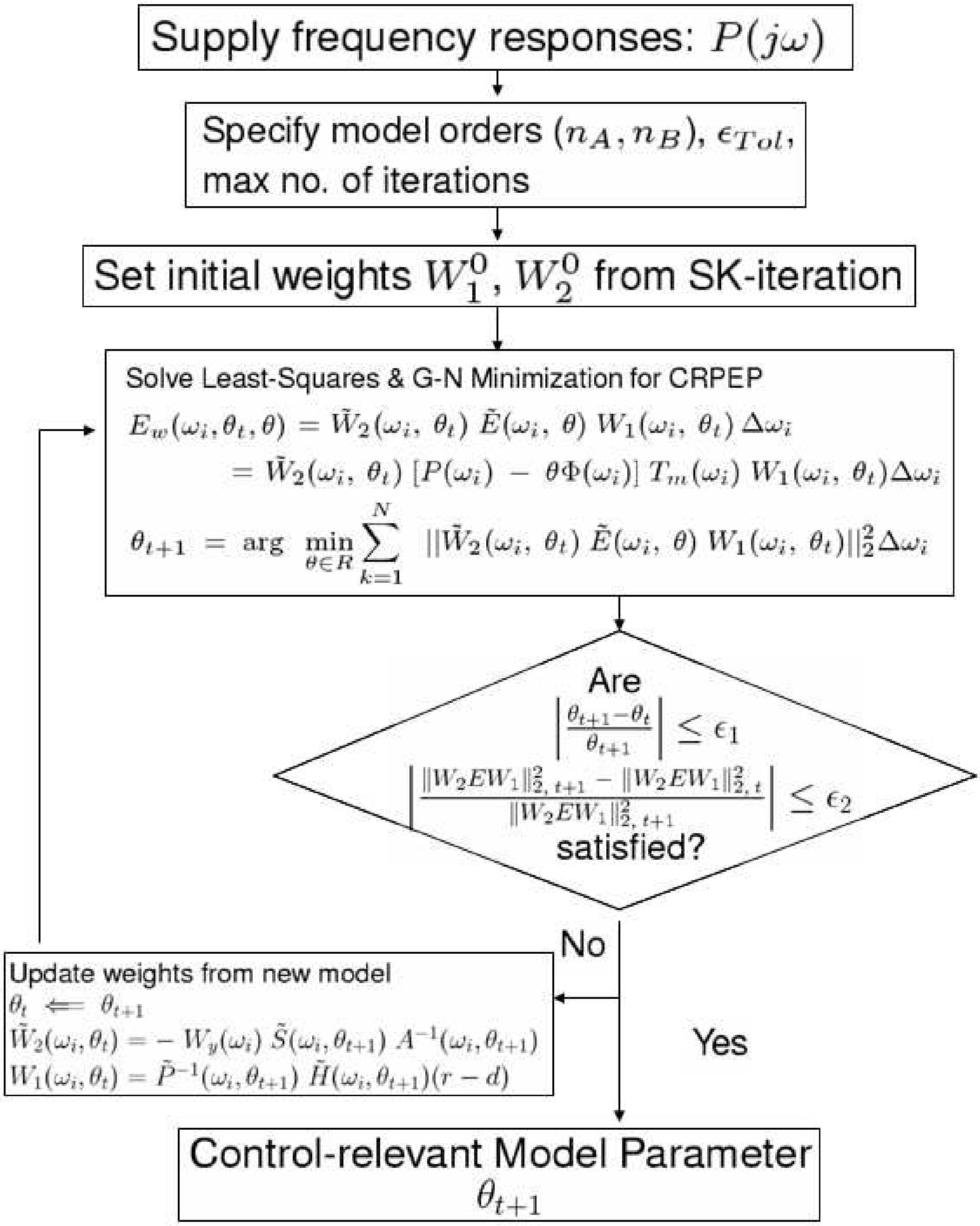


Figure 4: Flowchart for Control-relevant Parameter Estimation Algorithm

A consistent estimate of the frequency response is obtained from the observed input and output data via an Empirical Transfer Function Estimate (ETFE). Alternatively, frequency responses can be obtained through high-order ARX models and Spectral Analysis (SA). Then, frequency responses are approximated into parametric, multivariable systems using a discrete-time Matrix Fraction Description (MFD) model (de Callafon *et al.*, 1996).

The weighting functions are obtained by utilizing unconstrained Model Predictive Control (MPC);

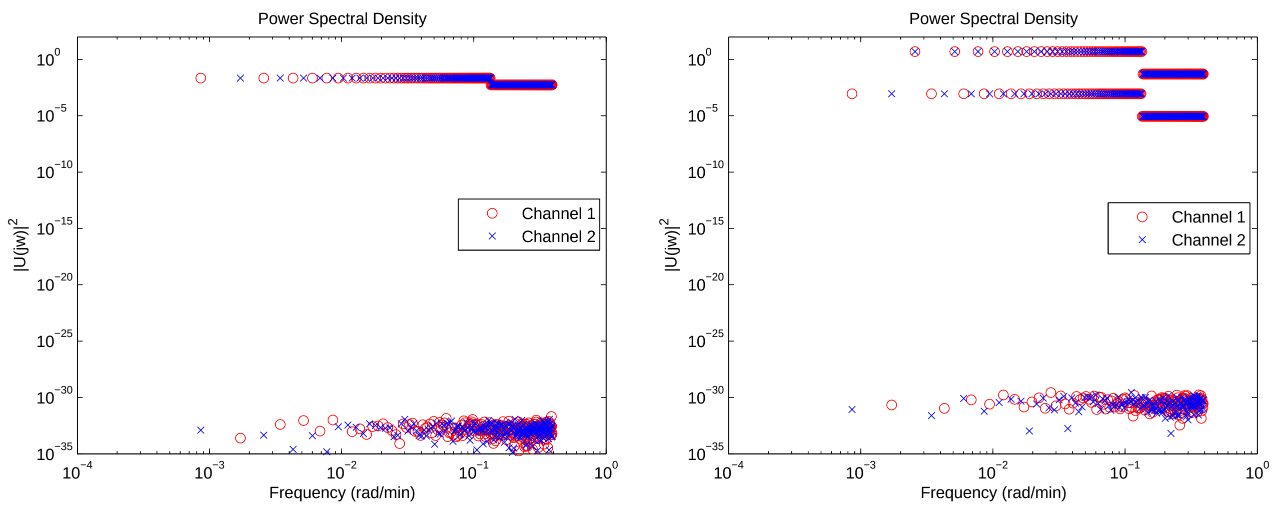
particularly, an output gain direction is considered as the input change in (25), i.e., r is collinear to an output direction vector $[u_j]$ that is corresponding to $[v_j]$ in the input design. The unweighted MFD model provides an initial model for the control-relevant weighting, and the overall procedure is implemented in a numerical algorithm as Figure 4. The detailed implementation of the control-relevant parameter estimation procedure is described in (Lee and Rivera, 2005).

5 High-Purity Distillation Column Case Study

The integrated methodology described in this paper is demonstrated in a case study of a linear distillation column (Jacobsen and Skogestad, 1994). The distillation has L/V configuration and operating variables of $y_D = 0.99$ and $x_B = 0.01$. Since this process is of the 41st-order, model order reduction is desirable. The datasets based on the two input signals (zippered and modified zippered spectra) are compared for their usefulness on the distillation process under noisy environments.

5.1 Open-Loop Test Experiments

From open-loop step responses, the dominant time constant range for this system can be estimated as τ_{dom}^L=15 and τ_{dom}^H=194 min. Coupled with user choices of δ=0, α_s=1, and β_s=1, these lead to acceptable choices, n_s=78, N_s=916, and T=8 min, for a series of identification testing signals that conform to the guidelines in Sections 2 and 3 (see Figure 5). A directional multisine input is applied with a modified zippered spectrum as v_2=[1 1] and γ=72 which is determined, based on a *priori* steady-state gain. The modified zippered signal is designed to excite the low gain direction $[1\ \ 1]$ as the state-space plot in Figure 6. The correlated harmonics in the modified zippered signal cause a higher input magnitude.

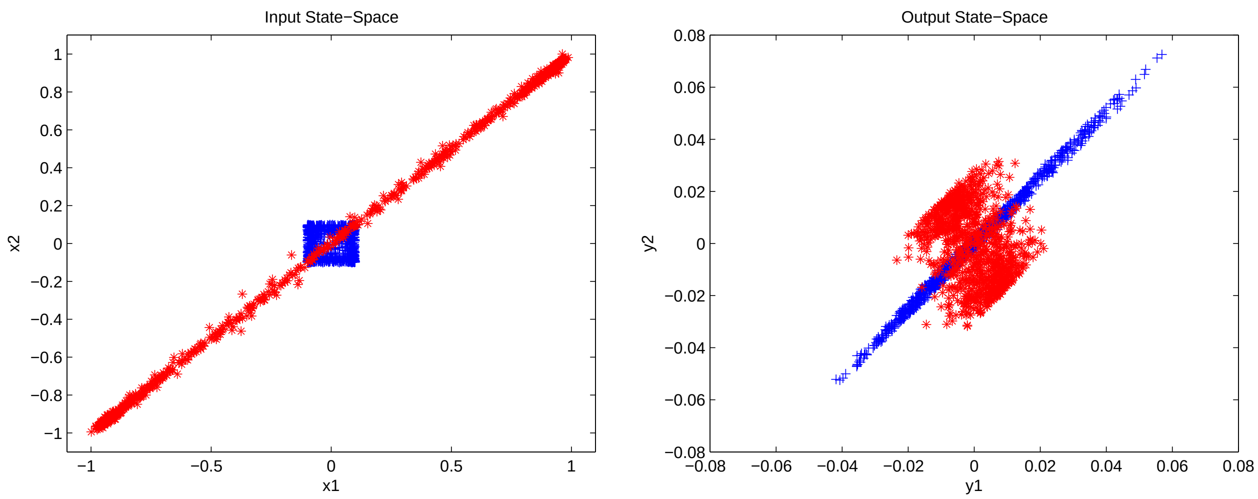


(a) Standard Zippered Case

(b) Modified Zippered Case

Figure 5: Input power spectral densities for Jacobsen-Skogestad distillation column: (a) a standard zippered spectrum and (b) a modified zippered spectrum

The output state-space plot gives a clear contrast between the two multisine signal designs. Figure 6 (b, blue +) shows a thin spread in the [1 1] high gain output direction despite the input state-space has



(a) Input State-Space (b) Output State-Space

Figure 6: Input (a) and Output (b) state-space plots for the linear distillation column: standard zippered spectrum (+:blue) and modified zippered spectrum ($*$:red)

the small square-type distribution. By the directional design via the modified zippered spectrum, the input state-space has a thin spread in [1 1] direction (Figure 6 (a), red $*$), though the resulting output state-space, Figure 6 (b, red $*$), has a diamond-shape spread. This indicates that the modified zippered spectrum generates a balanced output distribution; as a consequence, it will produce a model estimate with the improved gain directionality.

The correlated harmonics with the higher power level and directional adjustment comparatively promote the low gain to the high gain direction; therefore, the output span in the [1 -1] direction remains similar in the [1 1] direction (Figure 6 (b), red *). To demonstrate the effectiveness under noisy environments, white noise is added to the outputs of the tested data at [-1 *dB*] SNR as shown in Figure 7 in addition to estimating models from noise-free input-output data.

5.2 Control-Relevant Curvefitting

ETFE and SA are utilized to produce frequency responses from the datasets generated using the standard and modified zippered spectrum signals, respectively. Since the standard zippered spectrum has orthogonality from uncorrelated harmonics, unbiased estimates of the ETFE are naturally easy to compute. However, the ETFE cannot be estimated for the frequencies of a dataset from the modified zippered spectrum. Instead, SA is used for the modified zippered spectrum signals so that the low gain information excited by the correlated harmonics can be captured in the frequency responses. The weighting functions are defined by the setpoint change ([0.1 -0.1]) and closed-loop transfer functions using unconstrained MPC with a set of tuning parameter (PH=35, MH=10, Ywt=[1 1], and Uwt=[0.05 0.03]) for both noise-free and noisy conditions.

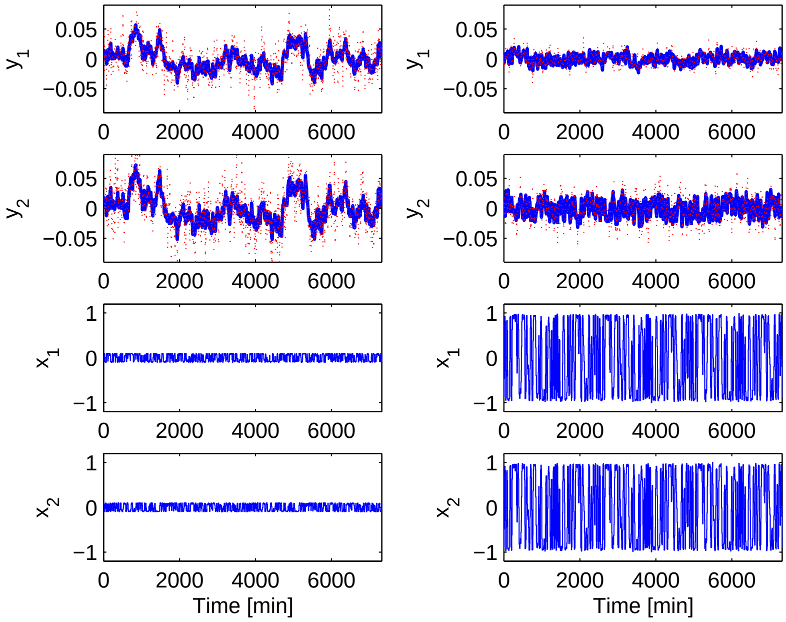


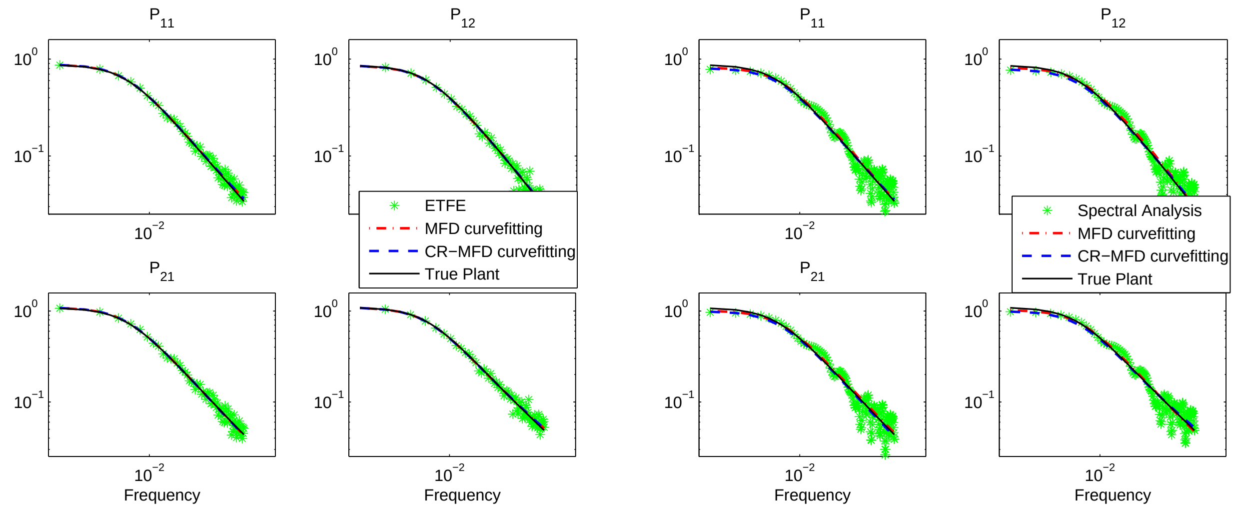
Figure 7: Time-domain sequences of the open-loop experiment for the linear distillation column: standard zippered spectrum (left; $CF(x)$=[1.34, 1.34]) and modified zippered spectrum (right; $CF(x)$=[1.31, 1.31]), solid (noise-free) and dotted (noisy)

5.2.1 Noise-Free Data Case

Figure 8 shows the curve-fittings of the weighted and unweighted models under noise-free conditions with the simplest MFD order [n_a=1, n_b=1, and n_k=1]. Both the unweighted and weighted MFD models have accurate fits to the frequency responses. Figure 9 shows $\rho(E_m H)$ of the MFD models; under noise-free condition, all the models except the unweighted model arised from the modified zippered signal have the low values. Figure 10, however, indicates that σ_{max} and σ_{min} are accurately estimated by all the models. The weighted model from the modified zippered signal shows the most precise estimate of σ_{max} and σ_{min}. In closed-loop setpoint tracking tests with MPC, all the MFD models display the equivalent results with efficient tracking performance (see Figure 11) since all the models have sufficiently accurate estimates of singular values.

5.2.2 Noisy Data Case

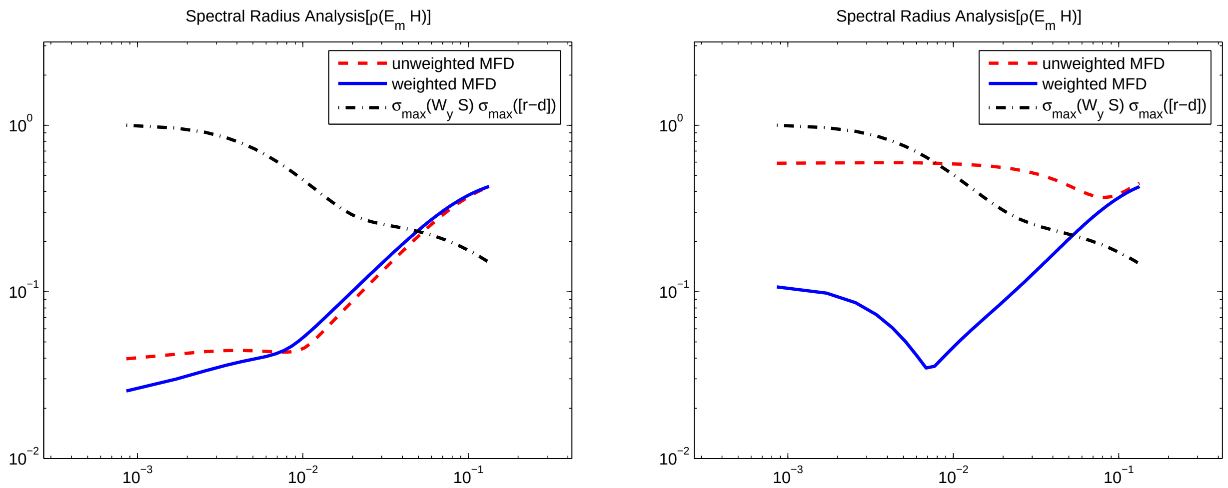
Figure 12 shows the curve-fittings of the weighted and unweighted models with the same MFD order [n_a=1, n_b=1, and n_k=1] as the noise-free conditions. Figures 13 and 14 reveal a significant contrast in the models. In particular, the control-relevant weighted model based on the data from the modified zippered signal has lowest $\rho(E_m H)$ (Figures 13) and a much closer estimate of the σ_{min} than any other model (Figure 14) while all the models closely estimate σ_{max}. In closed-loop setpoint tracking tests with MPC, the weighted model using the modified zippered input design is able to display the best result with fast and stable tracking performance without offset (see Figure 15).



(a) Standard Zippered Case with ETFE

(b) Modified Zippered Case with SA

Figure 8: Frequency-response curvefitting of data under noise-free conditions : (a) ETFEs by standard zippered spectrum and (b) SA by modified zippered spectrum using Hamming Window($W_r = 256$).



(a) Standard Zippered Case

(b) Modified Zippered Case

Figure 9: Small Gain Theorem Analysis: ETFEs from a standard zippered spectrum (a) and SA from a modified zippered spectrum (b) with parametric MFD [1 1 1] models under noise-free conditions.

6 Summary and Conclusions

In this paper, a novel integrated framework is presented for multivariable system identification and control system design. *A priori* knowledge of a system is efficiently utilized for generating informative multisine input signals and control-relevant parameter estimation. A modified zippered spectrum provides a powerful tool that is able to adjust the directions and power amplitudes of sinusoidal harmonics, promoting information content in the weak gain direction of a highly interactive system. A method for parametric

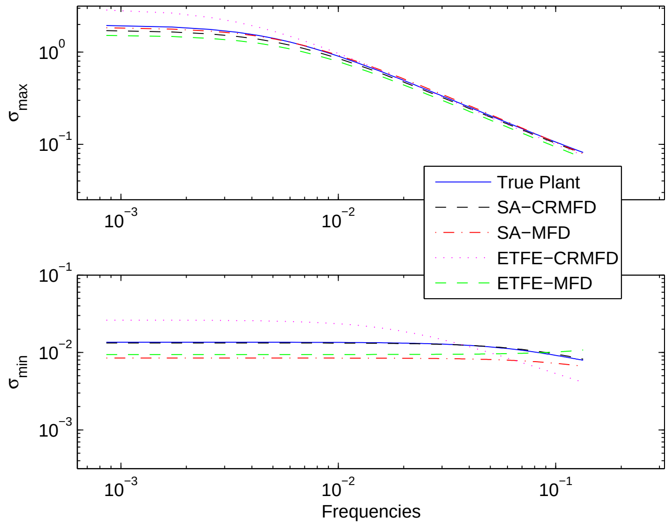


Figure 10: Singular values of the true plant and estimated models under noise-free conditions : SA= spectral analysis, CRMFD = weighted MFD model, and MFD = unweighted MFD model

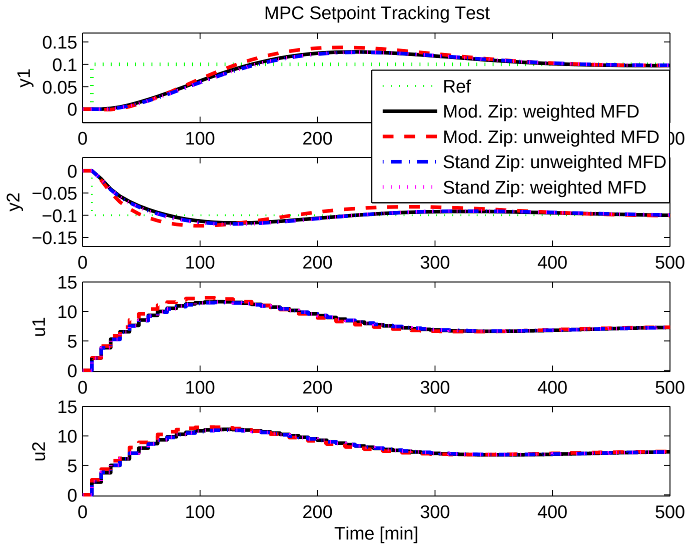
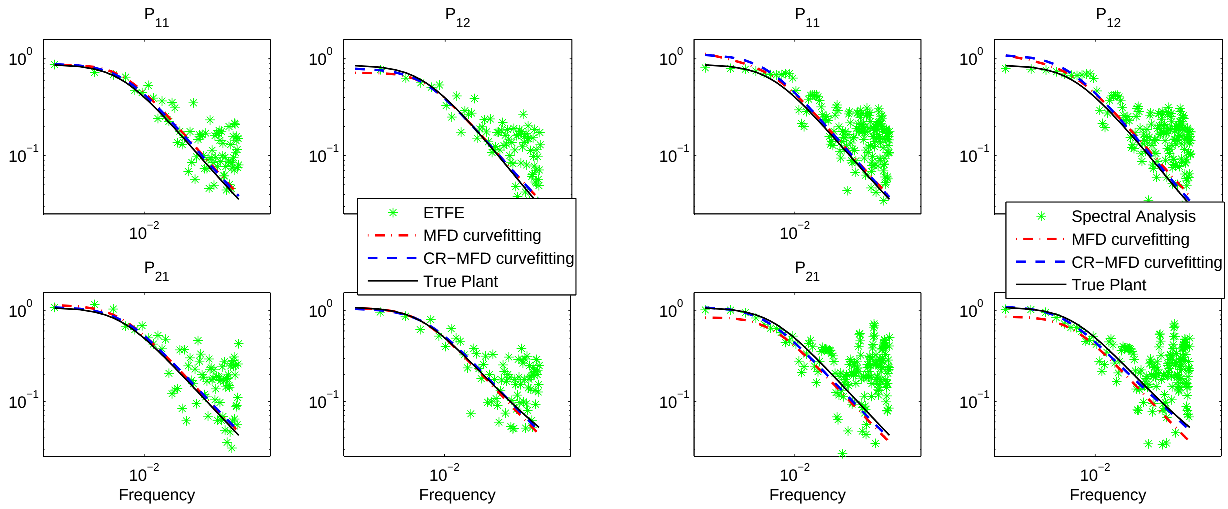


Figure 11: Setpoint MPC tracking tests with the MFD models under noise-free conditions for the linear distillation column: r=[0.1 -0.1]

model estimation via frequency-weighted curvefitting is achieved by the use of the full-polynomial MFD approach. The weighted curvefitter naturally capture the low gain direction resulting from the dataset using a directional signal design.

We see from the case study that a combined approach involving signal design and model estimation is superior to the conventional identification approaches for systems characterized by strong process interaction and ill-conditioning. The integrated methodology demonstrates its efficiency of estimating singular values precisely with desirable gain directionality under noise-free and noisy conditions. The future research will consider the development of a comprehensive identification test monitoring procedure that is



(a) Standard Zippered Case with ETFE

(b) Modified Zippered Case with SA

Figure 12: Frequency-response curvefitting of data under noisy conditions: (a) ETFEs by standard zippered spectrum and (b) SA by modified zippered spectrum using Hamming Window($W_r = 256$).

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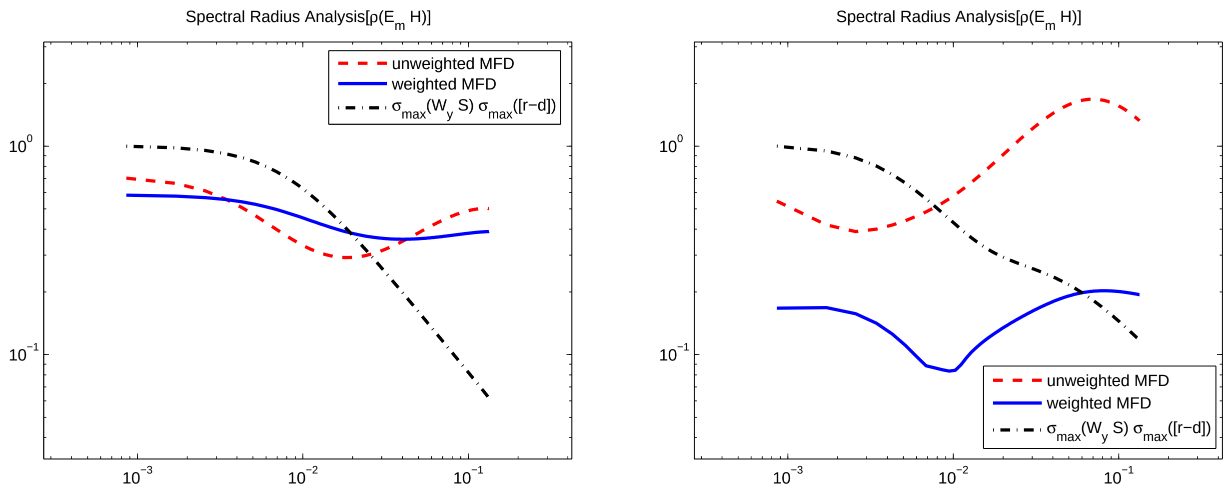

(a) Standard Zippered Case

(b) Modified Zippered Case

Figure 13: Small Gain Theorem Analysis: ETFEs from a standard zippered spectrum (a) and SA from a modified zippered spectrum (b) with parametric MFD [1 1 1] models under noisy conditions

able to improve the robustness, performance, and stability in a wide range of multivariable control system applications.

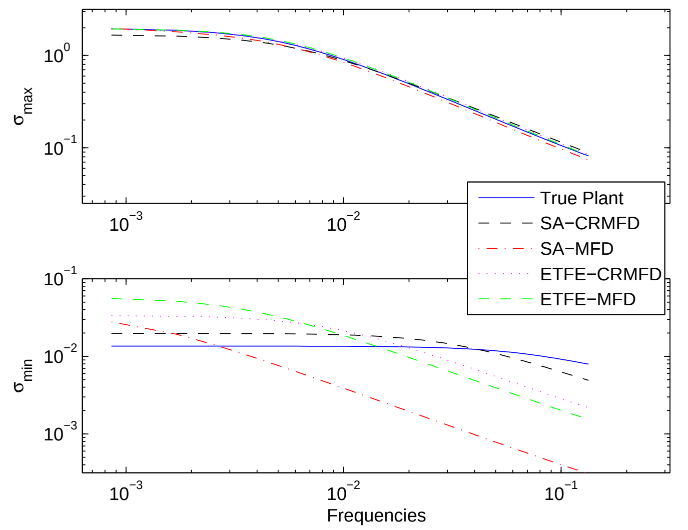

Figure 14: Singular values of the true plant and estimated models under noisy conditions : SA = spectral analysis, CRMFD = weighted MFD model, and MFD = unweighted MFD model

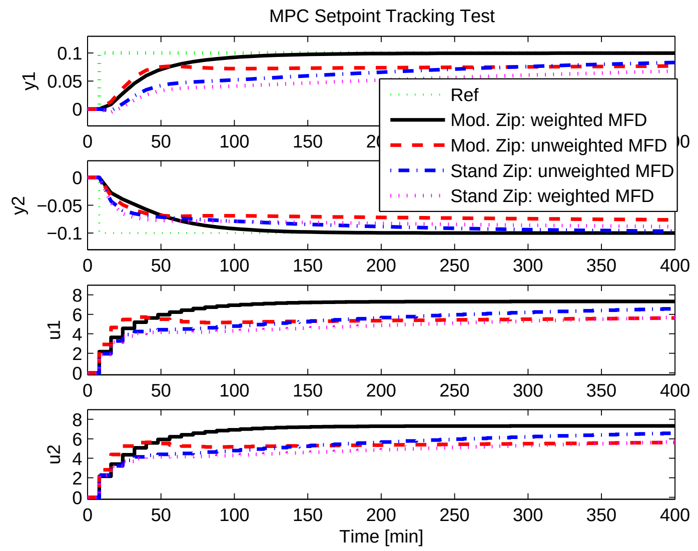

Figure 15: Setpoint MPC tracking tests with the MFD models under noisy conditions for the linear distillation column: r=[0.1 -0.1]

7 Acknowledgement

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