469f Nonlinear Mpc Using Multi-Parametric Nonlinear Programming Solutions

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Any nonlinear model predictive control (NMPC) problem can be restated as a multi-parametric nonlinear programming (mpNLP) problem by discretizing the dynamic states and identifying the initial values of those states as parameters. Of course, such a process can also be applied to the standard model predictive control problem, and has been by Tøndel, Johansen and Bemporad.[1] In both of these cases, successful solution of the parametric programming problem results in the MPC problem being solved offline for any possible initial state, thereby reducing the online calculations to simple tasks like table look-up and interpolation.

While the main gain for implementing standard MPC in this way is that it allows MPC to be applied to faster processes, the potential gains for NMPC are more profound. Besides computational burden being a much bigger hurdle for NMPC, NMPC is also hindered by the fact that nonlinear programming algorithms give no guarantee of finding a globally optimal solution. While no such guarantee exists for mpNLP either, mpNLP has a much better chance at identifying the global optimum for a given value of the parameters if the algorithm used is inherently exploratory (like the algorithm used in this work).

Parametric optimization problems are distinguished from their standard counterparts by the fact that some set of model parameters, which are held constant in the standard problem, are assumed to be uncertain or variable in the parametric problem. The parametric optimization task is then to calculate the optimal solution for all relevant values of the parameters. In mpNLP we have the result that, using an active set strategy and a regularity assumption, this problem can be transformed into a set of under-determined nonlinear equations which implicitly define a manifold of dimension equal to the number of parameters. Thus, one parameter problems yield paths of solution points, two parameter problems yield surfaces, etc. Also, predictor-corrector type algorithms may be applied to approximate these manifolds.[2] Note that such algorithms scale exponentially with the number of parameters but polynomially with the total number of variables.

In this work we propose applying mpNLP to the problem of NMPC. We show that an acceptable solution of the mpNLP problem (as mentioned above, this is the NMPC problem with all dynamic equations discretized and the initial values of the state variables identified as parameters) reduces the online tasks of NMPC to look-up and interpolation, and that full solution of the problem would result in globally, rather than locally, optimal control moves. We also demonstrate this technique by applying the mpNLP algorithm POPAK[3,4] to the NMPC example given in Martinsen et al., 2004[5].

Thus it is shown that mpNLP may be used to implement NMPC for problems with a small number of dynamic states (on the order of 10 or fewer), but few other restrictions. (Since mpNLP is polynomial time with respect to all variables other than the parameters/initial states, and the bulk of the calculation is done offline, there are no hard limits on things like horizon lengths, number of control variables and constraints. Also, since mpNLP algorithms have a much better chance of finding global optimums than an NMPC algorithm whose optimization step is done online, non-convexity is less of an issue when NMPC is implemented in this manner.)

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