

397c Constructing Tight Convex/Concave Relaxations of the Solutions of Parameter-Dependent Nonlinear Odes

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Optimization problems embedding initial value problems in nonlinear ordinary differential equations (ODEs) that depend on parameters are frequently encountered in chemical engineering as well as in many other engineering and scientific fields

$$\begin{aligned}\dot{\mathbf{x}}(t, \mathbf{p}) &= \mathbf{f}(t, \mathbf{p}, \mathbf{x}(t, \mathbf{p})), \quad \forall t \in (t_0, t_f] \\ \mathbf{x}(t_0, \mathbf{p}) &= \mathbf{h}(\mathbf{p})\end{aligned}\quad (S)$$

with $\mathbf{p} \in P$, $\mathbf{x}(t, \mathbf{p}) \in \mathbb{R}^{n_x}$, and $P \subseteq \mathbb{R}^{n_p}$ a nonempty convex set.

This paper is concerned with the problem of obtaining pointwise in time convex/concave bounds for the solutions \mathbf{x} of (S) on P . More precisely, the general goal is to construct auxiliary differential systems, the solutions of which underestimate/overestimate the solutions of the original differential system (S), and are guaranteed to be pointwise in time convex/concave on P . Note that the idea of constructing such auxiliary differential systems is not new to this paper for it has already been proposed by Singer and Barton [1]. However, in this former work, the authors considered pointwise in time affine relaxations only. The main contribution of this work lies in the development of a theoretical framework enabling the construction of general nonlinear relaxations for the solutions of parameter dependent nonlinear ODEs on subsets of a Euclidean space.

The practical application of these results is also closely considered. More precisely, a procedure for constructing such relaxations under mild assumptions is proposed, based on McCormick's relaxation technique [2]. Finally, several illustrative examples are presented which demonstrate the construction and practicality of the relaxations.

An important application of these results is in algorithms capable of solving dynamic optimization problems to guaranteed global optimality [3,4,5] as well as mixed-integer dynamic optimization (MIDO) [6] and hybrid discrete/continuous dynamic optimization [7] problems.

References

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