387c Optimal Projection of Nonlinear Volterra Models Onto the Laguerre Basis
Abhishek S. Soni and Robert S. Parker

One of the key impediments to the wide-spread use of nonlinear control in industry is the availability of suitable nonlinear models (1). At one end of the spectrum are the fundamental models, which for processes of realistic complexity often involve a hundred or more nonlinear differential equations. At the other end of the spectrum are the empirical models, which are obtained from process input-output data. Many of the widely used empirical model structures are linear, and in some cases this basic model formulation may not be able to adequately capture the nonlinear process dynamics. However, an important advantage of empirical models is that their structure can be chosen so as to facilitate the controller design problem. One of the commonly used nonlinear dynamic empirical model structures is the Volterra model. This work focuses on determining the optimal number of Laguerre filters and corresponding Laguerre pole for the projection of third-order Volterra models onto an orthonormal Laguerre basis.

Previously, we have developed a plant-friendly method for identifying third-order Volterra models using tailored input sequences (2). Nonlinear Volterra models are highly parametrized and this causes them to be noise-sensitive. One way to alleviate this difficulty is to project the kernels onto a small number of relatively smooth orthogonal functions. In the present work, the Laguerre functions are employed. The main advantage of the Laguerre basis is that in order to adequately describe a system, the number of Laguerre filters (L) required is much less than the Volterra model memory (M), *i.e.,* L<< M. Thus a Volterra-Laguerre model requires fewer parameters and is less noise-sensitive than the corresponding Volterra model of the same order.

Two important variables that need to be selected for the projection of Volterra models onto a Laguerre basis are the number of Laguerre filters and the Laguerre pole. As the number of Laguerre filters is increased the number of parameters involved in the estimation increases as well. The Laguerre pole is constrained to lie on the interval [0,1), and it controls the exponential rate of decay of the Laguerre functions. For nonlinear systems, the selection of both the number of Laguerre filters and the optimal pole has generally been carried out heuristically. A notable exception is (3), for infinite memory systems. In this work, the Akaike Information Criterion (AIC) (4), based on the mean log-likelihood of the estimates is used as a criterion to determine the quality of model fit. The AIC includes contributions from both the model size (set by the number of Laguerre filters) and model quality (characterized by both the number of Laguerre filters and the Laguerre pole) as measured by the sum-squared error between the Volterra and the Volterra-Laguerre model outputs. For a given set of data or case study, each combination of Laguerre filters and the corresponding Laguerre pole characterizes a Volterra-Laguerre model and has a unique AIC value associated with it. The model with the lowest AIC value represents the optimal balance between the number of parameters and model quality. Preliminary results are generated for a polymerization reactor case study (as in (2)) by discretizing the sample space for both the Laguerre filters and the pole. Ongoing studies focus on generalizing the results to Volterra-Laguerre systems of arbitrary order.

References

1. J. H. Lee. Modeling and identification for nonlinear predictive control: Requirements, current status and future research needs. In In F. Allgöwer and A. Zheng, editors, *Nonlinear Model Predictive Control: Assessment and Future Directions*. Birkhäuser, 1999.

2. A. S. Soni and R. S. Parker. Efficient Input Signal Design for Third-Order Volterra Model Identification. In *Proc. 2005 IFAC World Congress,* volume (accepted), Prague, Czech Republic, 2005.

3. R. J. G. B. Campello, G. Favier, and W. C. Amaral. Optimal expansions of discrete-time Volterra models using Laguerre functions. *Automatica,* 40:815--822, 2004.

4. H. Akaike. A new look at the Statistical Model Identification. *IEEE Trans. Aut. Control,* AC-19(6):716--723, December 1974.