369e Transient and Asymptotic Behavior of the Binary Breakage Problem

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Breakage is the process by which an initial particle population breaks to give more particles of smaller size. It is known in the literature by many names such as fragmentation or milling or scission or partition or disintegration or division as well as others. Breakage is encountered in a wide variety of physical phenomena and engineering applications, such as polymer degradation [1], coal combustion [2], grinding [3], crystallization [4], breakup of liquid droplets or air bubbles [5] as well as cell division of cells in a population [6]. A pure breakage process is mathematically described by a partial integro-differential equation, known as a population balance equation [7]. It is fully characterized by two functions. First, the breakage rate or breakage function, which describes the rate by which a particle of size x breaks to give smaller particles, and second, the breakage kernel (also known as partition probability density function in the context of cell population dynamics) which describes the probability that a mother particle will partition, upon breakage, into daughter particles of smaller sizes. In the case where each breakage event leads to the production of two daughter particles, the process is called binary breakage, while in the case where more daughter particles are produced, the process is known as multiple breakup. The general binary breakage problem with power-law breakage functions and two families of symmetric and asymmetric breakage kernels is studied in this work. A useful transformation leads to an equation that predicts self-similar solutions in its asymptotic limit and offers explicit knowledge of the mean size and particle density at each point in dimensionless time. A novel moving boundary algorithm in the transformed coordinate system is developed, allowing the accurate prediction of the full transient behavior of the system from the initial condition up to the point where self-similarity is achieved, and beyond if necessary. The numerical algorithm is very rapid and its results are in excellent agreement with known analytical solutions. In the case of the symmetric breakage kernels only unimodal, self-similar number density functions are obtained asymptotically for all parameter values and independent of the initial conditions, while in the case of asymmetric breakage kernels, bimodality appears for high degrees of asymmetry and sharp breakage functions. For symmetric and discrete breakage kernels self-similarity is not achieved. The solution exhibits sustained oscillations with amplitude that depends on the initial condition and the sharpness of the breakage mechanism, while the period is always fixed and equal to ln2 with respect to dimensionless time.

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