Segregation under Chaotic Flow in 2D Granular Systems

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Experiments reveal that an initially well mixed granular material composed of two distinct subclasses of particles, small and large or light and heavy, segregates radially into stable lobed patterns when rotated in various quasi two-dimensional, regular polygonal tumblers. The patterns are highly sensitive to the time-periodic flow, which in turn depends critically on the fill fraction and container shape. Simulations of a simple model reproduce the observed segregation patterns. KAM regions in Poincaré plots of the velocity field used to model the flow attract smaller (denser) particles and their spatial symmetries mirror those of the segregation patterns, suggesting that competition between the driving forces for radial segregation (percolation and buoyancy) and those for chaotic mixing play a key role.

This paper describes experiments and simulations of thin rotating tumblers (quasi-2D) operating in the continuous flow regime. As shown in Fig. 1(a), the bulk of the material moves in solid body rotation. A thin, lens-like region of material at the surface with thickness δ moves downhill in a rapid shear flow. This underlying flow can be described with a velocity field originally introduced by Khakhar *et al.* For a circular tumbler, particles follow the fixed trajectories shown in Fig. 1(b). If the shape of the tumbler is altered, the length and depth of the flowing layer varies with the container orientation, resulting in flow that is time-periodic. For time periodic flow, Poincaré sections are created by plotting the location of particles at the end of each period of flow (see Fig. 1(c) for an example of such flow in a hexagonal tumbler). These sections show the emergence of elliptic and hyperbolic periodic points. Particles placed at a periodic point will return there after an integer number of periods of flow. Elliptic points are surrounded by KAM regions, identified by rings of regular flow in the Poincaré section, while hyperbolic points are surrounded by regions of chaotic flow.

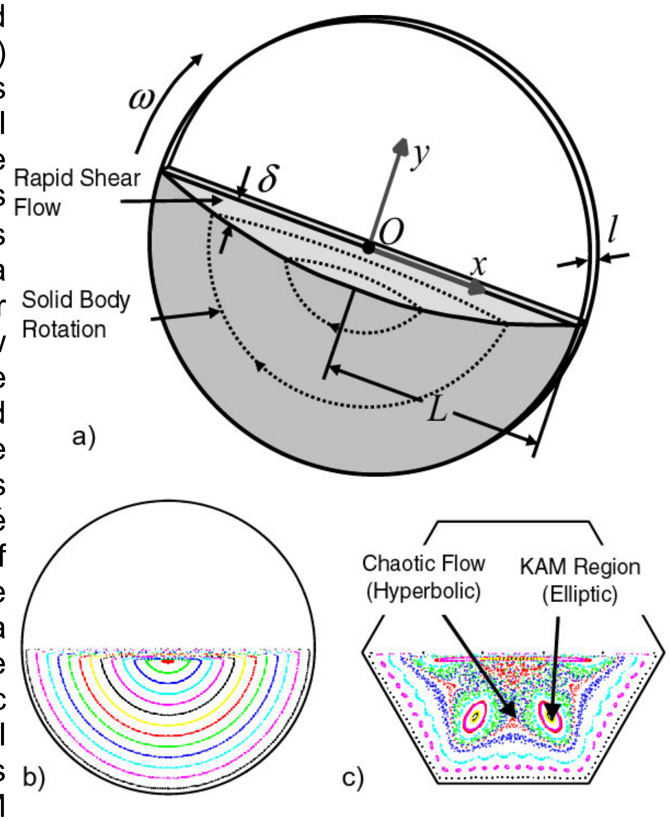

Figure 1 (a) Flow in a circular quasi-2D tumbler. (b) Poincaré section for a circular tumbler. (c) Poincaré section for a hexagonal tumbler with periodic points labeled.

We examine segregation of granular materials in quasi-2D tumblers using two complimentary methods. First, experiments are carried out with a mixture of round glass particles – 2/3 with diameters of 0.85 and 1/3 with diameters of 0.30 mm. The tumbler is rotated until steady patterns are reached (about 10 rotations). The second method uses computer simulation to study density segregation in a model where discrete particles are advected by a continuum velocity field, originally shown by Hill *et al*. Segregation is achieved by including a directed drift velocity based on the local particle concentration within the flowing layer which causes light and heavy particles to segregate. Both experiment and simulation are carried out in a variety of regular polygons with up to eight sides and with a wide range of fill fractions (the area occupied by particles relative to the area of the tumbler).

When mixtures of small and large particles in circular tumblers are rotated, the small particles percolate to the center of the tumbler, forming a radial core of small particles. However, in tumblers with non-circular geometry, the core is no longer simply radial. In particular, many geometries exhibit a core of small particles from which lobes of small particles extend. For instance, the half-full square tumbler of Fig. 2 has a pattern of small particles with two lobes pointing towards the corners of the tumbler. Note that the pattern produced by the density segregation model (right column) is identical to the experiment (left column). We show that segregation occurs in a nearly identical fashion regardless of the details of the force driving segregation. The Poincaré sections for the examples in Fig. 2 show elliptic points surrounded by KAM regions in the same locations as the lobes. In fact, Poincaré sections for half-full polygonal tumblers with an even number of sides show KAM regions with a period of *N*/2 (where *N* is the number of sides of the tumbler) exactly as the experiments do.

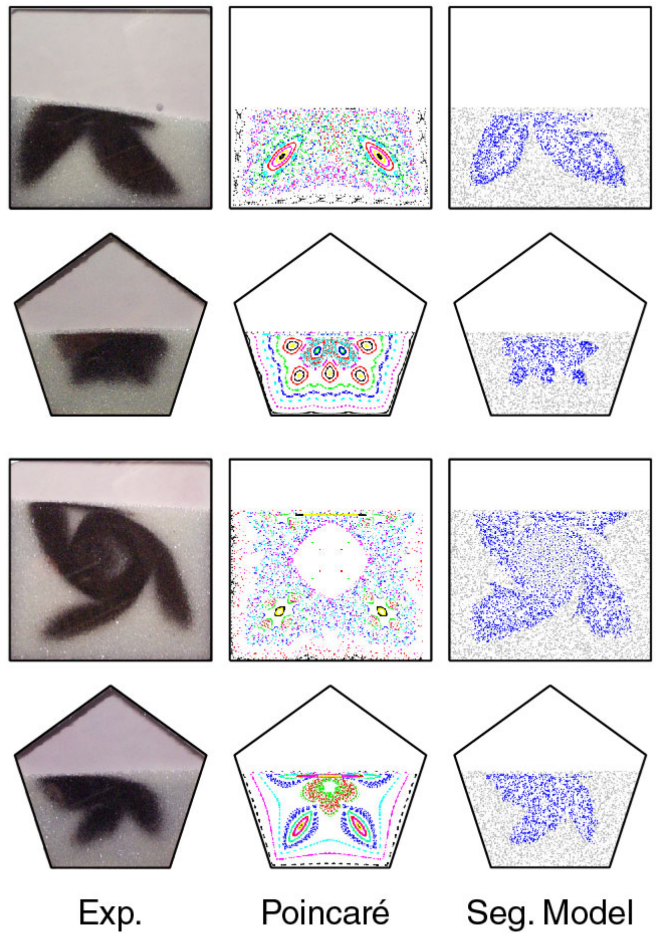

Figure 2 Left column – size segregation results from experiment. Center column – Poincaré sections from the continuum flow model. Right column – density segregation from simulations. Fill fraction from top to bottom: 50% (square), 50% (pentagon), 75% (square) and 60% (pentagon)

Polygonal tumblers with an odd number of sides, however, present a paradox. Consider the half-full pentagonal tumbler shown in Fig. 2. The Poincaré section shows elliptic points of period 2 near the center and elliptic points of period 5 about half way between the center and the edge. Despite the more complicated structure of the Poincaré section, the resulting segregation pattern is simple, resembling the radial core that occurs in a circular tumbler. However, it does correspond with the locations of the period 5 KAM regions. Similar patterns occur in triangular and heptagonal tumblers.

How do these patterns change when the fill fraction is increased? First consider the 75% full square shown in Fig. 2. In this case, three lobes extend from the core towards the corners where KAM regions exist in the Poincaré section. Note that when the fill fraction is increased enough (usually more than 55% full), a portion of the particles do not pass through the flowing layer at any point during rotation and maintain their original position. In a pentagonal tumbler, while the half-full case is simple, when the fill fraction is increased to 60%, the system reaches a state with 3 lobes extending from the center and corresponding KAM regions in the Poincaré sections. All of the polygonal tumblers exhibit states with numbers of lobes that vary from 2 to *N*-1.

To transition between states with different numbers of lobes, there must be changes in the nature of the underlying flow as the fill fraction of a tumbler is increased. The cases shown thus far suggest that the segregation pattern can be predicted based on the KAM regions observed in Poincaré sections. With this idea in mind, consider Poincaré sections of pentagonal tumbler with several different fill fractions (shown in Fig. 3). Beginning with the half-full case, the period 5 KAM regions move closer to the center as the fill fraction is increased 3%. The 53% and 56% full tumblers should produce segregation patterns similar to the half-full case in Fig. 2. Increasing the fill fraction further, period 3 regions form, giving rise to a segregation pattern with 3 lobes. As the fill fraction is increased more, these regions move towards the corners of the tumbler until it reaches a state resembling the pattern for a circular tumbler shown in Fig. 1 (at 65% full). This case displays a radial segregation pattern. Finally, as the fill fraction is increased beyond this point, the Poincaré sections show period 4 KAM regions that move from the core towards the edge of the tumbler. Experiments run in this range of fill fractions show segregation patterns with 4 lobes. The Poincaré sections in Fig. 3 show that between states with different numbers of lobes there are states where segregation patterns are just radial cores.

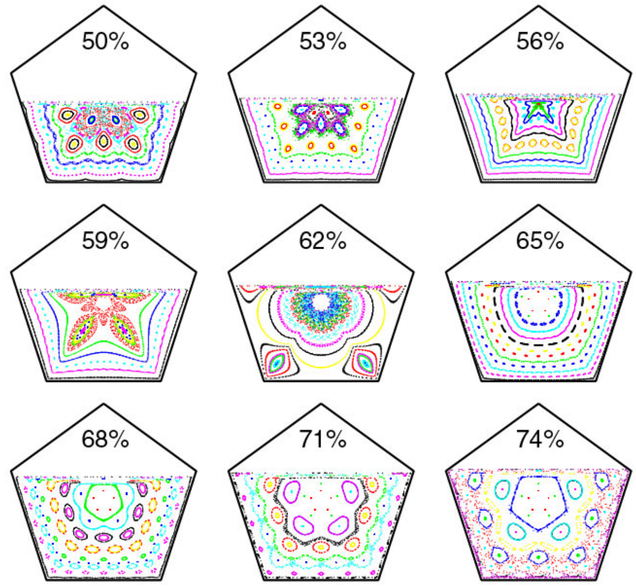

Figure 3 Poincaré sections for pentagonal tumblers with fill fractions of 50-74% in increments of 3%.

The work presented here makes it apparent that a strong correlation exists between the patterns observed in the underlying flow as indicated by Poincaré sections and the patterns formed by segregating mixtures of particles. The remaining challenges are to understand the dependence of the structure of Poincaré sections on fill fraction and geometry, and to understand how segregation patterns are shaped by the competition between radial separation and attraction to KAM regions. To address the latter, experiments with varying relative numbers of heavy to light particles would be instructional. We have conducted preliminary experiments showing that different segregation patterns form when the relative fraction of small particles is changed at constant fill fraction (same Poincaré section). Progress on the former challenge (understanding the structure of Poincaré sections) would be aided by interpreting the action of the underlying continuum flow as a mapping. This would allow for the calculation of the location of the periodic points. Knowledge of the location of these points as a function of fill fraction would aid in the prediction of segregation patterns without the use of Poincaré sections derived from simulations.

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Referenes:

Cisar, S.E., Umbanhowar, P.B., Ottino, J.M. (*to be submitted to Phys. Rev. E*)

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