337c Computing Three-Dimensional, Steady-State, Incompressible Flows Using a Parallel, Higher-Order, Mixed-Formulation, Galerkin Finite Element Method

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The computation of three-dimensional, steady-state, incompressible, flows using iterative solution approaches poses significant challenges for most algorithms. Our experience in solving for flows in melt crystal growth systems has shown that additional challenges are presented by the inherent coupling of thermal and flow fields, the representation of recirculating flows in enclosed geometries, and convergence issues for solving flows of moderate to high Reynolds or Rayleigh numbers.

In this presentaton, we compare the performance of two finite element methods for solution of three-dimensional, steady-state, incompressible flows representative of those found in melt crystal growth systems. The first approach employs standard equal-order interpolation using linear basis functions on tetrahedral elements using a pressure-stabilized, Petrov-Galerkin method (PSPG) that has been widely used for three-dimensional, finite-element analysis of incompressible flows. The second approach uses a new, Galerkin mixed-order formulation employing a higher-order, quadratic representation of the velocity field with a linear basis for pressure. This basis set is a modification of the Crouzeix--Raviart P2+-P1 element, containing 15 nodes, 45 velocity degrees of freedom, and 4 pressure degrees of freedom per tetrahedral element. Both algorithms employ Newton's method with an iterative linear solver and are implemented using MPI protocols on parallel computers. Several preconditioning methods are employed with GMRES via PETSc for the linear solver.

Metrics are presented comparing the utility of both approaches for solution of melt crystal growth flows and comments are provided on additional challenges posed by the free-boundary nature of the crystal growth problem.