337a A Novel Efficient Pseudospectral Method for the Dns of Turbulent Flow in a Wavy Channel

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In this work, we have used a previously developed efficient Helmholtz solver, based on a spectrally preconditioned biconjugate gradient algorithm, to perform Direct Numerical Simulations (DNS) in an undulating channel geometry. In contrast to the previous work (Dimitropoulos and Beris, 1997), we use a transformation involving the shear direction only to map the undulating geometry to a rectangular one so that a spectral approximation can be used. This is a much simpler domain transformation than the pseudoconformal mapping required in previous spectral method implementations of the solution of a Helmholtz equation in undulating channel geometry (Dimitropoulos et al., 1998) albeit it results to a non-orthogonal coordinate system. However, in principle, much more intense undulations can be handled since the pseudoconformal mapping ceases to provide an one to one transformation of the undulating geometry to a rectangular one at relatively modest degree of undulation (of the order 10%, depending on the aspect ratio). The Bi-CGSTAB (with preconditioning) is a very competitive iterative method for solving relevant classes of non-symmetric linear systems of equations since its convergence behavior is much smoother so that it often produces much more accurate solutions and in most cases it converges considerably faster than CG-S (Van der Vorst, 1992). The preconditioning strategy utilized is the iterative spectral preconditioning, developed based on the fast direct solver for solving the separable Helmholtz equations and the iterative scheme by Concus and Golub (1973). This iterative solver solves non-separable Helmholtz equations iteratively, iterating over the perturbation terms that come from the undulation of the geometry. We performed DNS of Newtonian turbulent flow through a wavy channel with both of the walls sinusoidal. Using the before mentioned non-orthogonal transformation, we have developed a spectral approximation to discretize the space and a time-splitting scheme based on a mixed implicit/ explicit time integration formulation involving a second order implicit multistep approximation (Adams-Moulton) and a second order explicit one (Adams-Bashforth). Thus, the three-dimensional and time-dependent undulating channel flow problem can be reduced to a series of Poisson and Helmholtz problems in a two-dimensional wavy geometry that can be efficiently solved using the Bi-CGSTAB spectral preconditioned solver developed in this work. Sample results will be shown in comparison to the literature.

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