A revised formulation of the Particle Bed Model for the investigation of the fluidization dynamics of solid suspensions

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ABSTRACT

In the present work, the equations of change and the closure relationships featuring in the original mono-dimensional Particle Bed Model (PBM) proposed by Foscolo & Gibilaro (1984) have been revised. The model, initially intended for analyzing the stability of homogeneous fluidized beds, has been extended to a multi-dimensional formulation so as to render it suitable for the study of the dynamics of both particulate and aggregative regimes of fluidization. In the equations of change, the pressure gradient is no longer shared by the two phases in proportion to their volume fractions, but features only in the continuous phase. Conversely, the "elastic" force is included, with opposite signs, in both fluid and solid linear momentum equations, so that the principle of action and reaction, to which the force is subjected, is fulfilled. Finally, the contributions of the fluid viscous stress tensor and of the solid stress tensor to the linear momentum conservation equations for the continuous and dispersed phases respectively are accounted for and no longer neglected. As for the closure relationships, the buoyancy is related to the weight of the fluidizing fluid displaced by the particle phase and to the local fluid acceleration, hence no longer being regarded as proportional to the fluid pressure gradient as proposed in the original PBM. Furthermore, a new constitutive equation is advanced for the drag force; this is expressed as the product of the drag force exerted on an unhindered particle, subject to the same volumetric flux of fluid, and a "corrective" function dependent on both bed voidage and particle Reynolds number. This revised equation is deemed more accurate than the original one proposed by Foscolo & Gibilaro particularly with reference to the intermediate flow regimes comprised between the viscous and the inertial ones. Finally, the "elastic" force is estimated by employing a rigorous approach which does not resort to equilibrium-based relations; the result, enhanced in accuracy and breadth of validity, considers "elastic" force and drag force proportional. The Revised Particle Bed Model has been first used to investigate the stability of homogeneous fluidized beds by means of linear analysis; moreover, it has been employed to simulate the fluidization dynamics of a gas-fluidized Geldart's Group B powder. The multi-dimensional model has been solved using the commercial CFD code CFX 4.4.

Keywords: Fluidization; Mathematical Modelling; Particle Bed Model; Fluidization Dynamics.

INTRODUCTION

Since its very first commercial applications, the technique of fluidization has attracted more and more the attention of the industrial world, which didn't fail to recognize and appreciate the potential offered by this innovative technology.

Albeit used extensively in commercial operations, nonetheless fluidization still poses a major challenge to engineers when tackling the design of new industrial plants. These, for their very nature, are highly dependent on their hydrodynamic behaviour which in turn is affected by the system geometry and size. Critical scale-up problems therefore arise, related to how accurately the performance changes with plant size can be accounted for throughout the design stage.

In this regard, CFD has proved a valuable research means; the aim is succeeding in simulating and investigating the behaviour of full-size units, so as to add insight into the passage from pilot plants to industrial ones, and render the latter less uncertain and risky. To this purpose, it is nevertheless critical that accurate models be developed, along with appropriate constitutive equations. Among the various models available today, Foscolo and Gibilaro's Particle Bed Model stands out for its valuable feature of presenting a good trade-off between accuracy and complexity. The Revised Particle Bed Model herein presented, whilst retaining such feature, revisits both equations of change and closure relationships featuring in the original PBM, with the aim to widen its range of employability and possibly enhance its predictive capabilities.

EQUATIONS OF CHANGE

The equations of change for both original and revised Particle Bed Model can be conveniently arranged in the following general form.

Fluid Phase Equations of Change

$$\frac{\partial}{\partial t}\left(\varepsilon \cdot \rho_f\right) + \vec{\nabla} \bullet \left(\varepsilon \cdot \rho_f \cdot \vec{U}_f\right) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\varepsilon \cdot \rho_f \cdot \vec{U}_f\right) + \vec{\nabla} \bullet \left[\varepsilon \cdot \rho_f \cdot \left(\vec{U}_f \otimes \vec{U}_f\right)\right] = +\vec{\nabla} \bullet \underline{\underline{\mathrm{T}}}_f - \vec{F}_{f\to p} + \vec{F}_f \tag{2}$$

Solid Phase Equations of Change

$$\frac{\partial}{\partial t}\left(\alpha \cdot \rho_p\right) + \vec{\nabla} \bullet \left(\alpha \cdot \rho_p \cdot \vec{U}_p\right) = 0 \tag{3}$$

$$\frac{\partial}{\partial t}\left(\alpha \cdot \rho_p \cdot \vec{U}_p\right) + \vec{\nabla} \bullet \left[\alpha \cdot \rho_p \cdot \left(\vec{U}_p \otimes \vec{U}_p\right)\right] = +\vec{\nabla} \bullet \underline{\underline{\mathrm{T}}}_s + \vec{F}_{f\to p} + \vec{F}_p \tag{4}$$

In these equations, ε and α are the fluid and solid volume fractions, ρ_f and ρ_p are the fluid and solid densities, and $\vec{U}_f$ and $\vec{U}_p$ are the fluid and solid velocities respectively. Furthermore, $\underline{\underline{\mathrm{T}}}_f$ and $\underline{\underline{\mathrm{T}}}_s$ represent the effective fluid and solid stress tensors respectively, whereas $\vec{F}_{f\to p}$ denotes the inter-phase force exerted by the continuous phase on the dispersed one. As for the terms $\vec{F}_f$ and $\vec{F}_p$, we refer to the following section.

CLOSURE RELATIONSHIPS

Fluid Phase Stress Tensor

$$\text{Original PBM:} \quad \underline{\underline{T}}_f = -P \cdot \underline{\underline{I}} \qquad (5)$$

$$\text{Revised PBM:} \quad \underline{\underline{T}}_f = -P \cdot \underline{\underline{I}} + \mu_f \cdot \left(\vec{\nabla}\vec{U}_f + \vec{\nabla}\vec{U}_f^{\,T} \right) \qquad (6)$$

where P is the fluid pressure, μ_f is the fluid viscosity, and $\underline{\underline{I}}$ is the unit tensor.

Solid Phase Stress Tensor

$$\text{Original PBM:} \quad \underline{\underline{T}}_s = \underline{\underline{0}} \qquad (7)$$

$$\text{Revised PBM:} \quad \underline{\underline{T}}_s = -P_s \cdot \underline{\underline{I}} + \mu_s \cdot \left(\vec{\nabla}\vec{U}_s + \vec{\nabla}\vec{U}_s^{\,T} \right) + \left(\xi_s - \frac{2}{3} \cdot \mu_s \right) \cdot \left(\vec{\nabla} \bullet \vec{U}_s \right) \cdot \underline{\underline{I}} \qquad (8)$$

where P_s is the solid pressure, μ_s is the shear solid viscosity, and ξ_s is the bulk solid viscosity.

Inter-phase Interaction Force

$$\text{Original PBM:} \quad \vec{F}_{f \to p} = \vec{F}_{S,V} + \vec{F}_{K,V} \qquad (9)$$

$$\text{Revised PBM:} \quad \vec{F}_{f \to p} = \vec{F}_{S,V} + \vec{F}_{K,V} + \vec{F}_{E,V} \qquad (10)$$

Fluid-phase and Solid-phase Remaining Forces

$$\text{Original PBM:} \quad \vec{F}_f = \varepsilon \cdot \rho_f \cdot \vec{g} \quad ; \quad \vec{F}_p = \alpha \cdot \rho_p \cdot \vec{g} + \vec{F}_{E,V} \qquad (11)$$

$$\text{Revised PBM:} \quad \vec{F}_f = \varepsilon \cdot \rho_f \cdot \vec{g} \quad ; \quad \vec{F}_p = \alpha \cdot \rho_p \cdot \vec{g} \qquad (12)$$

where $\vec{g}$ is the gravitational acceleration.

Buoyancy

$$\text{Original PBM:} \quad \vec{F}_{S,V} = -\alpha \cdot \vec{\nabla} P \qquad (13)$$

$$\text{Revised PBM:} \quad \vec{F}_{S,V} = -\alpha \cdot \rho_f \cdot \left(\vec{g} - \frac{D\vec{U}_f}{Dt} \right) \qquad (14)$$

Drag Force

$$\vec{F}_{K,V} = +\beta \cdot \left(\vec{U}_f - \vec{U}_p \right) \tag{15}$$

Original PBM: $$\beta = +\frac{3}{4} \cdot C_D \cdot \frac{\left\| \vec{U}_f - \vec{U}_p \right\| \cdot \rho_f \cdot \alpha}{D_p} \cdot \varepsilon^{-1.8} \tag{16}$$

Revised PBM: $$\beta = +\frac{3}{4} \cdot C_D \cdot \frac{\left\| \vec{U}_f - \vec{U}_p \right\| \cdot \rho_f \cdot \alpha}{D_p} \cdot \varepsilon^{-\phi(\varepsilon, \mathrm{Re})} \tag{17}$$

$$C_D = \left(0.63 + 4.8 \cdot \mathrm{Re}^{-\frac{1}{2}} \right)^2 \quad ; \quad \mathrm{Re} = \frac{\rho_f}{\mu_f} \cdot \varepsilon \cdot \left\| \vec{U}_f - \vec{U}_p \right\| \cdot D_p \tag{18}$$

The exponent ϕ of the "corrective" function depends both on the bed voidage and on the Reynolds number; such dependency is displayed in Figure 1.

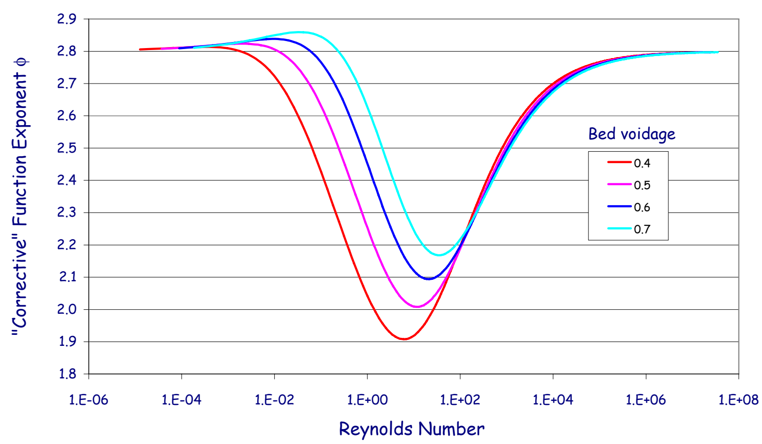

Figure 1: Exponent ϕ as a function of Re parameterized with respect to the bed voidage ε.

"Elastic" Force

Original PBM: $$\vec{F}_{E,V} = -3.2 \cdot D_p \cdot \alpha \cdot \left(\rho_p - \rho_f \right) \cdot \frac{\partial \varepsilon}{\partial z} \cdot \vec{g} \tag{19}$$

Revised PBM: $$\vec{F}_{E,V} = -\frac{2}{3} \cdot D_p \cdot \Omega\left(\varepsilon, \mathrm{Re} \right) \cdot \left[\vec{\nabla} \varepsilon \bullet \vec{n}_D \right] \cdot \vec{F}_{K,V} \tag{20}$$

where z is the upward vertical axis parallel to $\vec{g}$, $\vec{n}_D$ is the drag force versor, and Ω is a proportionality function which relates the "elastic" force to the drag force (its expression for brevity is herein omitted). Whereas equation (19) has been derived by drawing on equilibrium-based relationships, equation (20) is of general validity. In the revised formulation, the "elastic" force is no longer constant in direction and parallel to $\vec{g}$, but results instead parallel to the drag force.

STABILITY ANALYSIS

The revised PBM has been used to conduct a fluid-bed stability analysis on a wide range of Geldart's Group A gas-fluidized powders at different operating temperatures. In this analysis, the contributions of the solid stress tensor to the linear momentum conservation equation for the solid phase, and of the local acceleration of the fluid to the buoyant force have been neglected.

Values for the minimum bubbling voidage predicted by the revised PBM have been compared with those predicted by the original PBM and with experimental data.

Figure 2 reports the minimum bubbling voidage as a function of the mean particle diameter for different FCC powders (diameter: 137 – 26 μm, density: 1210 – 1420 kg/m^3); the experimental data refer to ambient conditions and have been obtained by Xie & Geldart (1995).

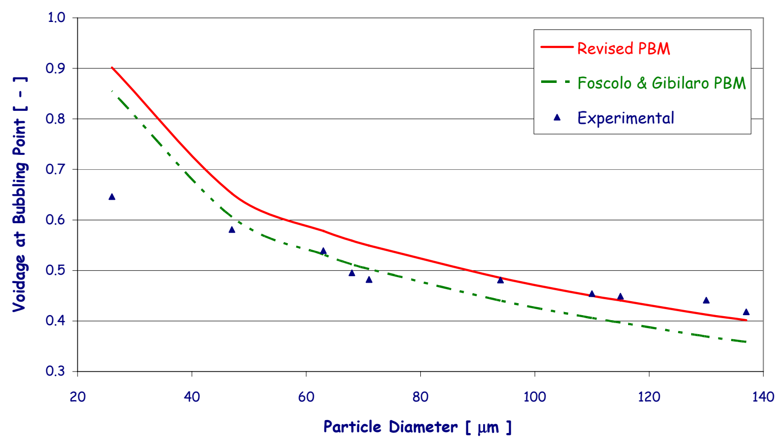

Figure 2: Minimum bubbling voidage as a function of mean particle diameter for different powders at ambient temperature.

A good agreement is found between predictions of the revised PBM and experimental data (error within ± 5 %) in the "high" range of mean particle diameters (greater than about 90 μm). Nevertheless, the more the mean particle diameter is reduced, the less accurate the predictions of the model become (the same holding true also for the original PBM). A possible explanation for the poor quality of the theoretical predictions obtained for very low mean particle diameters can be found in the role, altogether neglected in the present models, played by the inter-particle forces (IPFs), as discussed by Xie & Geldart (1995) and Lettieri (1999).

In Figure 3, the theoretical values of the minimum bubbling voidage are plotted against the experimental ones for a Ballotini powder (diameter: 62 μm, density: 2550 kg/m^3); the experimental data refer to operating temperatures varying from ambient conditions up to about 300 ºC and have been obtained by Lettieri (1999).

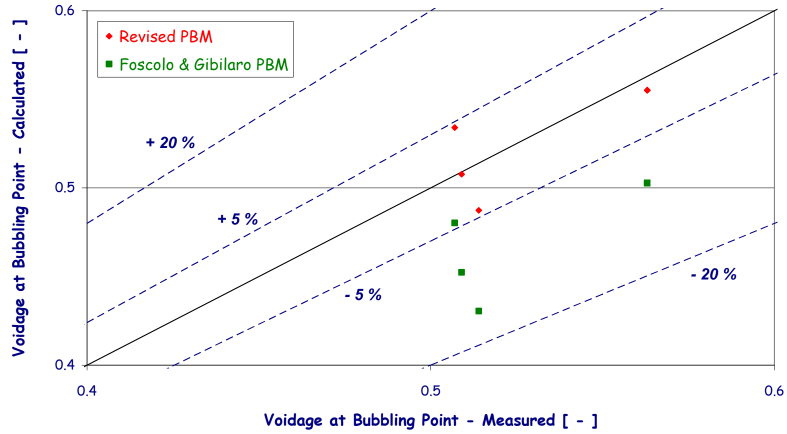

Figure 3: Comparison between theoretical and experimental minimum bubbling voidage for a Ballotini powder at different temperatures (20 – 300 °C).

Again a good agreement is found between predictions of the revised PBM and experimental data (error within ± 5 %). The revised PBM formulation proves better predictive than the original one, insomuch as the latter tends to underestimates the minimum bubbling voidage thus anticipating the onset of the bubbling regime of fluidization.

FLUIDIZATION DYNAMICS

The fluidization dynamics of a Geldart Group B powder has been simulated using the Revised Particle Bed Model. In the present work, we have considered a Ballotini powder (diameter: 350 μm, density: 2500 kg/m^3) fluidized by air; the superficial velocity has been assumed equal to 0.25 m/s. The mathematical model has been solved by using the commercial CFD code CFX 4.4. Figure 4 reports the results of the simulation (for the first three seconds) expressed in terms of bed voidage profile.

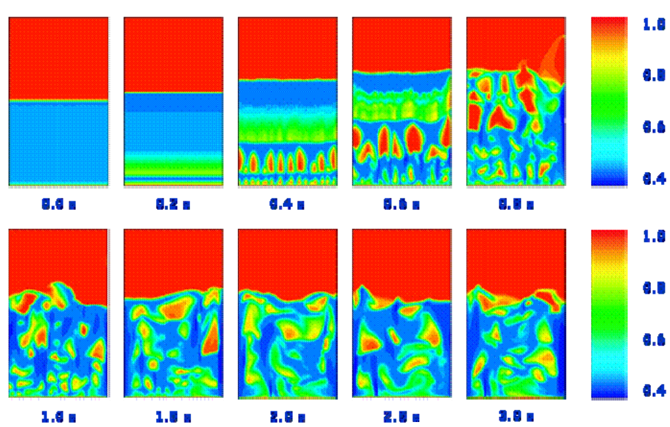

Figure 4: Voidage profile as a function of time for a Ballotini powder (diameter: 350 μm, density: 2500 kg/m^3) fluidized by air (superficial velocity equal to 0.25 m/s).

CONCLUSIONS

The original PBM has been revised and extended to a multi-dimensional formulation. The stability of homogeneous fluidized beds has been investigated; the results of the revised model have been compared with those of the original one and with experimental data. A good agreement has always been found between the latter and the revised PBM predictions. For much reduced mean particle diameters both original and revised model lose in accuracy. The revised PBM demonstrates also the capability of predicting the transition from homogeneous to bubbling fluidization at high temperatures when the role of the hydrodynamic forces remains dominant over the inter-particle forces. As a multi-dimensional application of the revised PBM, the fluidization dynamics of a Geldart's Group B powder has been simulated and the results provided in terms of bed voidage profile.

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