242i Estimation in Nonlinear Dynamic Systems Via Monte Carlo Sampling Versus Moving Horizon Estimation – Complementary or Competitive?

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Moving horizon estimation (MHE) has been among the most popular approaches for estimation in nonlinear dynamic systems, particularly in the presence of constraints [1,2]. Its properties have been widely studied and it has been shown to outperform extended Kalman filtering (EKF) by avoiding divergence and constraint violation. EKF extends Kalman filtering to nonlinear dynamic systems by linearizing the model at each time step, while assuming the noise and prior to be Gaussian. EKF does better than Kalman filtering, but may diverge and violate constraints. In addition to constraints, it is quite common for nonlinear dynamic systems to exhibit non-Gaussian distributions of the state variables or prior, as well as non-Gaussian errors due to outliers and sensor characteristics. MHE addresses some of these challenges by solving a constrained nonlinear programming problem over a moving horizon, usually of a finite size. The resulting estimates can be more accurate than EKF due to imposition of constraints and incorporation of the nonlinear dynamics in the objective function. However, MHE is significantly slower than EKF due to its non-recursive formulation. Furthermore, MHE relies on an "arrival cost" for each moving window that aims to capture prior knowledge from the model and measurements. Having a good estimate of the arrival cost (or prior in a Bayesian interpretation) is a key factor influencing the accuracy of MHE. Many methods have been explored for estimating the arrival cost including the use of a parallel EKF or unconstrained nonlinear filter, but all methods seem to assume the arrival cost to be Gaussian. As mentioned above, this assumption is often violated for many practical systems resulting in a loss of accuracy.

Recent developments in Bayesian estimation via sequential Monte Carlo (SMC) sampling have been used for solving the unconstrained and constrained dynamic estimation problem. SMC relies on an efficient propagation of the information contained in models and measurements via samples or particles, while avoiding linearization or assumptions of Gaussian or other fixed shape distributions. In our previous work [3,4,5], we have compared SMC with MHE and shown that the former can be at least as accurate as MHE, computationally more efficient, and can easily handle constraints and non-Gaussian errors. These properties are due to the recursive formulation of SMC, which permits the prior (analogous to the arrival cost in MHE) to be calculated more accurately and efficiently than MHE and EKF. Another important advantage of SMC is that it can readily provide regions of highest posterior density or error bounds for each estimate without any additional computation. Obtaining such bounds in MHE requires additional computation and may not be very accurate due to the Gaussian arrival cost.

This presentation will focus on whether SMC and MHE are competitive methods or may be combined to be complementary. We will discuss possible approaches for combining MHE with SMC, while providing a critical evaluation of the pros and cons of such a merger. Such approaches include using SMC for improving MHE by providing a more accurate arrival cost via the samples representing the prior at the beginning of each moving window. These samples may be used in MHE by fitting a multivariate distribution to obtain the arrival cost. Such an approach can be more accurate than some of the existing methods for estimating the arrival cost. Fitting non-parametric multivariate distributions can be even more beneficial, and may make MHE very similar to SMC in its accuracy. However, such an approach is not practically feasible due to the large increase in computational effort, particularly for high-dimensional problems. Another approach for combining SMC with MHE involves problems with multi-modal distributions. Most sampling based methods usually obtain the mean of the samples as the point estimate. This may not be very useful information for multimodal distributions, and the mode may be more meaningful. However, obtaining the mode from samples is often not very accurate. Here, combining SMC with MHE may make sense since MHE has the closed form of the distribution available, allowing more accurate estimation of the modes. The properties of SMC, MHE, and its potential combinations will be illustrated via examples from the literature, including those that MHE is known to find quite challenging [6].

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