Performance Assessment for Completely Unknown SISO and MIMO Systems Based on Gain and Phase Margins Using Modified Relay Feedback

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Abstract

A performance assessment procedure for single-input-single-output (SISO) and multi-input-multi-output (MIMO) systems based on modified relay feedback test is proposed in this paper. The well-known robustness measurements, gain and phase margins, are used to assess the performance of the control systems. The proposed method can on-line estimate the gain and phase margins for systems with both unknown controller parameters and process dynamics by a modified relay feedback scheme where a delay element is embedded between the relay and controller. The estimated results can be used to indicate the appropriateness of the controller parameters. When the retuning of controller is found necessary, a similar procedure can be applied to tune the PI/PID controller based on specifications of gain and phase margins. Performance assessment and controller design can be done simultaneously, which ensures a good performance of the control system.

Introduction

The proportional-integral-derivative (PID) controller is widely used in chemical process industries because of its simple structure and robustness to the modelling error. Despite the fact that numerous PI/PID tuning methods have been provided in the literature, many control loops are still found to perform poorly. Therefore, regular performance assessment and controller retuning are necessary. In process control, minimum variance has been used as a benchmark for assessing the closed-loop performance for decades (Harris et al, 1999). This criterion is a valuable measurement of the system performance, but it pays little attention to the traditional performance such as set-point tracking and disturbance rejection. Besides, another important factor of system performance, robustness, is not addressed directly.

Gain and phase margins have served as important measure of robustness for the single-input-single-output (SISO) system. It is also known from classical control theory that the phase margin is related to the damping of the system and thus can also serve as a performance measure. For the multi-loop control of multi-input-multi-output (MIMO) system, gain and phase margins can also be defined in the similar spirit as SISO system based on the effective open-loop process (EOP) (Huang et al., 2003) of the multi-loop system. Traditionally, under the assumption that the process model and controller parameters are known, the gain and phase margins are obtained numerically or graphically by trial-and-error use of Bode' plots. However, this assumption is not practical because the process model and controller settings may be unknown at the stage of performing the performance assessment. Thus, it is highly desirable to find a procedure for on-line monitoring of gain and phase margins. Recently, Ma and Zhu (2005) proposed a performance assessment procedure for SISO system based on modified relay feedback. Gain and phase margins are estimated by two relay tests where an ideal relay is used for the first test and a relay with hysteresis is used for the other. Due to a linear assumption about the amplitude of the limit cycle, their method may not give accurate results for processes with more complex dynamics such as process with RHP zero and oscillatory process.

In this paper, an on-line performance assessment procedure based on modified relay feedback test is proposed. This modified relay feedback structure embeds an additional delay between the relay and controller. The gain and phase margins are first used to assess the robust stability of the control systems. Furthermore, they are related to the time-domain control performance, in terms of integral of the absolute value of the error (IAE), for performance assessment. The proposed method can on-line estimate the gain and phase margins for systems with both unknown controller parameters and process dynamics by a modified relay feedback scheme. For multi-loop systems, the modified relay tests are conducted in a sequential manner to estimate the gain and phase margins of each loop. The estimated results can be used to indicate the appropriateness of the controller parameters. When the retuning of controller is found necessary, a similar procedure can be applied to tune the PI/PID controller based on the specifications of gain and phase margins. In this way, performance assessment and controller design can be done simultaneously, which ensures a good performance of the control system.

Performance assessment of SISO System

The proposed modified relay feedback structure is shown in Fig. 1 where G_c,

G, u_r, and y are the controller, process, relay output, and plant output, respectively. Moreover, a delay element, $e^{-\Delta s}$, is embedded between the relay and the controller. Compared with the conventional relay feedback, the most important features of this modified structure are that the controller is always connected in line with the process and an additional delay is embedded. As a result, it can assess the performance of the closed-loop system by on-line estimation of gain and phase margins to determine if a retuning of the controller is necessary.

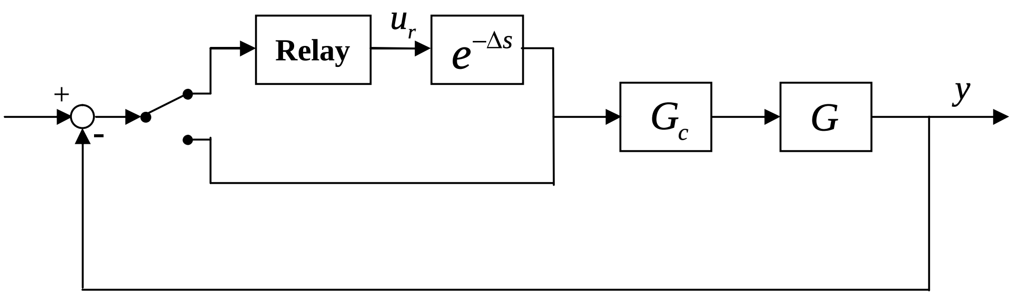

Fig. 1 Modified relay feedback system

Estimation of gain margin

Consider the modified relay feedback system as shown in Fig. 1. For the estimation of gain margin, the delay Δ is set as zero. Let the loop transfer function be $G_{LP}(s) = G_c(s)G(s)$. The system starts to oscillate and then attain a limit cycle. The oscillating point is the intersection of the Nyquist curve of $G_{LP}(s)$ and the negative real axis in the complex plane, as shown in Fig. 2. The phase crossover frequency of $G_{LP}(s)$ is $w_p = 2p/P_p$, where P_p is the period of the limit cycle. In addition, using the approximation of describing function, the amplitude of $G_{LP}(s)$ can be calculated as $\left|G_{LP}\left(jw_p\right)\right| = p\,a/4h$, where a is the amplitude of the limit cycle and h is the relay output magnitude. For more accurate estimation, $\left|G_{LP}\left(jw_p\right)\right|$ can be computed based on Fourier analysis as:

$$\left|G_{LP}\left(jw_p\right)\right| = \frac{\left|\int^{P_p} y(t)\,e^{-jw_p t}dt\right|}{\left|\int^{P_p} u_r(t)\,e^{-jw_p t}dt\right|} \tag{1}$$

Therefore, the gain margin, A_m, can be estimated as

$$A_m = \frac{1}{\left| G_{LP}\left(jw_p \right) \right|} \qquad (2)$$

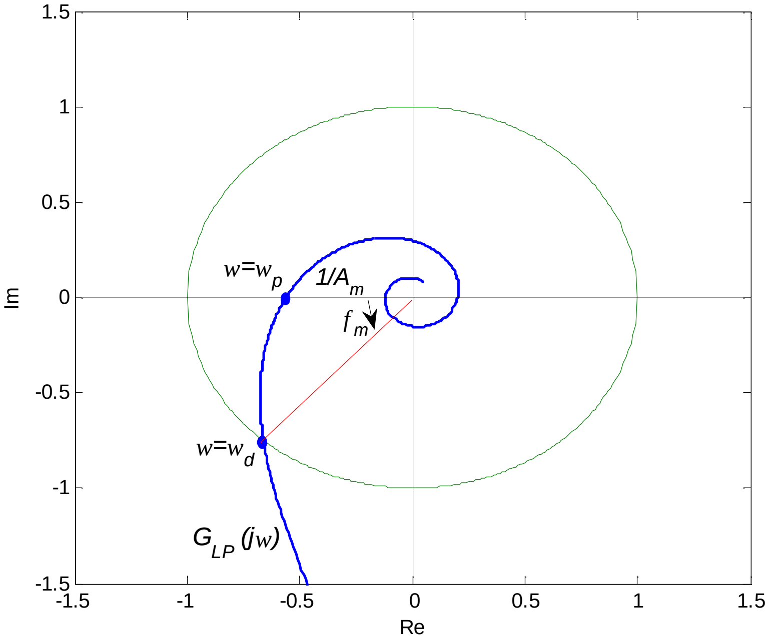

Fig. 2 Estimation of gain and phase margins

Estimation of phase margin

For the estimation of phase margin, the delay Δ is set as a non-zero value in order to extract the frequency information of $G_{LP}(s)$ at some frequency other than w_p. With a given value of Δ, the system oscillates with a period of P and we have $\mathbf{R}G_{LP}\left(jw \right) - \Delta w = -p$, where $w = 2p/P$. The desired frequency is the gain crossover frequency, w_d, which is the intersection of the Nyquist curve of $G_{LP}(s)$ and the unit circle in the complex plane, as shown in Fig. 2. Assume the desired value of Δ that make the amplitude of $G_{LP}\left(jw_d \right)$ equals unity is Δ_d and the period of the limit cycle is P_d, i.e. $\mathbf{R}G_{LP}\left(jw_d \right) - \Delta_d w_d = -p$ where $w_d = 2p/P_d$. Then, the phase margin, f_m, can be estimated as:

$$f_m = \mathbf{R}G_{LP}\left(jw_d \right) + p = \Delta_d w_d \qquad (3)$$

To find the value of Δ_d, an iterative procedure as the following is required.

Starting from an initial guess Δ^0, the value of Δ is updated by

$$\Delta^{i+1} = \Delta^i - g\left(\left|G_{LP}\left(jw\right)\right| - 1\right) \quad (4)$$

where $g > 0$ is the convergence rate and $\left|G_{LP}\left(jw\right)\right|$ is computed by

$$\left|G_{LP}\left(jw\right)\right| = \left|G_{LP}\left(jw\right)e^{-jw\Delta}\right| = \frac{\left|\int^{P} y(t)\, e^{-jwt} dt\right|}{\left|\int^{P} u_r(t)\, e^{-jwt} dt\right|} \quad (5)$$

When Eq.(4) converges, the resulting value of Δ is taken as Δ_d.

Assessment of IAE performance

With the estimated gain and phase margins, the robustness of the current system can be assessed. The recommended ranges of gain and phase margins are between 2 and 5 and between 30^o and 60^o. Nevertheless, gain and phase margins also reveal some implications about the IAE performance of the system. For set-point tracking, a control system with gain margin about 2.1 and phase margin about 60^o can achieve the optimal IAE performance (Huang and Jeng, 2002). There is no serious conflict between system servo performance and robustness. On the other hand, for disturbance rejection, trade-off has to be made between the IAE performance and robustness. For example, considering the system with first-order-plus-dead-time process controlled by PI/PID controller, the smaller the gain margin is, the lower the IAE value for regulation can achieve. However, the optimal IAE value occurs at a phase margin about 30^o to 50^o (Jeng et al., 2004). Therefore, based on the gain and phase margins, not only the system robustness, but also the possibility of achieving optimal IAE performance can be assessed.

PI/PID Tuning based on specification of gain and phase margins

After assessment, when the performance of the control system is found poor, retuning of the controller is needed. The modified relay feedback scheme can be applied for on-line tuning of PI/PID controller based on specification of gain and phase margins, designated as A_m^* and f_m^*, respectively.

Consider the PI controller of the following transfer function.

$$G_c(s) = k_c\left(1+\frac{1}{t_I s}\right) \tag{6}$$

For a given value of Δ, the parameters, k_c and t_I, can be found to satisfy the specification of phase margin. In other words, they can be found such that the following two equations hold.

$$f_m^* = \Delta w_d \quad \text{or} \quad P_d = \frac{2p\,\Delta}{f_m^*} \tag{7}$$

$$\left|G_{LP}\left(jw_d\right)\right| = \left|G_{LP}\left(jw_d\right)e^{-jw_d\Delta}\right| = \frac{\left|\int^{P_d} y(t)e^{-jw_d t}dt\right|}{\left|\int^{P_d} u_r(t)e^{-jw_d t}dt\right|} = 1 \tag{8}$$

However, the specification of gain margin may not be necessarily satisfied by such controller setting. In general, the gain margin of the resulting system, i.e. system with $f_m = f_m^*$, is a function of Δ value. Therefore, there exists a certain value of Δ which can make the gain margin of the resulting system meet its specification. According the analysis, an iterative procedure for tuning the PI controller is presented as follows:

1) Starting with a guessed value of Δ, i.e. Δ^0, adjust t_I by the following equation until it converges so that Eq.(7) holds.

$$t_I^{i+1} = t_I^i + g_1\left(P_d - \frac{2p\,\Delta}{f_m^*}\right) \tag{9}$$

2) Adjust k_c by the following equation until it converges so that Eq.(8) holds.

$$k_c^{i+1} = k_c^i - g_2\left(\left|G_{LP}\left(jw_d\right)\right| - 1\right) \tag{10}$$

3) Set $\Delta = 0$ and estimate A_m by Eq.(2).

4) Check if the estimated A_m equals A_m^*. If not, change the value of Δ by the following equation and go back to step 1) until $A_m = A_m^*$ holds.

$$\Delta^{i+1} = \Delta^i - g_3\left(A_m - A_m^*\right) \tag{11}$$

When the value of Δ converges, the resulting controller parameters are the desired ones.

For the tuning of PID controller, a similar procedure can be applied. The PID controller transfer function is given as

$$G_c(s) = k_c\left(1 + \frac{1}{t_I s} + t_d s\right) \tag{12}$$

The derivative gain, t_d, is usually chosen as a fixed ratio of the integral time t_I, Researchers Åström and Hägglund (1984) and Hang et al. (1991) have recommended that $t_d = t_I/4$. As a result, the procedure for PI controller tuning presented earlier can be applied directly. If t_d is not chosen as a fixed ratio of t_I, then the extra degree of freedom can be used for meeting another control requirement.

Performance assessment of MIMO System

Multi-loop SISO controllers are often used to control chemical plants which have MIMO dynamics. Ho et al. (1997) have defined the gain and phase margins based on the Gershgorin bands of the multi-loop system. However, according to their definition, the gain and phase margins of each one loop are independent of the controllers in the other loops so that the interactions between loops are not considered. In this paper, the gain and phase margins is defined in the similar spirit as SISO system based on the effective open-loop process (EOP) of the multi-loop system (Huang et al., 2003). The ith EOP describes the effective transmission from the ith input to the ith output when all other loops are closed. With the formulation of EOP, the multi-loop control system can be considered as several equivalent SISO loops.

Consider the multi-loop control system of a 2x2 multivariable process with the following process and controller transfer function matrices, $G(s)$ and $G_c(s)$.

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}; \qquad G_c(s) = \begin{bmatrix} g_{c1}(s) & 0 \\ 0 & g_{c2}(s) \end{bmatrix} \tag{13}$$

The mathematical definition of the two EOPs, $\mathbf{g}_1(s)$ and $\mathbf{g}_2(s)$, are given as

$$\begin{aligned} \mathbf{g}_1(s) &= g_{11}(s) - g_{12}(s)\left[g_{22}(s)\right]^{-1} g_{21}(s)h_2(s) \\ \mathbf{g}_2(s) &= g_{22}(s) - g_{21}(s)\left[g_{11}(s)\right]^{-1} g_{12}(s)h_1(s) \end{aligned} \tag{14}$$

where

$$h_i(s) = \frac{g_{ci}(s)g_{ii}(s)}{1 + g_{ci}(s)g_{ii}(s)}; \qquad i = 1, 2 \tag{15}$$

Based on the EOPs of Eq.(14), the loop transfer functions of the equivalent loops are $G_{LP,1}(s) = g_{c1}(s)\mathbf{g}_1(s)$ and $G_{LP,2}(s) = g_{c2}(s)\mathbf{g}_2(s)$. Therefore, as the case of SISO system, the gain and phase margins of each loop can be estimated by sequentially using of the proposed modified relay feedback system. Fig. 3 shows the modified relay feedback scheme for the estimation of gain and phase margins of the first loop, where loop 1 is under relay mode and loop 2 is under control mode. Then, the modes of loops are switched to estimate the gain and phase margins of the second loop. This procedure can be extended to an *n*-loop system, where the modified relay tests are conducted in a sequential manner to estimate the gain and phase margins of each loop.

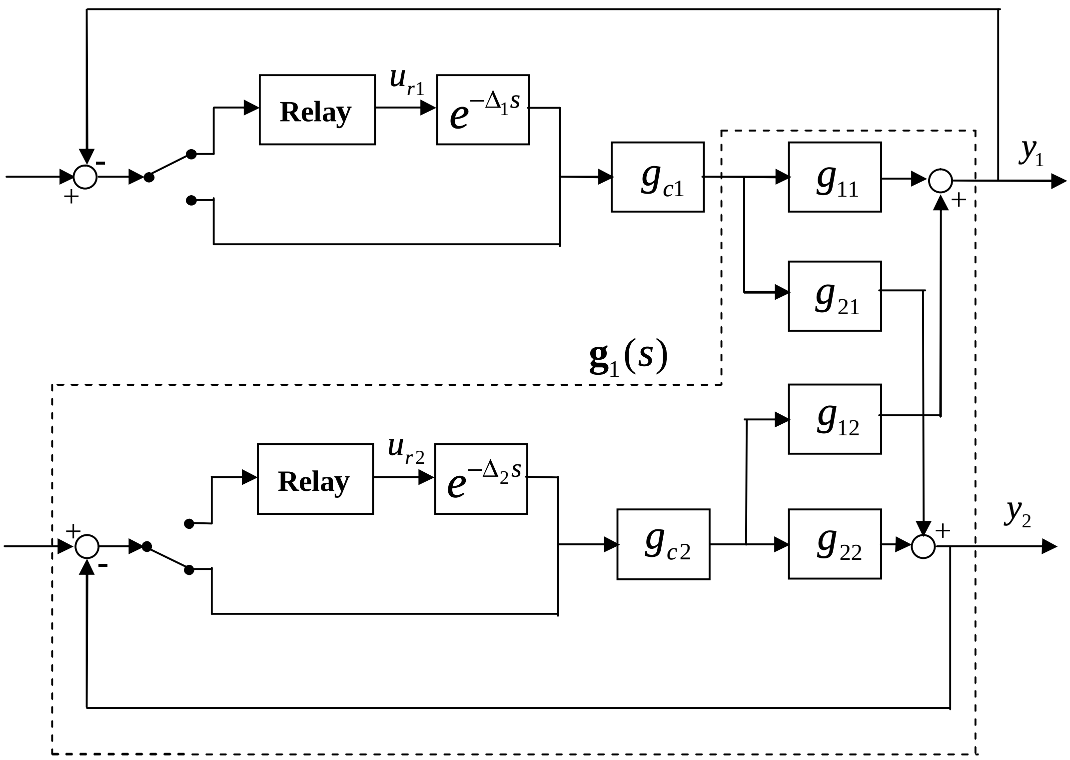

Fig. 3 Modified relay feedback scheme for a 2x2 multi-loop system

When retuning of the controller is found necessary, a similar procedure like the case of SISO system can be applied to tune the controller $g_{ci}(s)$ to meet the specification of gain and phase margins of the ith loop. If gain and phase margins of more than one loop are simultaneously specified, the controller tuning needs to go through an iterative procedures due to the interaction nature of multi-loop system. In that case, gain and phase margins should be carefully specified to ensure the convergence of the controller parameters.

Conclusions

A performance assessment procedure for SISO and MIMO systems based on modified relay feedback test is proposed in this paper. The proposed method can on-line estimate the gain and phase margins for systems with both unknown controller parameters and process dynamics. The estimated results can be used to assess the performance of the closed-loop system. When the retuning of controller is found necessary, a similar procedure can be applied to tune the PI/PID controller based on the user-specified gain and phase margins. Performance assessment and controller design can be done simultaneously, which can ensure a good performance of the control system.

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