OPTIMAL DESIGN OF BATCH-STORAGE NETWORK UNDER RANDOM FAILURES AND WASTE TREATMENT PROCESSES

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ABSTRACT

The purpose of this study is to find the analytic solution of determining the optimal capacity (lot-size) of batch-storage network to meet the finished product demand under random failures of operating time and/or batch material. The superstructure of the plant consists of a network of serially and/or parallel interlinked batch processes and storage units. The production processes transform a set of feedstock materials into another set of products with constant conversion factors. Final product demand flow is susceptible to short-term random variation of cycle time and batch size as well as long-term variation of averaged trend. Some of production processes have random variation of product quantity. The spoiled materials are treated through regeneration or waste disposal processes. All other processes have only random variation of cycle times. The objective function of optimization is minimizing the total cost composed of setup and inventory holding costs as well as the capital costs of constructing processes and storage units. A novel production and inventory analysis, PSW (Periodic Square Wave) model, provides a judicious graphical method to find the upper and lower bounds of random flows. The advantage of PSW model comes from the fact that the model provides a set of simple analytic solution in spite of realistic description of the random material flow between processes and storage units and consequently the computation burden is significantly reduced. The resulting simple analytic solution can greatly enhance the proper and quick investment decision at the early plant design stage confronted with highly uncertain business environment.
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Introduction

The plant structure is composed of a batch-storage network that can cover most supply chain components such as raw material purchase, production, transportation and finished product demand. Optimal design of batch-storage network has been studied by Yi and Reklaitis using Periodic Square Wave (PSW) model (2002, 2003, 2004). In this study, waste treatment processes are added to the network. The processes in this study will be classified into three types according to random characteristics. The first type processes possess the uncertainty only in operating time. The second type processes possess uncertain product batch quantity because of random quantity of off-spec materials. The third type processes possess the uncertainty in both operating time and batch quantity. Considering the uncertainties in customer order time and quantity, product demand corresponds to type 3 process. Some parts of production processes have the characteristics of type 2 process. Type 2 processes are closely related with waste treatment processes. Raw material purchase, transportation and some parts of production processes are in the category of type 1. Most processes have uncertainties in both operating time and batch quantity but, considering the complexity of research, we will exclude joint uncertainty of operating time and batch quantity from the research scope of this study except that the product demand has both uncertainties as given parameters. This study will divide the demand variation into long-term trend and short-term randomness. The mid-term seasonality can be considered as a part of long-term trend or can be incorporated into the model by decomposing the periodic signal into sum of periodic square waves. Long-term trend of demand is averaged monthly or yearly and therefore the near future value of long-term trend is relatively accurately predictable. The far future prediction error of long-term trend can be reduced as it approaches to near future. To accommodate long-term trend into production plan, chemical plants have used multiperiod formulation since half century ago. In this study, we will enlarge the PSW model capability to the multiperiod formulation. Note that multiperiod formulation is non-periodic opposed to the basic assumption of PSW model. Short-term random change of any processes will be treated with the PSW model and a judicious graphical method. The great advantage of using PSW model to deal with uncertainty exists in no further computational increase thanks to analytical solutions. Thus, overall computation time is determined by the multiperiod formulation to compute average flow rates through the network. The computation time of the multiperiod formulation suggested in this study takes about 7 times of the same size linear programming problem (Yi and Reklaitis 2004).

Definition of Parameters and Variables

We follow the definition of parameters and variables in Yi and Reklaitis (2003). A chemical plant, which converts raw materials into final products through multiple physicochemical processing steps, is composed of a set of storage units (*J*) and a set of batch processes (*I*). Note that storage index $j \in J$ is superscript and process index $i \in I$ is subscript. We assume that one storage unit stores one material and therefore the storage index *j* corresponds to material index. Transportation processes are considered as a subset of batch processes without loss of generality. Each storage is involved with five types of material movement, purchasing from suppliers ($k \in K(j)$), shipping to consumer demand ($m \in M(j)$), discharging to waste disposal sink ($n \in N(j)$), feeding to production processes and producing from production processes.

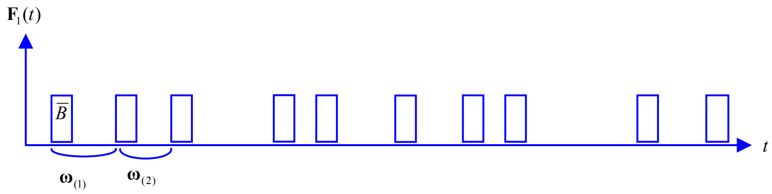

Figure 1. Flow of Type 1 Process

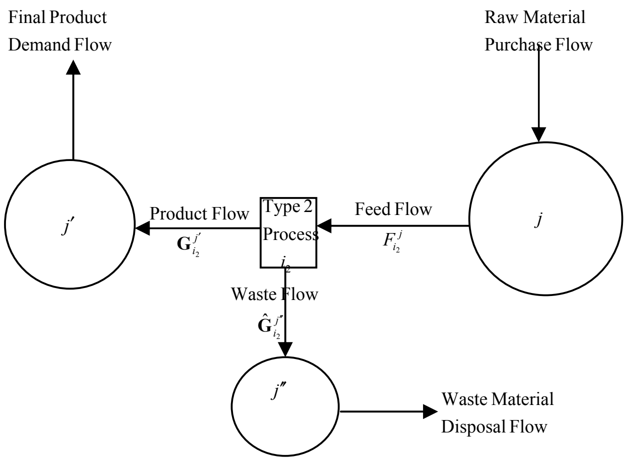

Figure 2. Schematic Diagram of Type 2 Process

We will consider 3 types of processes prone to random failures in this study. Figure 1 and 3 show the flows of type 1 and 2 process respectively. Figure 2 shows a typical configuration of type 2 process. The type 1 process does not include batch material loss or batch size change but only include random operating time loss or random increase of idle time. The type 2 process includes batch material loss and the spoiled material should be treated specially through waste treatment processing routes. The failed batch in type 2 process consumes not only feedstock materials but also production time equivalent to processing cycle time. We put assumptions that the batch size of type 1 process and the cycle time of type 2 process are not random though they are unknown. It is very unlikely that, in these modernized society, raw material purchase and transportation processes accompany frequent batch material losses and therefore, they are exclusively considered as type 1 process in this study. Production processes have both types. Mixing or blending processes usually do not have batch material loss and therefore, correspond to type 1 process. However, many reaction processes commonly have batch material loss and correspond to type 2 process. Note that the spoiled material in failed batch of type 2 process undergoes regeneration or waste treatment processing steps. In this study, the set of type 1 production processes will be denoted by $i_1 \in I_1$ and the variables or parameters related with type 1 production process will have subscript i_1. The set of type 2 production processes will be denoted by $i_2 \in I_2$ and the variables or parameters related with type 2 production process will have subscript i_2. Note that $I_i \cup I_2 = I$ and $I_i \cap I_2 = \varnothing$. Each production process requires multiple feedstock materials of fixed composition ($f_{i_1}^j$ or $f_{i_2}^j$) and produces multiple products with fixed product yield ($g_{i_1}^j$ or $g_{i_2}^j$). For type 2 process, the spoiled material of failed batch goes to the storage units other than the storage units where the product goes according to fixed waste yield ($\hat{g}_{i_2}^j$).

Note that, for the same storage unit *j* and process i_2, $g_{i_2}^j \hat{g}_{i_2}^j = 0$.

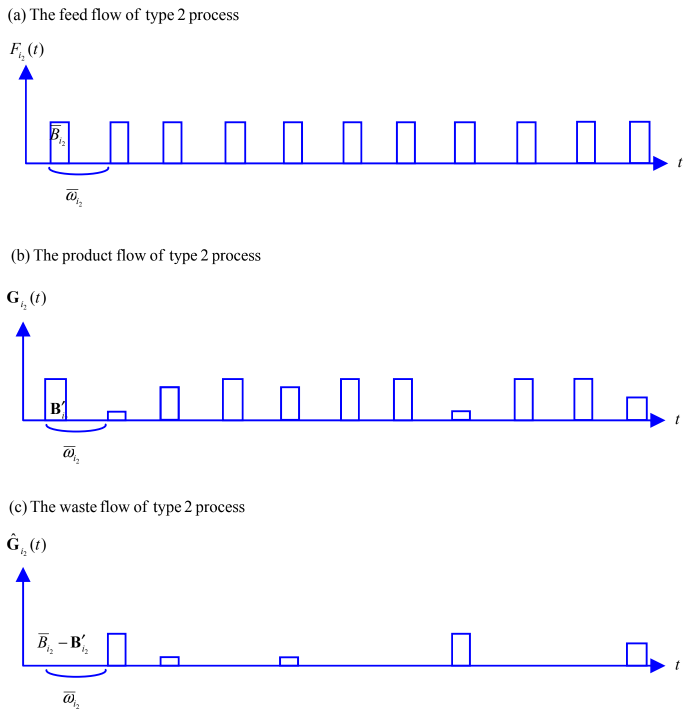

Figure 3. Flow of Type 2 Process

The material flow from process to storage (or from storage to process) is represented by the Periodic Square Wave (PSW) model when there is no failure (Yi and Reklaitis, 2003). Each production process is supposed to produce a batch of product during every cycle time ω_i. The cycle time of a production unit is composed of a storage operation time $`x_i\omega_i$ (or $x_i'\omega_i$) and other processing time. Note that the left quotation mark on the variable represents the variable is defined for the feeding flow to a production process and the right quotation mark on the variable represents the variable is defined for the discharging flow from a production process. The processing initiates at the start-up time $`t_i$ (or $t_i'$). Therefore, the material flow representation of PSW model for production process is composed of four variables: the batch size $B_i$, the cycle time $\omega_i$, the storage operation time fraction $`x_i$ (or x_i'), the start-up time $`t_i$ (or t_i'). The material flows of raw material purchased, waste disposal and finished product sold are also represented by four variables $B_k^j, \omega_k^j, x_k^j, t_k^j$, $B_n^j, \omega_n^j, x_n^j, t_n^j$ and $B_m^j, \omega_m^j, x_m^j, t_m^j$ respectively. Note that $\omega_{i_1}, \omega_k^j, \omega_n^j$, ω_m^j and B_{i_2} will be considered as random variables. For the convenience of presentation, the variables without superscript and subscript, B, ω and x represent the batch size, cycle time and storage operation time fraction of any process of raw material purchase, production or finished product demand. The cycle times of type 1 processes are random variables, which are denoted as bold character $\boldsymbol{\omega}_{(l)}$ where (*l*) is the order of batch as is shown at Figure 1. Suppose that $\boldsymbol{\omega}_{(l)}$ have identically independent distribution functions with respect to (*l*). $\overline{\omega}$ is the mean of $\boldsymbol{\omega}_{(l)}$ and $\overline{B}$ is (mean) batch size and both are unknowns. For a given convergence limit $0 < \varepsilon_1 << 1$ and a confidence level $0 < \delta_1 < 1$, choose the least integer η so that $P\left\{\left|\frac{1}{\eta}\sum_{l=1}^{\eta}\boldsymbol{\omega}_{(l)} - \overline{\omega}\right| < \varepsilon_1\right\} \geq 1-\delta_1$. From Chebychev's inequality, $\eta \geq \frac{Var(\boldsymbol{\omega})}{\delta_1 {\varepsilon_1}^2}$, that is, $\eta = \text{int}\left[\frac{Var(\boldsymbol{\omega})}{\delta_1 {\varepsilon_1}^2}\right] + 1$. Define $\underline{\underline{\omega}}$ as an actual operating time so that $\underline{\underline{\omega}} \leq \boldsymbol{\omega}_{(l)}$. Then, $\underline{\underline{\omega}} = \alpha\overline{\omega}$ where availability α is defined as:

$$\alpha \equiv \frac{\text{Actual Operating Time}}{\text{Actual Operating Time} + \text{Average Idle Time}} \quad \text{for Type 1 process,}$$

$$\alpha_{i_2} \equiv \frac{\text{Average On - spec Product Quantity}}{\text{Feed Quantity}} \quad \text{for Type 2 process} \tag{1}$$

Note that, from the assumption, $\boldsymbol{\omega}_{i_2} \equiv \overline{\omega}_{i_2}$ for type 2 processes. The feed flow to the type 2 process is assumed to have no uncertainty, that is, it has constant cycle time $\overline{\omega}_{i_2}$ and batch size $\overline{B}_{i_2}$. However, the batch size of discharging flow from type 2 process is random. The succeeded batch $\mathbf{B}'_{i_2(l)}$ goes to product storage unit and the failed batch $\left(\overline{B}_{i_2} - \mathbf{B}'_{i_2(l)}\right)$ goes to waste storage unit as shown at Figure 3. The availability of type 2 processes is defined as the ratio of total succeeded batch quantity over total feed batch quantity for sufficiently large number of batch $\eta_{i_2} = \text{int}\left[\frac{Var\left(\mathbf{B}'_{i_2}\right)}{\delta_1 {\varepsilon_1}^2}\right] + 1$. In other words, the product batch $\mathbf{B}'_{i_2(l)} \leq \overline{B}_{i_2}$ will satisfy $P\left\{\left|\frac{1}{\eta_{i_2}}\sum_{l=1}^{\eta_{i_2}}\mathbf{B}'_{i_2(l)} - \alpha_{i_2}\overline{B}_{i_2}\right| < \varepsilon_1\right\} \geq 1-\delta_1$ for type 2 process. α_{i_2} can have any real value in $0 < \alpha_{i_2} \leq 1$ but α_{i_2} should be chosen so as for $\alpha_{i_2}\eta_{i_2}$ to be an integer in this study. This is required for the graphical analysis to find the upper and lower bounds of flow as are shown at Figure 4 and 5. We can relax this requirement with more complicated graphical analysis in the future study. Define long cycle time $\tilde{\omega} \equiv \eta\overline{\omega}$ and total failure duration in long cycle time $d \equiv (1-\alpha)\tilde{\omega}$. $\tilde{\omega}$ has the meaning of the least period within which all random effects diminish with proper confidence level. η is called number of batch in long cycle time. It has the meaning of the least number of batch during which all random effects diminish with proper confidence level. Note that α and η are given parameters. Both parameters are estimated from the past operating history and/or the characteristics of the same kind of process built in other plant. For type 1 processes, $\underline{\underline{\omega}}$ is an unknown to be determined. For type 2 processes, $\overline{\omega}_{i_2} (= \underline{\underline{\omega}}_{i_2})$ is an unknown.

As a third type of process, the random characteristics of finished product demand will be defined differently. Both the batch size and cycle time of demand flow are assumed to be random variables. For a given convergence limit $0 < \varepsilon_1 << 1$ and confidence levels $0 < \delta_1, \delta_2, \delta_3, \delta_4 < 1$, choose the least integer η_m^j so that $P\left\{\left|\overline{\left(\mathbf{B}_m^j / \boldsymbol{\omega}_m^j\right)_{\eta_m^j}} - D_m^j\right| < \varepsilon_1\right\} \geq 1-\delta_1$

where $\overline{\left(\mathbf{B}_m^j/\boldsymbol{\omega}_m^j\right)}_{\eta_m^j} \equiv \frac{\sum_{l=1}^{\eta_m^j} \frac{\mathbf{B}_{m(l)}^j}{\boldsymbol{\omega}_{m(l)}^j}}{\eta_m^j}$. From Chebychev's inequality, $\eta_m^j = \text{int}\left[\frac{Var\left(\mathbf{B}_m^j/\boldsymbol{\omega}_m^j\right)}{\delta_1 \varepsilon_1^{\,2}}\right]+1$. Then, choose the least real number $\tilde{\omega}_m^j$ so that $P\left\{\sum_{n=1}^{\eta_m^j} \boldsymbol{\omega}_{m(l)}^j \ge \tilde{\omega}_m^j\right\} \le \delta_2$ and Choose maximum batch size $\overline{\overline{B}}{}_m^j$ so that $P\left\{\mathbf{B}_m^j \le \overline{\overline{B}}{}_m^j\right\} \ge 1-\delta_3$. Then, minimum number of batch in long cycle time $\gamma_m^j \equiv \text{int}\left[\frac{D_m^j \tilde{\omega}_m^j}{\overline{\overline{B}}{}_m^j}\right]+1$ and adjust $\tilde{\omega}_m^j = \frac{\gamma_m^j \overline{\overline{B}}{}_m^j}{D_m^j}$. Choose $\underline{\underline{\omega}}{}_m^j$ so that $P\left\{\boldsymbol{\omega}_{m(l)}^j \ge \underline{\underline{\omega}}{}_m^j\right\} \ge 1-\delta_4$. Then, $\alpha_m^j \equiv \frac{\gamma_m^j \underline{\underline{\omega}}{}_m^j}{\tilde{\omega}_m^j}$ and $d_m^j \equiv (1-\alpha_m^j)\tilde{\omega}_m^j$. Note that η_m^j, $\tilde{\omega}_m^j$, $\overline{\overline{B}}{}_m^j$, γ_m^j, $\underline{\underline{\omega}}{}_m^j$, α_m^j and d_m^j are given parameters for demand flow.

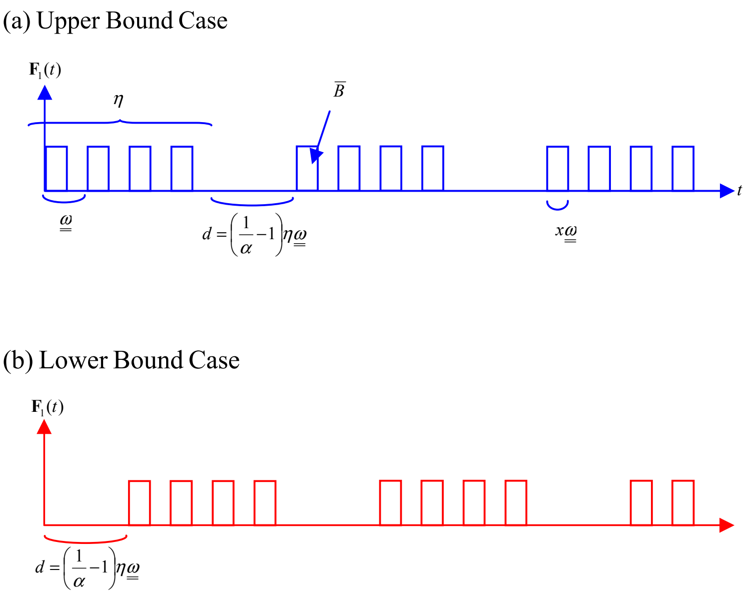

Figure 4. Two Extreme Cases of Random Failure

What is needed in this study is maximum, minimum and average inventory levels rather than actual inventory level. Maximum inventory level will be used to compute the storage size. Minimum inventory level will be used for the optimization constraint so that inventory level should be nonnegative. Average inventory level will be used to compute the inventory holding cost of the optimization problem. Instead, the upper/lower bound and average of accumulated material flow for each stream connected to the storage unit will be computed. Denote $\mathbf{F}_1(t)$ as random flow of a type 1 process. Note that the flow has constant average flow rate *D* measured during a long cycle time. $\mathbf{F}_1(t)$ has two extreme flow cases as can be seen at Figure 4. The upper bound of accumulated flow $\int_0^t \mathbf{F}_1(t)dt$ occurs when all the operating time failures occur at the end of repeated long cycle times.

The lower bound of accumulated flow occurs when all the operating time failures occur at the start of repeated long cycle times. Note that we can define these two extreme cases with probabilistic distributions and confidence levels, too. Figure 5 shows the corresponding accumulated flow patterns. We can easily find the upper and lower bounds of $\int_0^t \mathbf{F}_1(t)dt$.

$$D\left[t+(1-x)\underline{\underline{\omega}}+\left(\frac{1}{\alpha}-1\right)\eta\underline{\underline{\omega}}\right] \geq \int_0^t \mathbf{F}_1(t)dt \geq D\left[t-\left(\frac{1}{\alpha}-1\right)\eta\underline{\underline{\omega}}\right] \quad (2)$$

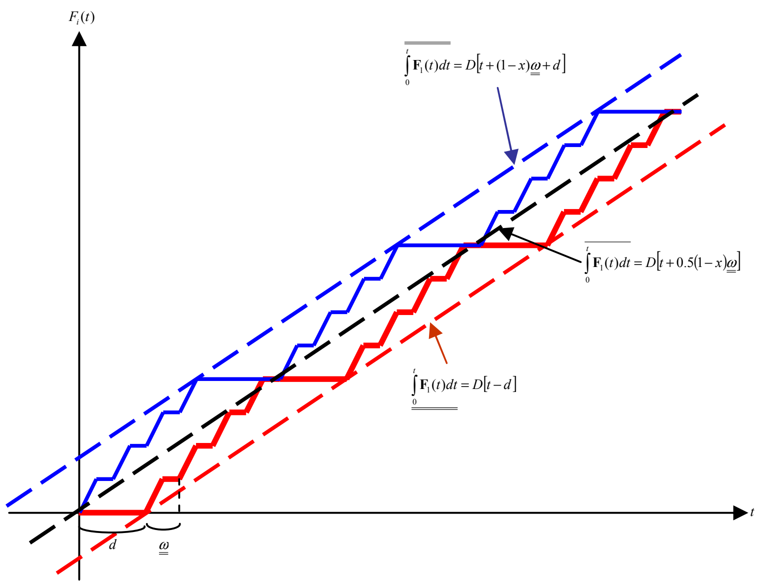

Figure 5. Flow Accumulation Functions for Two Extreme Cases.

Note that $d=\left(\frac{1}{\alpha}-1\right)\eta\underline{\underline{\omega}}$ for type 1 processes. The average of accumulated flow $\overline{\int_0^t \mathbf{F}_1(t)dt}$ is simply chosen to be in the middle of the upper and lower bounds.

$$\overline{\int_0^t \mathbf{F}_1(t)dt} = D\left[t+0.5(1-x)\underline{\underline{\omega}}\right] \quad (3)$$

This can be justified from the fact that $\overline{\int_0^t \mathbf{F}_1(t)dt}$ is also the average of periodic flow with the period of $\overline{\omega}=\underline{\underline{\omega}}+\frac{d}{\eta}$. Note that the average flow rate *D* in Eq. (3) should be multiplied by $f_{i_1}^j$ for feed flow or $g_{i_1}^j$ for product flow. Denote $F_{i_2}^j(t)$, $\mathbf{G}_{i_2}^j(t)$ and $\hat{\mathbf{G}}_{i_2}^j(t)$ as the feed, product and waste flows of a type 2 process. The analysis to find the upper and lower bounds of $\mathbf{G}_{i_2}^j(t)$ and $\hat{\mathbf{G}}_{i_2}^j(t)$ is the same as that of $\mathbf{F}_1(t)$ and therefore, Equations (2) and (3) can be used with different denotations. The feed flow of type 2 process does not have any failures. The analysis to find the upper and lower bounds of $\int_0^t F_{i_2}^j(t)dt$ follows those of Yi and Reklaitis (2003). The average of $F_{i_2}^j(t)$ is selected as the middle of upper and lower bounds (Yi and Reklaitis, 2003).

The upper/lower bounds and average of finished product demand flow follows those of $\mathbf{F}_1(t)$. Note that there can be other choices of average flows depending on the properties of randomness for specific problems.

The upper bound of inventory level $\overline{\overline{V^j}}$ is computed by adding the upper bounds of all incoming flows and subtracting the lower bounds of all outgoing flows from initial inventory. The lower bound of inventory level $\underline{\underline{V^j}}$ is computed by adding the lower bounds of all incoming flows and subtracting the upper bounds of all outgoing flows from initial inventory. The average inventory level $\overline{V^j}$ is computed by the averages of accumulated flows. The objective function for the design of the batch-storage network is to minimize the annualized total cost consisted of the raw material procurement cost, the setup cost of processes, the waste disposal cost, the inventory holding cost of storage units and the capital cost of the processes and storage units at a given availability and number of batch in long cycle time of each process. The constraints of optimization are no depletion of all storage units, that is $\underline{\underline{V^j}} \geq 0$.

The solution procedure to solve the Kuhn-Tucker conditions of the optimization problem and the overall computation procedure is the same as those in Yi and Reklaitis (2003). At first, optimal average flow rates should be obtained by numerically solving the second level problem. Then, analytical solutions of cycle times, batch sizes, storage sizes and initial startup times can be calculated by using simple equations.

Example Plant Design

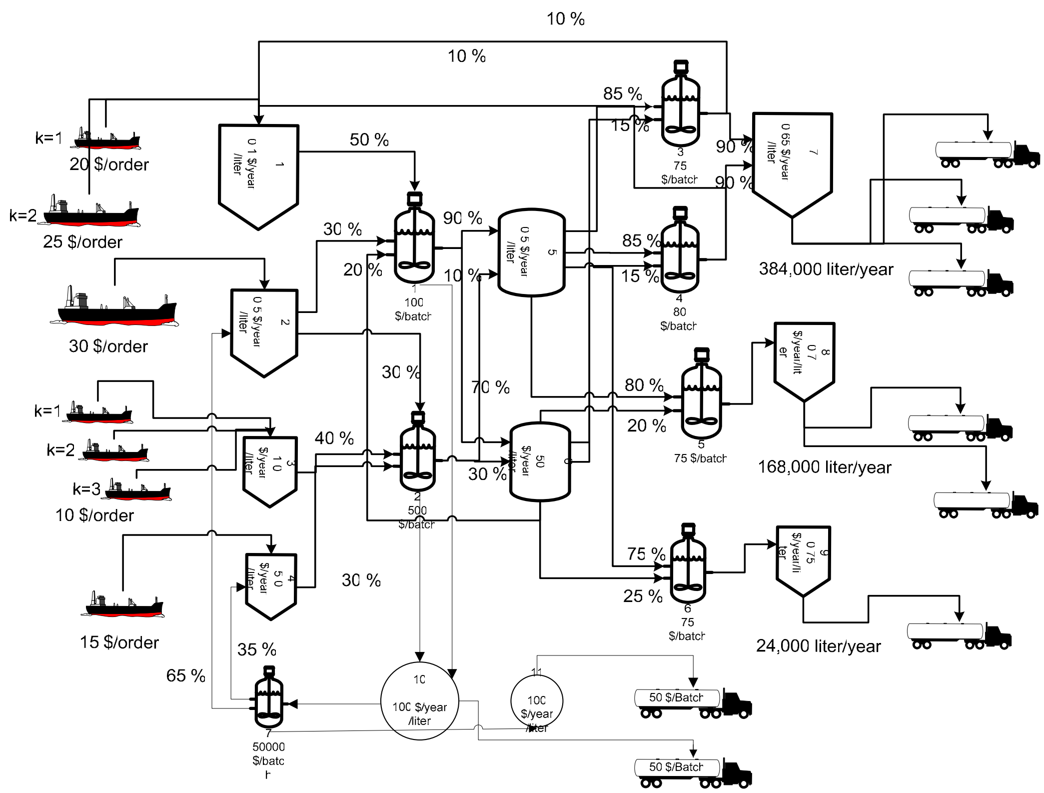

Figure 6. An Example Plant Design Problem

Suppose the plant that produces 3 finished products from 4 raw materials as shown at Figure 6. Figure 6 also includes most input data for computation. A design problem without process I7, storage J10 and J11 was studied in Yi and Reklaitis (2003). process I1 and I2 are type 2 processes and the waste materials of failed batches are collected in Storage J10. The waste material J10 can be disposed through the waste disposal process J10 or can be regenerated through the Process I7 to raw materials J2 and J4. The process I7 is also type 2 process and the waste material goes to storage J11. The waste material J11 is disposed through the waste disposal process J11. All the other processes are type 1 processes.

Figure 7 shows the dependency of optimal solutions: batch size, cycle time, cost of process and storage size, on availability. 6 cases were considered depending on type of process and number of batch in long cycle time. The marks of diamond, square and triangle were used for type 1 process. The marks of x, x with vertical bar and circle were used for type 2 process. The marks of diamond and x used $\eta_i = 10$. The marks of square and x with vertical bar used $\eta_i = 20$. The marks of triangle and circle used $\eta_i = 30$. All optimal solutions return to the solutions in Yi and Reklaitis (2003) when the availability is 1. Type 1 process and type 2 process show different graph patterns. The cycle time and cost of process of type 2 process are bigger than those of type 1. The batch size and storage size of type 2 are smaller than that of type 1. Increasing η_i reduces batch size and cycle time but increases cost of process and storage size. The effect of η_i is relatively insignificant compared to availability except storage size. The storage size of type 2 process has a maximum with respect to availability.

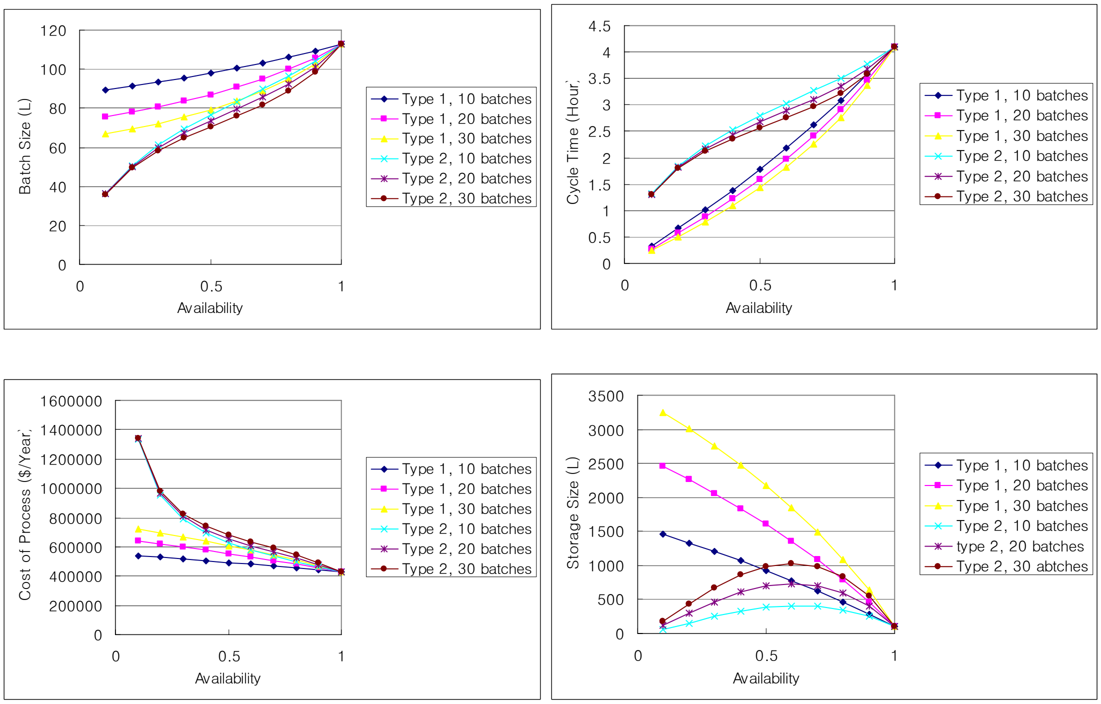

Figure 7. The Variation of Optimal Solutions With Respect To Availability

Conclusion

This article determines the optimal size of batch processes and storage units interconnected in network structure when the processes are bound to random failures of operating time and material spoilage. PSW model was judiciously used to find the upper and lower bounds of flows susceptible to short-term random variation of cycle time or batch size. Multiperiod formulation was combined with PSW model to count on long-term trend of product demand. Instead of usual definition of random properties such as mean and variance, availability and number of batch in long cycle time were introduced as input parameters. The availability is widely used in process reliability analysis such as FMEA. The number of batch in long cycle time is proportional to variance. These parameters were more practical and easier to estimate based on human perception. The main sources of random failures were operating time loss and batch material loss. Waste regeneration and disposal processes were installed to treat failed batch materials. The optimization problem was consisted of minimizing the sum of setup cost, capital cost of processes, inventory holding cost and price of materials under the constraints of meeting random product demand and no depletion of materials. PSW model with judicious graphical analysis of accumulated flows provided great flexibility to accommodate the random variation of various types of material flows into one optimization formulation resulting in analytical solutions. Analytical solutions greatly reduce computational burden which is the unique achievement of this study. The remained variables that could not be solved analytically were average flow rates. Concave cost minimization network flow problem should have been solved for obtaining optimal average flow rates. The computation time to solve this problem with well-known algorithm was about 7 times of the same size linear programming problem.

Even though the average flow rates can be obtained by other methods such as ordinary planning model, the optimality of other variables are still valid.

Acknowledgement

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