231c Boundary-Integral Calculations for the Emulsion Flow through a Granular Material

Alexander Z. Zinchenko and Robert H. Davis

Emulsion flow through a granular material is a problem of great practical relevance (oil filtration through sand, for example). In most practical cases, the emulsion drop sizes are comparable to the pore sizes, and so an emulsion cannot be treated as a pseudo-single-phase fluid, but microstructural simulation methods must be developed instead. We have developed a robust and very accurate boundary-integral methodology to simulate low-Reynolds number motion of three-dimensional deformable drops through a granular material with tight constrictions (the inner constriction diameter being from two to several times smaller than the non-deformed drop diameter) from first principles. Our method incorporates Hebeker representation for the solid particle boundary-integral contributions (as a combination of a single-layer and double-layer potentials), novel semi-analytical desingularization techniques for the boundary integrals (especially important when drops move through constrictions with high resistance and nearly coat solid surfaces, with extremely small surface clearance), and unstructured fixed-topology meshes on both solid and drop surfaces (with several thousand triangular elements per surface). The quality of drop triangulations is maintained by "passive mesh stabilization"(Zinchenko and Davis: JFM 2002, vol.455, p.21; Phil. Trans. Roy. Soc. Lond. A 2003, vol.361, p.813). Detailed results are obtained for single-drop motion between two spherical and disk-like solid particles rigidly held in a uniform flow, and through a hole between three solid spheres in contact. For a two-particle constriction, an interesting pattern of "collar formation" on the drop surface is observed. For all the cases with drop-to-medium viscosity ratio of 4, a critical capillary number is accurately obtained, below which the drop remains trapped in the constriction and is not able to pass. The temporal dynamics of drop motion (including the time spent in the constriction) for supercritical conditions is accurately determined. The effect of the viscosity ratio (from 0.25 to 10) on the squeezing process is also analyzed. Interestingly, for a two-sphere constriction and symmetric initial conditions, the decrease in the viscosity ratio in some range tends to immobilize the drop in the constriction, while, for a three-sphere constriction, the trend is always the opposite. The ability of our algorithm to maintain a trapped state virtually indefinitely (which could not be achieved without novel desingularization techniques) is particularly important for successful simulations of multidrop motion past large random arrays of solid obstacles; in such simulations,some drops move through the constrictions, while others are trapped at the pore throats. Our first long-time simulations for 10-20 deformable drops squeezing between 10-20 rigidly-held solid spheres in a periodic box (featuring multipole acceleration, Zinchenko and Davis JCP 2000, vol.157, p.539) will be also demonstrated. The ultimate goal is to calculate the flux of the droplet phase through a granular material by time averaging. Such simulations are much more demanding than for pure multidrop systems (without solid obstacles), strong lubrication effect being among the reasons.