221d Electrostatic Bounds on the Hydrodynamic Friction and Mobility of Arbitrarily Shaped Bodies in Stokes Flow

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In this talk we demonstrate rigorous inequalities for the hydrodynamic translational friction and mobility matrices $\boldsymbol{\zeta}$ and $\boldsymbol{\mu}$ of an arbitrarily shaped rigid particle in terms of the electrostatic capacitance C of a conducting particle of identical shape. Specifically, we derive the scalar and matrix inequalities $1/3 tr\, \boldsymbol{\zeta}^{-1} < 1/3 tr\, \boldsymbol{\mu} < C^{-1}$ and $2/3\, \boldsymbol{\zeta}^{-1} < C^{-1}\mathbf{I}$, where all quantities are normalized by the corresponding values for a sphere, and the mobility matrix is evaluated in the center-of-mobility reference frame. These bounds are obtained using a variational approach with the energy-dissipation functional expressed in terms of the induced-force distribution on the surface of the particle. To relate the hydrodynamic problem to the solution of the corresponding electrostatic problem, the trial force field is expressed in terms of the charge distribution on the equipotential particle surface. This procedure yields the first rigorous bounds on hydrodynamic friction that apply to bodies with translation--rotation coupling. In addition, we establish a rigorous, sharp bound on the effective (scalar) Brownian diffusion coefficient of an arbitrarily shaped particle.

We demonstrate that the error of the approximation $tr\, 1/3\, \boldsymbol{\zeta}^{-1}=C^{-1}$, corresponding to our scalar bound, is quadratic in the deviation of the trial induced-force field from the exact form---which explains why this relation is highly accurate for many particle shapes. Our numerical results confirm that the approximation is accurate for a variety of objects, including helices with translation--rotation coupling. Since the electrostatic problem involves a scalar potential rather than a vector velocity field, it is much simpler to solve than the corresponding hydrodynamic problem. Thus, our results facilitate evaluation of the mobility and diffusion coefficients of non-spherical particles (e.g., macromolecules).