209b Preconditioning for the Simultaneous Solution of Gas-Solid Flows

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Introduction

For single phase flows, the introduction of preconditioning allowed the use of simultaneous solution algorithms for low-Mach calculations. For gas-solid flows, preconditioned simultaneous solution algorithms have only been introduced recently. Although simultaneous solution algorithms have clear advantages over sequential solution algorithms, e.g. a fully implicit treatment of the source terms, the convergence behaviour obtained for gas-solid flows is poor compared to the convergence behaviour obtained for single phase flows.

In the present paper, the origin of this convergence slow down is revealed and several possibilities to improve the convergence are discussed.

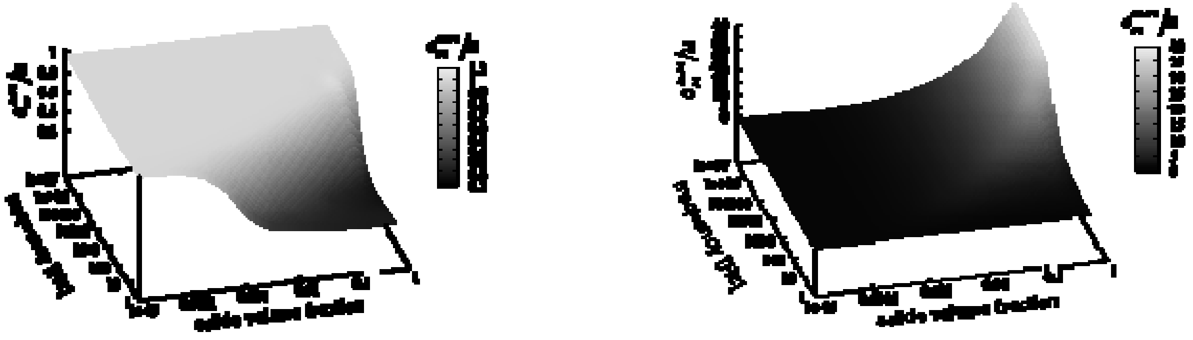
Mixture speed of sound behaviour

Experimental observations show that the propagation speed of gas phase pressure waves in a gas-solid mixture, the so-called mixture speed of sound, strongly depends on the solid volume fraction and the frequency of the pressure waves, i.e. the sound frequency. At high frequencies, the mixture speed of sound equals the single gas phase speed of sound, independent of the solid volume fraction. With decreasing frequency and increasing solid volume fraction, the mixture speed of sound gradually decreases to less than one thirtieth of the high frequency mixture speed of sound. Gas-solid interactions are at the origin of the complex mixture speed of sound behaviour. An eigenvalue analysis of the Eulerian-Eulerian gas-solid flow models shows that these models correctly capture this complex behaviour.

Preconditioning and numerical mixture speed of sound behaviour

When applying preconditioning, a numerical mixture speed of sound is introduced. The latter is scaled according to a reference velocity. To remove the stiffness in the stream wise direction, the latter should be somewhat larger, but of the same order of magnitude, than the convective velocity in the stream wise direction.

So far, two preconditioners have been presented in the gas-solid flow literature. The preconditioners of De Wilde et al. (2002) and Mao et al. (2003) are straightforward extensions of the single gas phase preconditioner of Weiss and Smith (1995), in which the influence of the gas-solid interactions is not taken into account. An eigenvalue analysis of the thus preconditioned gas-solid system reveals that, when using the



(a) (b)

Figure 1. Numerical mixture speed of sound resulting from preconditioning as a function of the frequency and the solid volume fraction.

Non-filtered model A. Preconditioner based on Weiss and Smith (1995):

(a) based on the single gas phase speed of sound (De Wilde et al., 2002; Mao et al., 2003)

(b) based on the low frequency mixture speed of sound (De Wilde et al., 2005).

latter type of preconditioner, the numerical mixture speed of sound that is obtained, depends on the solid volume fraction and on the frequency (Figure 1 (a)). In fact, the behaviour of the numerical mixture speed of sound is very similar to the behaviour of the mixture speed of sound, i.e. the numerical mixture speed of sound decreases with increasing solid volume fraction and decreasing frequency. As the preconditioner of De Wilde et al. (2002) or Mao et al. (2003) is based on the single gas phase preconditioner (Weiss and Smith, 1995), the high frequency numerical mixture speed of sound is scaled properly, i.e. according to the reference velocity. At low frequencies, on the other hand, the numerical mixture speed of sound becomes small compared to the convective velocity (Figure 1 (a)). Hence, whereas at high frequencies the behaviour remains subsonic even after preconditioning, at low frequencies, a supersonic behaviour is induced by the preconditioner of De Wilde et al. (2002) or Mao et al. (2003). A numerically induced supersonic behaviour of a subsonic flow problem is to be avoided. Furthermore, neither the discretization scheme, the integration algorithm, or the location of the pressure boundary condition allow a combination of high frequency subsonic and low frequency supersonic behaviour. With respect to the latter, a pressure boundary condition imposed at the outlet may become inefficient. A straightforward solution to avoid supersonic behaviour at low frequencies being induced by the preconditioner – i.e. without altering the functional form of the preconditioner of De Wilde et al. (2002) or Mao et al. (2003) – is to increase the value of the reference velocity to which the high frequency mixture speed of sound is rescaled, namely from the convective velocity to a multiple of the convective velocity. This, however, drastically reduces the step size and the gain in convergence speed of applying preconditioning.

In another approach, the preconditioner of De Wilde et al. (2005) accounts for the low frequency limit mixture speed of sound behaviour by defining the reference velocity using the low frequency mixture speed of sound equation. The latter is a function of the solid volume fraction, but not of the frequency. An eigenvalue analysis shows that with this preconditioner, the low frequency numerical mixture speed of sound is now properly scaled (Figure 1 (b)) and, moreover, becomes independent of the solid volume fraction. At high frequencies, however, the preconditioner of De Wilde et al. (2005) induces a frequency and solid volume fraction dependency of the numerical mixture speed of sound. The numerical mixture speed of sound then increases with increasing frequency and increasing solid volume fraction to a maximum of about thirty times the convective velocity (Figure 1 (b)). Hence, although no supersonic behaviour is induced by the preconditioner of De Wilde et al. (2005), the gain of applying preconditioning is limited as the step size is to be scaled according to the largest velocity.

Alternative acceleration techniques

Preconditioning accounting for the gas-solid interaction source terms

A preconditioner is derived that allows to account for the gas-solid interaction source terms. A gas-solid interaction history term is added to the preconditioner that partially eliminates the influence of the gas-solid interaction source terms. The degree to which the gas-solid interaction history can be incorporated in the preconditioner depends on the correlation of the updates of the variables during iteration. The correlation is shown to be significant up to a distance of about 40 iterations. Hence, this new preconditioner cannot completely eliminate the effects of the gas-solid interactions, in particular the low frequency effects.

Multi-grid approach

The gas-solid flow equations are solved on a spatial and on a temporal mesh. As a consequence, a filter frequency is introduced. Phenomena that occur on a smaller spatial or temporal scale than that of the mesh, i.e. phenomena with a characteristic frequency higher than the filter frequency of the mesh, can not explicitly be calculated.

By using a multi-grid approach, alternating calculations on meshes with a different filter frequency, the preconditioner and the step size can be optimized for each grid, according to its filter frequency. On a coarse mesh, only the low frequency behaviour is calculated and the step size can be scaled according to the low frequency numerical mixture speed of sound behaviour (De Wilde et al., 2005), allowing to transfer the slow low frequency waves rapidly through the domain. Hence, compared to single phase calculations, an even more important gain in convergence speed is expected from using a multi-grid approach for gas-solid flow calculations.

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