APPLICATION OF DRIFT-FLUX MODEL TO PHASE HOLDUP IN LIQUID-SOLID CIRCULATING FLUIDIZED BEDS

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Abstract

Solids circulation rate and solids holdup are experimentally investigated in a liquid solid circulating fluidized bed covering a wide range in solids and liquid flow rates, particle size and density. The experimental data as well as the data reported in literature are analyzed using Drift-flux model (Zuber and Findlay, 1965). The distribution parameter for the data is found to range from 0.78 to 0.99. The weighted average drift velocity is well correlated to the terminal velocity of the particle. Solids holdup predicted using the model compares satisfactorily with the experimental holdup.

Key words:

Liquid-Solid Circulating Fluidized Bed - Drift-Flux model – Distribution Parameter – Weighted average drift velocity – Solids holdup.

Introduction

It is well recognized that the phase holdup in two-phase flow is due to (a) the velocity and concentration profiles across the cross-section of the duct and (b) the local relative velocity between the phases caused by gravitational effects. Drift-flux model takes into account the first effect by a distribution parameter and the second effect by the weighted average drift velocity. The model provides an insight into the resistances to the motion of a phase due to the presence of the second phase. Though conceptually the model is applicable to flowing two-phase systems wherein one of the phases is dispersed in the other, and is satisfactorily validated to the flow of gas-liquid systems, (Zuber and Findlay, 1965). Its application to liquid-solid systems especially the circulating fluidized beds has not been so far attempted an aspect which forms the subject matter of this paper.

When a liquid flows through a bed of solids at velocities exceeding the terminal velocity, particles begin to entrain from the bed, necessitating recirculation of the entrained solids to the bottom of the bed to maintain the solids inventory in the bed. Such an operation of liquid-solid contacting is termed liquid-solid circulating fluidized bed (LSCFB).

The interest in the study of liquid-solid circulating fluidized beds in recent years arises from their potential applications to biochemical processes, protein purification processes, non-catalytic and catalytic liquid-solid reactions as well as to physical processes such as binary solids mixing and ore dressing. Specific examples of industrial application of LSCFB include continuous protein recovery from unclarified fermentation broth through adsorption and desorption using anion-exchange resins (Pirozzi et al., 1989; Jin et. al., 1997, Zhu et. al. 2000, Lan et al, 2002), synthesis of linear alkyl benzene from benzene and I-dodecene using zeolite catalysts (Liang et. al. 1995), and cesium removal from high radioactive liquid waste using potassium titanium hexacyanoferrate (Feng et al, 2003).

The advantages associated with LSCFB to make them suitable for the aforementioned applications are the effective contact between the phases resulting in enhanced heat and mass transfer rates between the phases, high liquid throughout per unit cross-section, adaptability to continuous operation and control of solids circulation rate and solids volume fraction through a choice of solids and liquid flow rates.

Earlier Literature

Earlier work relating to the study of solids circulation rate and solids holdup in LSCFB is limited. Differentiating the liquid-solid circulating fluidization regime into initial and fully developed circulating fluidization zones, Zheng et al (1999) presented experimental data indicating the influence of primary and auxiliary liquid flow rates and the particle density on the particle velocity and the solids holdup in the riser of LSCFB. Based on the data, Zheng and Zhu (2000) presented a correlation relating solids holdup to the total liquid flow rate and the solids flux, both quantities defined as dimensionless parameters. The correlation however predicts their solids holdup data with a standard deviation exceeding 35%. Liang et al (1997) attempted to relate the bed voidage using Richardson-Zaki slip velocity correlation and found substantial deviation from their experimental data. The authors attributed the difference to the non-uniform radial concentration profiles. Kuramoto et al (1998) reported the in-applicability of Richardson-Zaki slip velocity correlation to LSCFB attributing to the formation of particle aggregates and channeling of the liquid phase. This paper presents experimental data on solids holdup and solids circulation rate in LSCFB covering a wide range in experimental conditions and material properties. The data are analyzed applying the drift-flux model, detailing the significance of the model parameters and their relationship to the operating variables. .

Experimental

Experiments are conducted using a Plexiglas riser column of 94mm i.d. and 2400mm high (Figure 1). The base of the riser has two distributors, one each for the primary and the auxiliary liquid flows into the riser. Primary liquid flows through a multi-tubular distributor consisting of 21 s.s. tubes of 10mm i.d., 12.9 mm o.d. and 172 mm long occupying 39.5% of the cross-section of the bed and extending 110 mm into the bed. The auxiliary liquid distributor is a perforated brass plate with 2 mm openings to give 7.4% free area, ensuring uniform liquid distribution across the riser cross-section. Auxiliary liquid flow serves to fluidize the particles at the base of the riser, and facilitates and regulates the solids flow into the riser thus serving as a non-mechanical control value for the flow of solids into the riser. The combined flow of the primary and auxiliary liquid enables the particles to move concurrently to the top of the riser, where they are separated from the liquid and returned to the base of the riser via particle storage vessel and particle flow rate measuring device. The device is a graduated tube to give the height of solids collected in a known time, when the solids flow into the storage vessel is temporarily arrested. The graduated tube is pre-calibrated for the chosen liquid-solids system, and gives the weight of solids collected per unit time. Tap water is used as the fluidizing liquid in all the experiments. Liquid flow is metered through calibrated orifice-meters. The provision of dual liquid flows into the riser enables the control of the liquid flow rate and the solids circulation rate independently by adjusting the auxiliary and primary liquid flow.

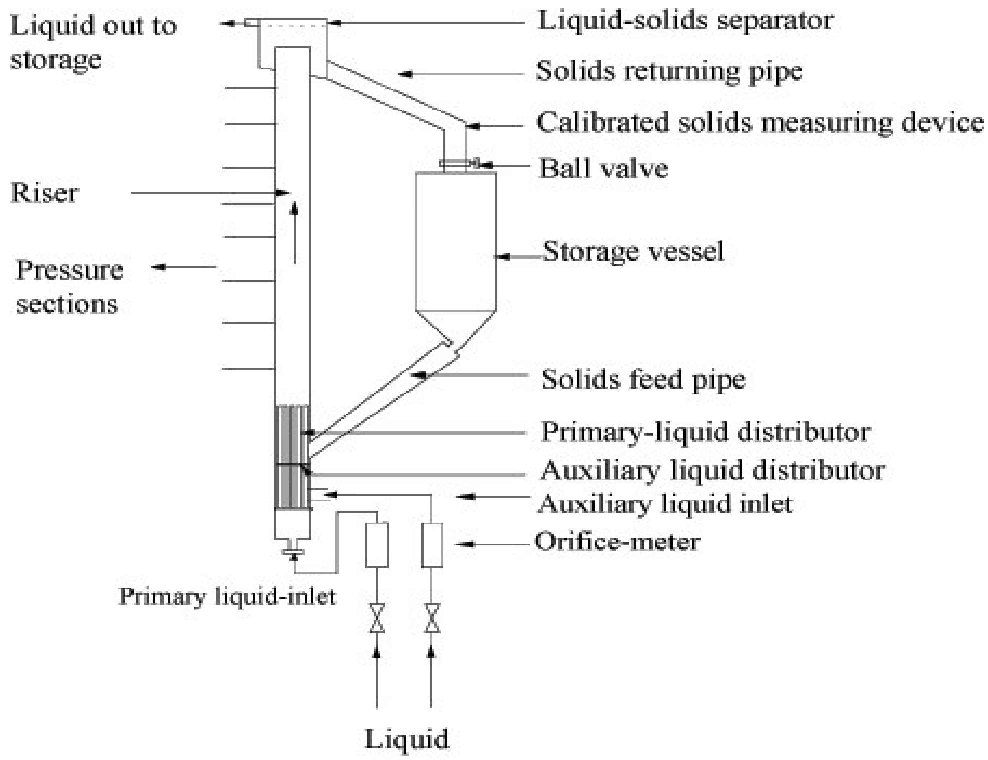

Fig.1. Schematic diagram of the experimental setup

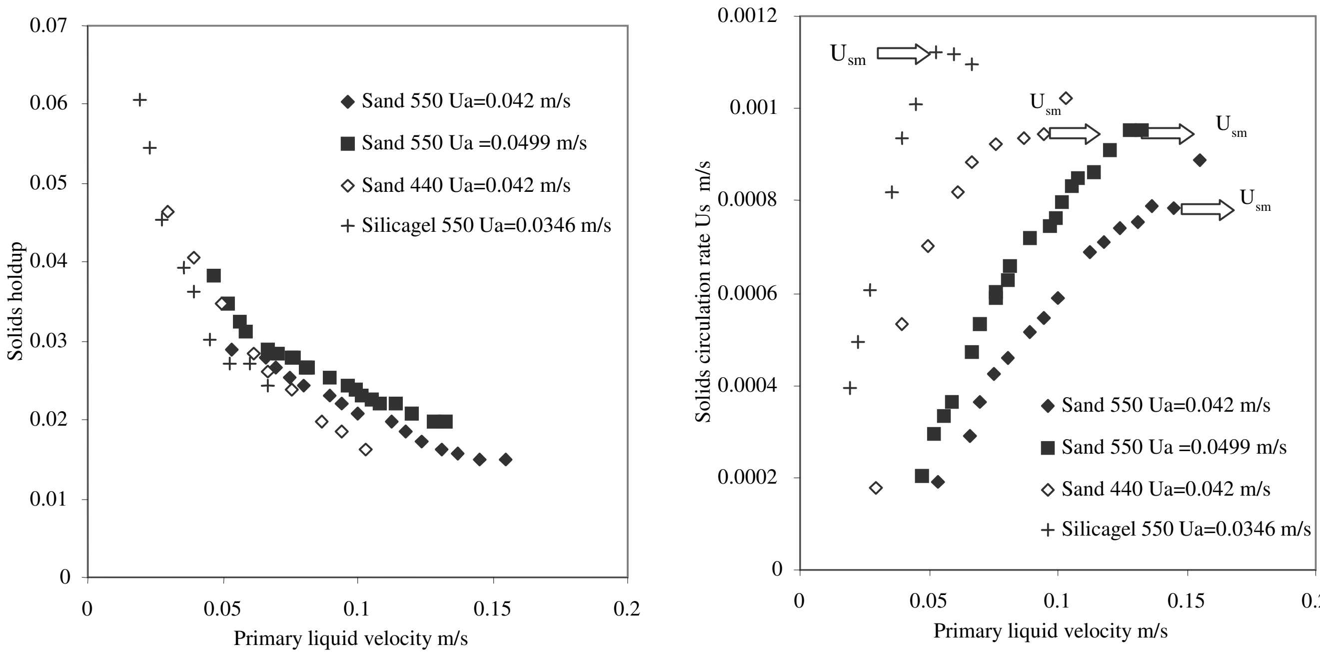

Fig 2: Typical variation in (a) the solids holdup and (b) Solids circulation rate with particle characteristics and liquid flow rates.

All the experiments are carried out at ambient temperature of 28±1°c. The solids holdup is determined from the measured pressure gradient at different locations along the length of the riser, as,

$$-\frac{\Delta p}{\Delta L} = (\rho_s \varepsilon_s + \rho_l \varepsilon_l) g; \quad \varepsilon_s + \varepsilon_l = 1 \qquad \text{----------} \ (1)$$

For the given solids of known size and density, the primary and auxiliary liquid flow rates are varied to study their effect on the solids circulation rate and solids holdup. The range of variables covered and the physical properties of the solids used in the present study are given in table 1.

Results and Discussion:

<u>Solids Circulation rate and Solids holdup</u>:

Figure 2 represents the typical variation of solids holdup, ε_s and the solids circulation rate, U_s with the primary liquid flow rate, U_f and the auxiliary liquid flow rate, U_a. It is to be noted that U_s increases with an increase in U_f and attains a maximum, U_{sm} at high liquid rates. An increase in U_f however decreases the solids holdup and ε_s attains a minimum at solids circulation rate corresponding to $U_{sm.}$ Noting that the auxiliary liquid controls solids throughput into the column, the figure illustrates the increase of U_s and ε_s with an increase in U_a. A comparison of the data, presented in figure 2, for the two sizes of sand shows that a decrease in particle size, under identical experimental conditions, increases U_s and decreases $\varepsilon_{s.}$ Likewise, as shown in the figure, a decrease in particle density increases U_s and decreases the solids holdup. The variation of solids holdup with the solids circulation rate at constant solids throughput rate (i.e. at constant U_a values) and at constant liquid throughput rate (i.e. at constant U_l values) is depleted in figure 3. It is seen that for a given solids feed rate ε_s decreases with increase in $U_{s,}$ as at constant $U_{a,}$ an increase in U_s is realized with an increase in U_f or a decrease in particle size or particle density. On the other hand, if the total liquid throughput rate, U_l is held constant, ε_s increases with an increase in U_s as the increase in U_s can only be effected by an increase in U_a (resulting in a decrease in U_f to maintain U_l constant) or a decrease in particle size or particle density, the factors that result in large solids holdup.

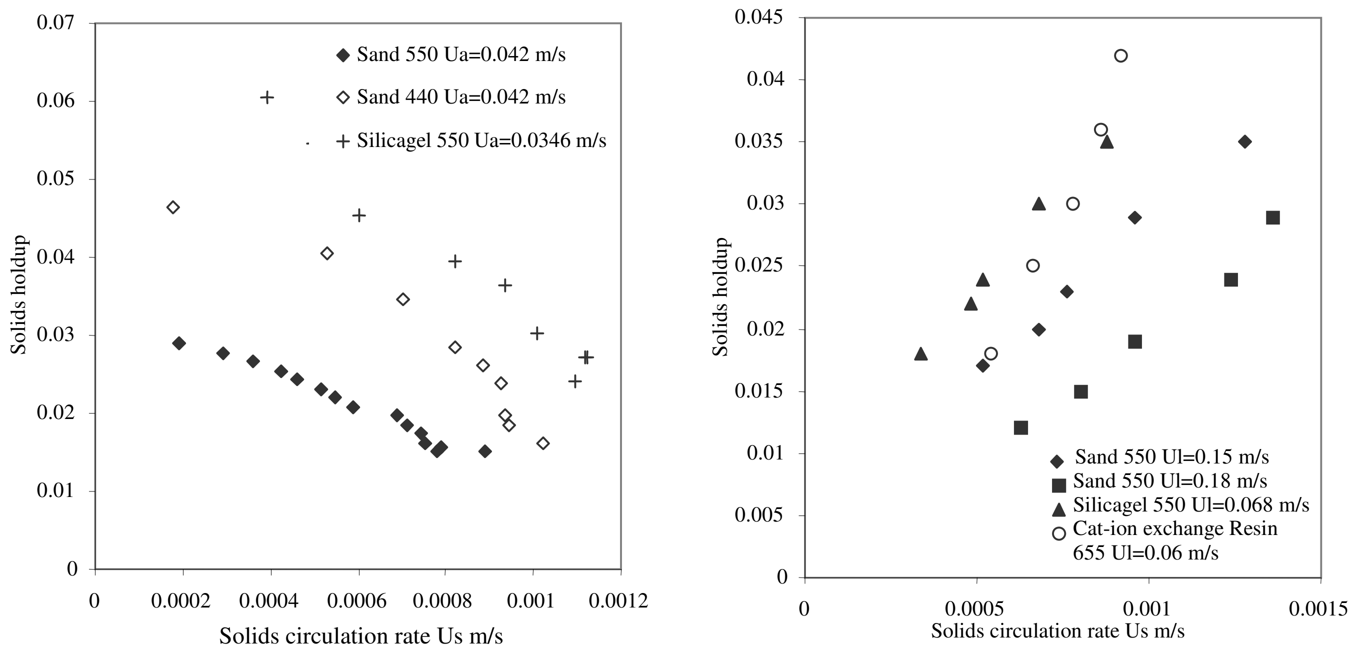

Fig. 3 Typical variation of solids holdup with solids circulation rate for a (a) constant Auxiliary liquid flow rate and (b) a constant total liquid rate.

Based on the observations, the experimental data are correlated as

$$\varepsilon_s = 0.105\left(\frac{U_a}{U_t}\right)^{0.53}\left(\frac{\Delta\rho}{\rho_l}\right)^{-0.063}\exp\left(-\frac{U_f}{U_t}\right), \quad \sigma = 0.12 \text{ --- (2)}$$

$$\frac{U_s}{U_{sm}} = 1.5\left(1 - 2.3\exp\left(\frac{-U_l}{U_t}\right)\right) \qquad \sigma = 0.14 \text{ --- (3)}$$

where the terminal velocity of the particle, U_t is defined as (Perry,1998),

$$U_t = \left[\frac{4gd\Delta\rho}{3\,\rho_l c_d}\right]^{\frac{1}{2}}; \; c_d = \left(\frac{24}{Re}\right)\left(1 + 0.1\,Re^{0.7}\right) \text{ for } 0.1 < Re < 1000 \text{ -- - - (4)}$$

and the Root Mean Square deviation, σ is given by

$$\sigma = \left\{\frac{1}{N}\sum_{N=1}^{N=N}\left(\frac{X_{exp} - X_{pred}}{X_{exp}}\right)^2\right\}^{\frac{1}{2}} \qquad \text{--- (5)}$$

The maximum solids circulation rate, U_{sm} and the corresponding minimum in solids holdup, $(\varepsilon_s)_{min}$ are found to be dependent upon the auxiliary liquid flow rate and the particle characteristics as

$$U_{sm} = 0.1 U_a \, Ga^{-0.2}\left(\frac{\Delta\rho}{\rho_l}\right)^{-0.3}, \; \sigma = 0.13 \quad \text{--- (6)}$$

$$(\varepsilon_s)_{min} = 0.0283\left(\frac{U_\alpha}{U_t}\right), \qquad \sigma = 0.16 \quad \text{--- (7)}$$

Application of drift-flux model

The drift-flux model takes into account the effect of non-uniform radial particle concentration and liquid velocity distributions in the riser, as well as the effect of the local relative velocity between the two phases. The relative velocity between the phases in two-phase flow is expressed as (Lapidus and Elgin, 1957)

$$U_{sl} = \frac{U_s}{\varepsilon_s} - \frac{U_l}{(1-\varepsilon_s)} \qquad \text{--- (8)}$$

Rearranging,

$$j_{sl} = U_{sl}\,\varepsilon_s\,(1-\varepsilon_s) = U_s - \varepsilon_s\, j \qquad \text{--- (9)}$$

j_{sl} is the drift-flux and represents the volumetric rate at which particles move forward through unit area of a plane normal to the riser-axis already traveling with the flow at a velocity, j. Equation (9) is applicable at any local point in the flow of two-phase flow. Extending the analysis to relate to cross-sectional averages, Wallis (1969) and Zuber and Findlay (1965) defined a distribution parameter

$$C_o = \frac{<\overline{\varepsilon_s j}>}{<\varepsilon_s><j>} = \frac{\left(\frac{1}{A}\int_A \varepsilon_s\, j dA\right)}{\left[\frac{1}{A}\int_A \varepsilon_s dA\right]\left[\frac{1}{A}\int_A j dA\right]} \qquad \text{--- (10)}$$

and weighted average drift velocity

$$\mathrm{u_{sj}} = \frac{j_{sl}}{<\varepsilon_s>} \qquad \text{--- (11)}$$

Equation (9) is now written as

$$\frac{<U_s>}{<\varepsilon_s>} = C_0 <j> + \mathrm{u_{sj}} \qquad \text{--- (12)}$$

Hence, the distribution parameter, C_o represents an empirical factor correcting the one-dimensional homogeneous theory to account for the fact that the particle volume fraction and the liquid velocity profile across the riser cross-section can vary independently of one another. If the local relative velocity between the phases is zero, the drift velocities of both the phases are zero, and the two phases have the same velocity which is equal to j.

C_o and u_{sj} are evaluated for the experimental data of the present study and that of the literature, by plotting $\frac{<U_s>}{<\varepsilon_s>}$ against <j>=(Q/A). Figure 4 presents typical data wherein it is seen that the change in weighted mean velocity of the particle with a change in the average volumetric flux density could well be represented as a straight line for a given material and the auxiliary liquid flow rate. The slopes of the lines in the velocity-flux plane reflect the effect of the non-uniform radial velocity and particle concentration profiles. While the slopes of the lines are constant for the given material, the intercept, corresponding to <j>=0, is found to decrease with increase in the auxiliary liquid flow rate (Table 2). This dependency of u_{sj} on U_a is explained, recognizing that the solids concentration in the riser depends upon the auxiliary liquid flow and that the rise velocity of solid particle is affected by the presence of other particles. In the absence of interaction from the other particles, the drift velocity should be the terminal velocity of the particle in an infinite medium. Zuber and Hench (1962) reported the dependency of the weighted average drift velocity on the dispersed phase holdup for the gas-liquid systems.

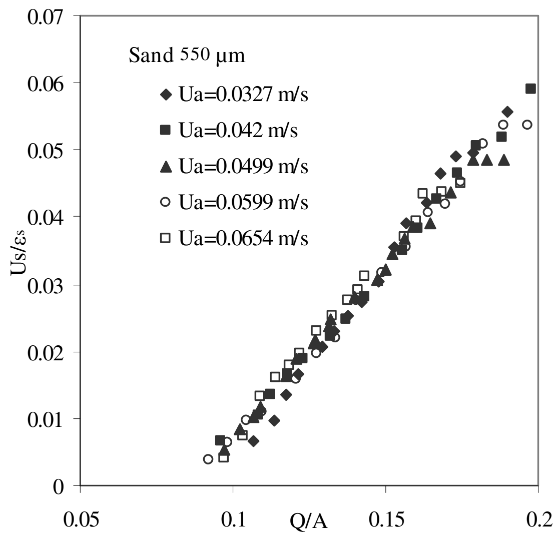

Fig 4 : Typical variation of the weighted mean particle velocity with the average volumetric density

Table 1 lists C_o and u_{sj} values for the experimental data of the present study and that reported in literature. The following observations are made from the data: The value of C_o is less than 1, suggesting that the particle concentration near the wall is higher than the average concentration, i.e. for axisymmetrical distribution, the particle concentration is the lowest at the center of the riser and increases radially to reach the highest value at the wall.

The range in C_o from 0.78 to 0.99 for the liquid-solid up flow in LSCFB compares with its range from 1.0 to 1.5 for liquid-vapor up flow. The difference lies in that the solids move slower compared to the bulk liquid in LSCFB. In bubbly flow the bubbles rise faster to the bulk liquid. Further, the concentration and velocity profiles in bubbly flow are convex upwards, while the concentration profile is concave and the velocity profile is convex in LSCFB.

The weighted average drift velocity, u_{sj} depends upon the auxiliary liquid flow rate (Fig.4) and the particle characteristics (Table 2). The data are satisfactorily correlated as

$$u_{sj} = 0.8u_a^{-0.2}\, U_t^{1.2} \qquad \text{--- (13)}$$

The predicted values, $(u_{sj})_{pred}$ are listed in Table 1. The observation that u_{sj} is approximately equal to U_t agrees well with the observation reported for gas-liquid up flow. When $u_{sj} <<< j>$, the effect of the local relative velocity on ε_s may be neglected when compared to the effects of non-uniformity in velocity and concentration profiles.

Equation (12) facilitates the estimation of average solids holdup, ε_s in the riser from knowledge of C_o and u_{sj}. Equation (12) can be rewritten as

$$\frac{<U_s>}{<\varepsilon_s>}\frac{1}{<j>} = C_0 + u_{sj} \qquad \text{--- (14)}$$

Further, as $[<U_s>/<j>]=(Q_s/Q)$, the average input volumetric flow concentration of particles, equation (14) is rearranged to give

$$\varepsilon_s = \frac{\left[\frac{<U_s>}{<j>}\right]}{C_o + u_{sj}/<j>} \qquad \text{--- (15)}$$

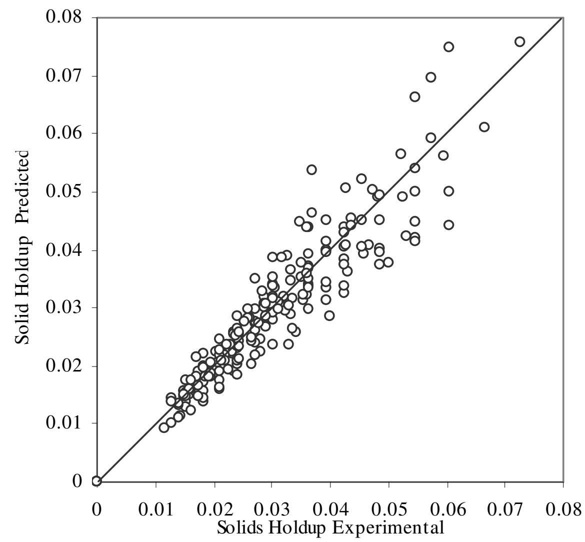

Fig. 5 Comparison of solids holdup (experimental) with Solids holdup (predicted using equation 15.) (350 data points)

Figure (5) compares the experimental ε_s with ε_s predicted using equations (13) and (15) choosing appropriate values of C_o.

Summary and Conclusions

Liquid-Solid Circulating Fluidized Beds offer a number of advantages with respect to fluid-solid contacting, and find wide application industrially ranging from non-catalytic fluid-solid reactions to protein purification processes. Solids circulation rate and solids holdup are the important parameters characterizing the interaction between the phases. Both the parameters are found to depend on the flow rates of the phases and the particle characteristics such as the particle size and density. Correlations to predict the two parameters presented in the paper cover a wide range in experimental conditions.

Table 1 Experimental details and Model parameters.

Material & Density kg/m^3	Size, d µm	U_t m/s	Reynolds Number Re	Co	(u_{sj})exp m/s	(u_{sj})pred m/s Eqn. (13)	Author(s)
Sand 2700	550	0.089	49	0.82	-0.08	-0.08	Present Study
Sand 2700	440	0.070	31	0.82	-0.06	-0.062	-do-
Sand 2700	300	0.045	13.4	0.85	-0.037	-0.041	-
Cat-ion exchange Resin 1325	655	0341	22.3	0.81	-0.0248	-0.0249	-
Cat-ion exchange Resin 1325	550	0.028	15.18	0.93	-0.0189	-0.023	-
Cat-ion exchange Resin 1325	463	0.0222	10.25	0.94	-0.0186	-.0.0196	-
Silica 1650	550	0.0458	25	0.79	-0.037	-0.041	-
Silica 1650	337	0.0248	8.4	0.78	-0.019	-0.02	-
Blue Stone 2850	337	0.055	18.54	0.89	-0.045	-0.049	-
Glass 2490	508	0.075	38.14	0.95	-0.083	-0.078	Zheng et. al.
Glass 2460	405	0.053	23.3	0.83	-0.039	-0.059	Liang et.al
Glass 2490	182	0.0198	3.62	0.98	-0.0175	*	Kuramoto et. al
Glass 2490	93	0.0065	0.59	0.99	-0.006	*	Kuramoto et. al

* Different solids feed mechanism.

Table 2 Typical variation of u_{sj} with U_a (material Sand 550 µm)

U_a m/s	0.0327	0.042	0.0499	0.0599	0.0654
u_{sj} m/s	-0.0871	-0.0851	-0.0829	-0.0801	-0.0761

Drift-flux model is applied to the experimental data of the present study and the data reported in literature covering a range in Re from 0.60 to 49 and liquid rates from 0.04 to 0.4 m/s. It is found from the analysis that the distribution parameter ranges from 0.78 to 0.99 suggesting that the solids concentration is minimum at the riser core and is maximum at the wall of the riser. The weighted average drift velocity is found to depend on the terminal rise velocity of the particle: it is a weak function of auxiliary liquid flow rate that controls solids input into the riser.

The analysis shows that the drift-flux model can be satisfactorily applied to predict the phase holdup in LSCFB, with the model parameters exhibiting similarities with those of gas-liquid up flow though ducts, except that the radial particle concentration profile is concave in the former and is convex in the gas-liquid up flow.

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Notations

A	cross sectional area of the riser (m^2)
C_d	drag Coefficient (-)
C_o	distribution parameter (-)
d	mean particle size, μm, m
Ga	Galileo Number ($d^3 g\, \rho_l^2 / \mu_l^2$) (-)
$\Delta P/\Delta L$	pressure drop at the test section, N/m^3
g	acceleration due to gravity, m/s^2
j	superficial velocity of the two phases (m/s)
j_{sl}	volumetric particle drift flux (m/s)
N	number of data points
Q	volumetric flow rate of the both the phases (m^3/s.)
Q_s	particle volumetric flow rate (m^3/s)
Re	particle Reynolds number, ($\rho_l U_l d_p/\mu_l$)
U_a	auxiliary liquid velocity, m/s
U_f	primary liquid velocity, m/s
U_l	superficial total liquid velocity, (= $U_f + U_a$), m/s
U_S	solids circulation rate expressed as superficial solids velocity, m/s
U_{sl}	relative velocity between two phases m/s.
u_{sj}	weighted average drift flux velocity m/s.
U_{sm}	maximum possible solids circulation rate, (m/s)
U_t	particle terminal velocity, m/s
X	parameter

Greek letters

ε_l	bed voidage (-)
ε_s	solids holdup (-)
σ	root mean square deviation

μ_l liquid viscosity (kg/m s)
ρ_l liquid density (kg/m^3)
ρ_s particle density (kg/m^3)
$\Delta\rho$ = (ρ_s - ρ_l)
< > cross-sectional average quantity

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