On the relative motion of two spherical bubbles rising in line and interacting by a laminar wake

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Key words: Bubble rise velocity; Bubble-liquid interaction; Bubble–bubble interaction; History force; Quasi-steady drag force, Wake attraction impulse.

The dynamics of bubble pairs rising in line in a Newtonian uniform flow, at high and intermediate Reynolds numbers Re, is different from the single bubble dynamics due to their interaction. During the free rise, a wake is carried up behind the leading bubble. The wake attraction effect on the trailing bubble culminates with coalescence at the time of contact, for pure liquids. The coalescence phenomena may have an overwhelming effect on the bubble size distribution evolution. Obtaining accurate expressions for the velocities and forces on interactive bubbles is a valuable purpose for studying the local dynamic of bubble pairs. A key subject in bubbly flow is the understanding of how both phases interact locally.

In this study, a simplified expression for computing the motion of two equally sized bubbles rising in line in pure water at intermediate Reynolds number (i.e. developing a laminar wake) and for small Weber number $We \ll 1$ (spherical bubble) is obtained. Starting from the motion equation for the leading bubble as a single undisturbed bubble and considering the motion of the trailing bubble as quasi-steady state (c. fr. Katz and Meneveau, 1996, referred further as KM), and taking into account the attraction effect of the wake, we obtain an expression different from those obtained in previous works. The simplified expression is compared with experimental measurements from KM, and it is observed that the trailing bubble velocity is predicted, for dimensionless separation distances $2 \le x/d \le 7$, with better accuracy than other more sophisticated models reported in the literature.

The leading bubble moving at the steady state condition, where the mechanical equilibrium condition, expressed by the drag and the buoyancy forces balance is satisfied, gives its terminal velocity in a Newtonian uniform flow and behaves as an isolated bubble (KM). If F_{dk} is the drag force for the *k*-th bubble, with *k*=1 for the leading bubble, *k*=2 for the trailing bubble and *k*=02 for a hypothetical isolated bubble traveling at the trailing bubble velocity, the following general equation is proposed:

$$F_{dk} = -C_{dk}\frac{\pi}{4}d^2\frac{1}{2}\rho\left(U_{bk} - U_{ref}\right)^2 \tag{1}$$

where U_{ref} is a liquid reference velocity, coincident with the uniform liquid velocity U_L for $k = \{1,02\}$ and coincident with the average wake velocity $\bar{w}_z$ for *k*=2. ρ, C_{dk}, d and U_{bk} are the liquid density the steady drag coefficient (inversely proportional to the Reynolds number), the bubble diameter and the *k*-th bubble velocity, respectively.

The vertical component of the linear momentum balance on the trailing bubble moving in the wake of the leading bubble could be expressed as (Ramírez-Muñoz et al. 2005):

$$F_{WA} + F_{d2} + F_g = 0 \tag{2}$$

where F_{WA} is the vertical Lagrangian wake impulse on the trailing bubble, F_{d2} is the quasi-steady drag force on its surface and F_g is the buoyancy force. The wake impulse can be modeled from the Batchelor (1967) asymptotic wake expression. The drag reduction effect is modeled by referring the velocity change to the averaged wake velocity, which was not weighted here by a $\gamma < 1$ factor, differently to the approach followed by Ramírez-Muñoz et al. (2005), where γ is introduced in order to find the “true” relative velocity for computing the drag effect in non-uniform flows. The present simplifying assumption was chosen since we would like to keep the problem formulation as simple as possible and without additional parameters. However, it should be remarked that the drag effect might be underestimated.

Eq. (2) can be set dimensionless by dividing it by the quasi-steady drag force for a hypothetical bubble F_{d02}, with relative velocity $U_{b2} - U_L$:

$$\frac{F_{d2}}{F_{d02}} = \frac{F_{d1}}{F_{d02}} - \frac{F_{WA}}{F_{d02}} = \frac{1}{\beta}\frac{F_{d1}}{F_{d02}} \tag{3}$$

where:

$$\frac{1}{\beta} = 1 - \frac{F_{WA}}{F_{d1}} \tag{4}$$

Thus, the quasi-steady force balance on the trailing bubble can be highly simplified, provided that the multiplier β function is known or a plausible assumption can be established on it. In order to obtain a simplified expression for the β function, the asymptotic wake solution was applied to its definition and plotted in Figure 1. This plot was compared to the dimensionless trailing bubble rise velocity U_b from the experimental data from KM for $Re_1 = 21.5$, where:

$$U_b = \frac{U_{b2} - U_L}{U_{b1} - U_L} \tag{5}$$

and $U_L = 0$ for bubbles in a quiescent liquid.

It is apparent in Figure 1 that there should be a proportionality between β and U_b, such that one is able to propose, in particular:

$$\beta \approx U_b \tag{6}$$

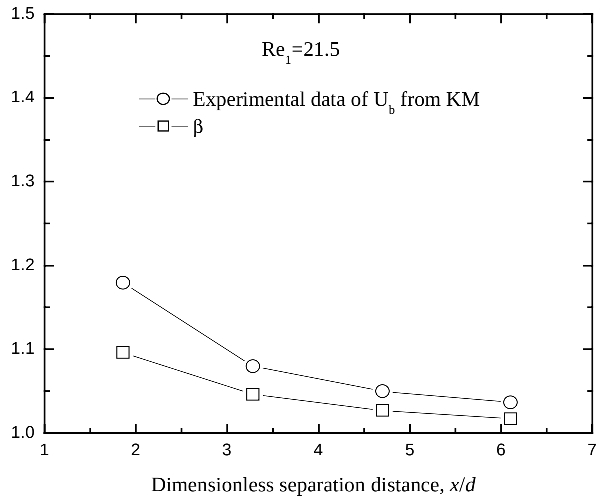

Figure 1. Comparison of β versus U_b for different separation distance.

This assumption on the β behavior overestimates its contribution and this might be appropriated in view of the drag effect underestimation, as pointed out above. From substitution of Eq. (6) in Eq. (3) and consideration of the drag force terms, an expression for U_b was obtained:

$$U_b = \frac{W + \sqrt{W^2 + 4}}{2} \tag{7}$$

where:

$$W = \frac{\bar{w}_z - U_L}{U_{b1} - U_L} \tag{8}$$

Eq. (7) enhances the wake effect on the trailing bubble and gives the right asymptotic behavior $U_b = 1$ when the wake influence is negligible ($W \rightarrow 0$).

Discussion and results

The three main features of this work are (i) consideration of the wake effect not only by its influence on the quasi-steady drag force, but also by the wake attraction force on the trailing bubble, (ii) the introduction of a multiplier β function and (iii) its observed similarity behavior to the trailing bubble rise velocity. The inclusion of the β function in the present development retains the physical meaning of the wake attraction force on the trailing bubble motion, as given by the original balance in Eq. (2). Neglecting this effect drives to the value $\beta = 1$ and to a steady state balance on the trailing bubble. The quasi-steady state naturalness of this process, as pointed out by KM, resides in the fact that the approaching velocity between both bubbles is only a function of their dimensionless separation distance at a given Reynolds number. Thus, quasi-steady state models for two bubbles rising in line should consider the wake attraction force F_{WA} on the trailing bubble and not only the quasi-steady drag force F_{d2} as done by Zhang and Fan (2003). The quasi-steady drag force is by itself, unable to represent the whole quasi-steady state behavior (Ramírez-Muñoz et al., 2005).

A comparison of the dimensionless trailing bubble rise velocity U_b with the correspondent experimental data from KM, is done in Figures 2 and 3 at Reynolds numbers $Re_1 = 3.06$ and $Re_1 = 35.4$, respectively. Plots for the model developed by Zhang and Fan (2003), considering buoyancy, quasi-steady drag, added mass and history forces are also shown in Figures 2 and 3. It should be pointed out that while their model does not include the wake attraction force, it does include the history force, whose significance has been discussed by the KM experimental observations as a minor effect. This fact is apparent in the better predictions of the present model, considering only the wake attraction force, together with the main forces of quasi-steady drag and buoyancy.

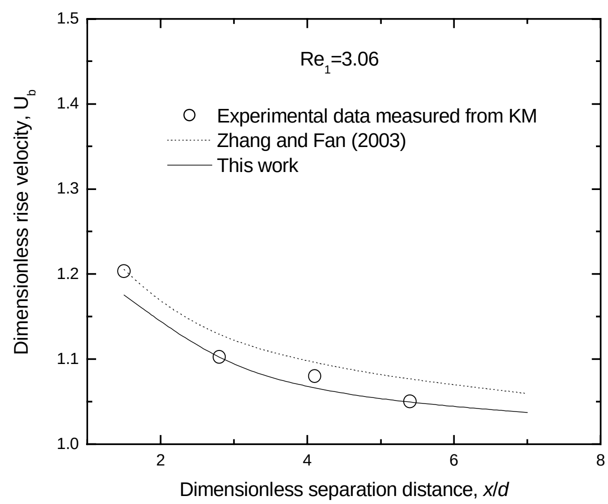

Figure 2. Trailing bubble rise velocity at $Re_1 = 3.06$, $Cd_1 = 5.43$.

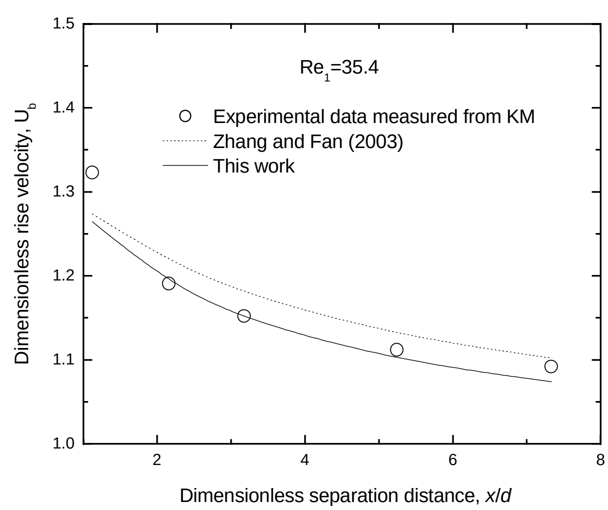

Figure 3. Trailing bubble rise velocity at $Re_1 = 35.4$, $Cd_1 = 1.10$.

Conclusions

The wake effect on the trailing bubble rise velocity, as considered in this work, was accounted for not only by a quasi-steady drag force, but also and very meaningfully, by a wake impulse force generated by the leading bubble motion and acting on the trailing bubble. This wake attraction force was transformed by defining a multiplier β function greater than 1. The multiplier β function was evaluated for the asymptotic wake velocity solution and a qualitative behavior similar to the experimental U_b function was observed, driving to the plausible assumption $\beta \approx U_b$. The main result from this assumption, when incorporated to the balance equation, was a dimensionless trailing bubble rise velocity whose performance is shown to fit better the experimental results from KM than more sophisticated models which do not account for the wake attraction force. However, care should be taken in using Eq. (7), since it was built-up on considering a similarity assumption and not only on fundamental principles. By this time, Eq. (7) has only been tested within the range $3 \leq Re \leq 36$.

Acknowledgements

The authors would like to acknowledge *CONACyT*, Mexico, for an academic scholarship granted to one of them (JRM).

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