TA012
DIELCTRIC PROPERTY MEASUREMENTS AND TECHNIQUES

USEFUL RELATIONSHIPS BETWEEN DIELECTRIC PROPERTIES AND BULK DENSITY OF POWDERED AND GRANULAR MATERIALS

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When the permittivities, or dielectric properties, of granular or powdered solid materials are important, some reliable relationship between the permittivity and the bulk density of the air-particle mixture is required. Examples of such applications include radio-frequency or microwave dielectric heating of these materials and the use of correlations between permittivity and water content of hygroscopic materials for sensing moisture content. Also, relationships between the permittivities of pulverized or granular samples of materials and those of the solid materials have long been of interest for use in determining the permittivities of the solids from measured permittivities of the particulate samples. Dielectric mixture equations have been developed and investigated for this purpose [1-3]. An extrapolation procedure, based on the linearity with density of functions of the real and imaginary parts of the permittivity of pulverized and granular materials, has also been used to obtain estimates of the permittivity of the solid material [4, 5]. Both of these methods, calculation by dielectric mixture equations and the extrapolation procedure, will be described for estimating the permittivity of solids from measured permittivity data on granular or powdered samples of those materials.

Many different dielectric mixture equations have been proposed to represent the effective permittivity of mixtures of dielectric materials, and the characteristics and performance of these equations for various applications have been discussed in the literature [6-11]. The complex permittivity relative to free space is represented here as $\varepsilon = \varepsilon' - j\varepsilon''$, where ε' is the dielectric constant and ε'' is the dielectric loss factor. Dielectric mixture equations for a two-phase mixture can be used to calculate the dielectric properties of a solid material from the dielectric properties of an air-particle mixture of air and pulverized or granular particles of the solid. To use the mixture equations, one needs to know the permittivity of the pulverized sample at its bulk density (air-particle mixture density), ρ, and the specific gravity or density of the solid material, ρ_2. The fractional part of the total volume of the mixture occupied by the particles (volume fraction), v_2, is then given by ρ / ρ_2.

Several well-known dielectric mixture equations have been studied to relate the permittivities of the solid material to the permittivities of granular or pulverized samples of the same material [5, 12-15]. Two of these equations are the Complex Refractive Index mixture equation [16-22],

$$(\varepsilon)^{1/2} = v_1(\varepsilon_1)^{1/2} + v_2(\varepsilon_2)^{1/2} \qquad (1)$$

and the Landau and Lifshitz, Looyenga equation [4, 13],

$$(\varepsilon)^{1/3} = v_1(\varepsilon_1)^{1/3} + v_2(\varepsilon_2)^{1/3} \qquad (2)$$

where ε represents the complex permittivity of the mixture, ε_1 is the permittivity of the medium in which particles of permittivity ε_2 are dispersed, and v_1, and v_2 are the volume fractions of the respective components, where $v_1 + v_2 = 1$.

Each of these equations can be used to calculate the complex permittivity of the solid material by substituting $1 - j0$ for ε_1, the complex permittivity of air, and solving for ε_2. The corresponding equations are:

Complex Refractive Index (1)

$$\varepsilon_2 = \left(\frac{\sqrt{\varepsilon} + v_2 - 1}{v_2} \right)^2 \tag{3}$$

Landau & Lifshitz, Looyenga (2)

$$\varepsilon_2 = \left(\frac{\varepsilon^{1/3} + v_2 - 1}{v_2} \right)^3 \tag{4}$$

Essentially linear relationships between functions of the real and imaginary components of the complex permittivity of particulate materials such as pulverized coal, wheat, and whole-wheat flour and their bulk densities were identified previously [4, 23, 24]. These findings were based on earlier work in which Klein [25] observed the linearity of $\sqrt{\varepsilon'}$ with the bulk density of granular coal, and in which Kent [26], working with fish meal, found that ε' and ε'' were both quadratic functions of the bulk density of the fish meal as follows:

$$\varepsilon' = a\rho^2 + b\rho + 1 \tag{5}$$

$$\varepsilon'' = c\rho^2 + d\rho \tag{6}$$

where ρ represents the density of the air-particle mixture, a,b,c and d are constants for a given particulate material and ε' and ε'' have values of 1 and 0, respectively, for air alone ($\rho = 0$). Equations (5) and (6) are equivalent to expressing the relative complex permittivity as a quadratic function of bulk density, $\varepsilon = a^* \rho^2 + b^* \rho + 1$, where the constants $a^* = a - jc$ and $b^* = b - jd$ are complex numbers.

The linearity of $(\varepsilon')^{1/2}$ with bulk density was confirmed with data on pulverized coal and this relationship is expressed as

$$(\varepsilon')^{1/2} = m\rho + 1 \tag{7}$$

Equations (5) and (7) are equivalent if $a = m^2$ and $b = 2m$. Thus, measurement of the dielectric constant of a particulate material at one bulk density, along with the $(\rho = 0,\ \varepsilon' = 1)$ intercept, provides information on the dielectric constant at all densities, including that of the solid material if its density is known. Examining the expression for the loss factor (6), the square can be completed by adding a constant, e, to each side, and a linear function is obtained.

$$(\varepsilon'' + e)^{1/2} = c^{1/2}\rho + e^{1/2} \tag{8}$$

where $e = d^2 / 4c$. Thus, to describe the density dependence of the complex permittivity of a particulate material, one needs to obtain the values for $a,b,c,$ and d. Measurement of the permittivity of the particulate material at a given density establishes slope m of (7), thus determining values for a and b.

Measurements at a few additional densities are necessary for determination of c and d, as explained previously [12, 27]. It is interesting to note that (5) and (6) are consistent with the Complex Refractive Index mixture equation (1), when used for the real part of the complex permittivity. Since $v_2 = \rho / \rho_2$, where ρ is the density of the air-particle mixture, and ρ_2 is the density of the particles, this substitution in (1) yields the following for an air-particle mixture:

$$(\varepsilon')^{1/2} = \frac{(\varepsilon_2')^{1/2} - 1}{\rho_2} \rho + 1 \tag{9}$$

which is equivalent to (7), where $m = (\sqrt{\varepsilon_2'} - 1)/\rho_2$. In an analogous manner, it can be shown that the linearity of the cube root of the dielectric constant of an air-particle mixture with its density is consistent with the Landau and Lifshitz, Looyenga mixture equation (2) [4],

$$(\varepsilon')^{1/3} = \frac{(\varepsilon_2')^{1/3} - 1}{\rho_2} \rho + 1 \tag{10}$$

Applications of these techniques have been presented elsewhere for coal [4, 5, 13, 23, 27], wheat and wheat flour [4, 14, 15, 24], plastics [12, 13], and coal and limestone and their mixtures [28-30]. As a result of these studies, the Landau & Lifshitz, Looyenga equation generally provided the best relation between the permittivities of the solid and particulate materials. Therefore, a relationship based on this equation can be recommended for correcting permittivities of granular and powdered materials, with permittivities in the same range as these materials, for changes in bulk density as follows:

$$\varepsilon_b = \left[\left((\varepsilon_a)^{1/3} - 1\right)\rho_b / \rho_a + 1\right]^3 \tag{11}$$

where ε_a is the complex permittivity of the air-particle mixture at a given density ρ_a, and ε_b is the permittivity of the mixture at a different density ρ_b.

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