Reduced-order observers for high dimensional chemical processes

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1. Introduction

Monitoring chemical processes in the most reliable and cost effective manner is a vital issue for the process industries. This goal can be achieved by the use of observers which estimate many of the process variables from the available measurements. Different observer design techniques have been proposed and extensively investigated over the last four decades. The differences between these approaches stem from the property if the model is linear or nonlinear and the computation procedure used for the calculation of the observer gain [5, 6, 9]. However, problems can arise for systems where the degree of observability varies strongly from one state to another. This is almost always the case if the plant is correctly described by a distributed system which is approximated by a set of ordinary differential equations or if the system has a high dimension but only a few measurements are available. While it should be theoretically possible to design an observer which correctly estimates the values of all states, assuming that the plant is observable, this is not the case in practice due to plant-model mismatch as well as the presence of measurement noise [1,10]. In fact, observers will usually attenuate sensor noise for most systems where more than a few states need to be estimated from each measurement.

This work addresses these issues by presenting two observer design techniques, both of which can be applied with modifications to either linear or nonlinear systems. The difference between the two presented observer design methodologies is that the first one is ideally suited for systems without inputs, while the second one takes the input-to-state behavior into account in addition to the state-to-output behavior. These new design techniques can be applied to distributed system [2], but are also applicable to lumped systems. The main idea behind the presented observer designs is that instead of trying to reconstruct the values of all states of a system, only states having a significant degree of observability will be included in the observer. This results in low-order observers which are relatively insensitive to measurement noise while at the same time allows reconstruction of states which contribute most to the state-to-output behavior. It is important to point out that the resulting low-order observers are different from the traditional definition of a reduced-order observer, i.e. the proposed technique designs observers for the modes which are most observable instead of reconstructing all states except the measured ones as is the case for traditional reduced-order observer designs.

The two presented observer designs are based upon reducing the original system via a projection. The two methods differ from each other in that for systems without inputs, a singular value decomposition of the observability covariance matrix is used for computation of the projection, whereas the second observer design technique relies on a projection which balances the input-to-state and the state-to-output behavior.

The proposed methods for observer design have been applied to two examples: one distillation column with five trays and another one with 30 trays. It is shown that it is possible to design a low-order observer which reconstructs the values of all states of the system with a low number of measurements. Additionally, the observer designs are performed for the original nonlinear system as well as for a linearized version.

2. Reduced-order observer

The presented work proposes two types of observers. The first kind is restricted to system without input. In this approach the linear system described by (1) is

$$\dot{x} = Ax \tag{1a}$$

$$y = Cx \tag{1b}$$

reduced through a projection based on the singular value decomposition of the observability covariance matrix [8]. The states important for the state-to-output behavior are retained in the reduced system and the observer is designed for the resulting low-order system. In order to obtain a reduced-order system, the observability covariance matrix (W_O) is calculated for the system described by (1) and the matrix is transformed

$$\overline{W_O} = T^{-1} W_O T \tag{2}$$

such that

$$\overline{W_O} = \begin{pmatrix} \sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & 0 & \vdots & \vdots \\ \vdots & 0 & \sigma_3 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{pmatrix}, \ \sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \geq \sigma_n \tag{3}$$

where T is the transformation matrix such that the columns of T consists of the singular vectors of W_O and the $\sigma' s$ are the singular values of W_O. Accordingly, the state space transformation $x = T\overline{x}$ results in the following transformed system:

$$\dot{\overline{x}} = T^{-1} A T \overline{x} = \overline{A}\overline{x} \tag{4a}$$

$$y = CT\overline{x} \tag{4b}$$

This system can be partitioned into more important $(\overline{x}_1)$ and less important states $(\overline{x}_2)$

$$\begin{pmatrix} \dot{\overline{x}}_1 \\ \dot{\overline{x}}_2 \end{pmatrix} = \begin{pmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{pmatrix} \begin{pmatrix} \overline{x}_1 \\ \overline{x}_2 \end{pmatrix} \tag{5a}$$

$$y = \begin{pmatrix} \overline{C}_1 & \overline{C}_2 \end{pmatrix} \begin{pmatrix} \overline{x}_1 \\ \overline{x}_2 \end{pmatrix} \tag{5b}$$

The less important states $(\overline{x}_2)$ correspond to the small singular values and can be truncated:

$$\dot{\bar{x}}_1 = \bar{A}_{11}\bar{x}_1 \quad (6a)$$
$$\bar{y} = \bar{C}_1\bar{x}_1 \quad (6b)$$

The reduced order observer is designed for the system given by (6) and results in

$$\dot{\hat{x}}_1 = \bar{A}_{11}\hat{x}_1 - L(\hat{y} - y) \quad (7a)$$
$$\hat{y} = \bar{C}_1\hat{x}_1 \quad (7b)$$
$$x = T\hat{x}_1 \quad (7c)$$

where L is the gain of the observer.

For nonlinear systems

$$\dot{x} = f(x) \quad (8a)$$
$$y = h(x) \quad (8b)$$

it is possible to compute the observability covariance matrix [8] and the transformation matrix T is computed by singular value decomposition of the observability covariance matrix. The transformed nonlinear system is given by:

$$\dot{\bar{x}} = T^{-1}f(T\bar{x}) = \bar{f}(\bar{x}) \quad (9a)$$
$$y = h(T\bar{x}) = \bar{h}(\bar{x}) \quad (9b)$$

The states not contributing to the state-to-output behavior in (9) can be truncated and the reduced system is given by

$$\dot{\bar{x}}_1 = PT^{-1}f(T\bar{x}) = P\bar{f}(\bar{x}) \quad (10a)$$
$$\bar{x}_2 = \bar{x}_{2,SS}(0) \quad (10b)$$
$$\bar{y} = h(T\bar{x}) = \bar{h}(\bar{x}) \quad (10c)$$

where P is a projection matrix of the form $[I \quad 0]$ and its rank is equal to the dimension of the reduced-order model. The corresponding nonlinear observer

$$\dot{\hat{x}}_1 = P\bar{f}(\hat{x}) - L(\hat{y} - y) \quad (11a)$$
$$\hat{x}_2 = \hat{x}_{2,ss}(0) \quad (11b)$$
$$\hat{y} = h(T\hat{x}) = \bar{h}(\hat{x}) \quad (11c)$$
$$x = T\hat{x}_1 \quad (11d)$$

contains the observer gain L which is computed based on the linearized version of the nonlinear model (8), similar to an extended Luenberger observer.

The second type of observer proposed in this work is applicable to systems with inputs. A linear system

$$\dot{x} = Ax + Bu \quad (12a)$$
$$y = Cx \quad (12b)$$

described by (12) can be reduced by balanced truncation such that the states contributing the most to the input-output behavior of the system are retained in the reduced system. This model reduction is performed by first transforming the system

$$\dot{\bar{x}} = TAT^{-1}\bar{x} + TBu = \bar{A}\bar{x} + \bar{B}u \quad (13a)$$

$$y = CT^{-1}\bar{x}\ ,\qquad (13b)$$

partitioning the states of the balanced system

$$\begin{pmatrix}\dot{\bar{x}}_1\\ \dot{\bar{x}}_2\end{pmatrix} = \begin{pmatrix}\bar{A}_{11} & \bar{A}_{12}\\ \bar{A}_{21} & \bar{A}_{22}\end{pmatrix}\begin{pmatrix}\bar{x}_1\\ \bar{x}_2\end{pmatrix} + \begin{pmatrix}B_1\\ B_2\end{pmatrix}u \qquad (14a)$$

$$y = \begin{pmatrix}\bar{C}_1 & \bar{C}_2\end{pmatrix}\begin{pmatrix}\bar{x}_1\\ \bar{x}_2\end{pmatrix} \qquad (14b)$$

and truncating the states x_2:

$$\dot{\bar{x}}_1 = \bar{A}_{11}\bar{x}_1 + \bar{B}_1 u \qquad (15a)$$
$$\bar{y} = \bar{C}_1\bar{x}_1 \qquad (15b)$$

A reduced-order observer is the designed for the truncated system:

$$\dot{\hat{x}}_1 = \bar{A}_{11}\dot{\hat{x}}_1 + \bar{B}_1 u - L(\hat{y} - y) \qquad (16a)$$
$$\hat{y} = \bar{C}_1\hat{x}_1 \qquad (16b)$$
$$x = T^{-1}\hat{x}_1 \qquad (16c)$$

The extension to nonlinear systems

$$\dot{x} = f(x,u) \qquad (17a)$$
$$y = h(x) \qquad (17b)$$

is similar to what has been presented for the observer without inputs. The main difference is that the coordinate transformation $\bar{x} = Tx$ is computed such that the observability and controllability covariance matrices are diagonal and equal. The transformed system is given by:

$$\dot{\bar{x}} = Tf(T^{-1}\bar{x},u) = \bar{f}(\bar{x},u) \qquad (18a)$$
$$\bar{y} = h(T^{-1}\bar{x}) = \bar{h}(\bar{x}) \qquad (18b)$$

After truncation the system is given by:

$$\dot{\bar{x}}_1 = PTf(T^{-1}\bar{x},u) = P\bar{f}(\bar{x},u) \qquad (19a)$$
$$\bar{x}_2 = \bar{x}_{2,SS}(0) \qquad (19b)$$
$$\bar{y} = h(T^{-1}\bar{x}) = \bar{h}(\bar{x}) \qquad (19c)$$

where the rank of P is equal to the dimension of the reduced-order nonlinear system. The observer for the nonlinear system is given by:

$$\dot{\hat{x}}_1 = P\bar{f}(\hat{x},u) - L(\hat{y} - y) \qquad (20a)$$
$$\hat{x}_2 = \hat{x}_{2,ss}(0) \qquad (20b)$$
$$\hat{y} = h(T\hat{x}) = \bar{h}(\hat{x}) \qquad (20c)$$
$$x = T\hat{x}_1 \qquad (20d)$$

3. Illustrative examples

3.1 Reduced-order design for systems without inputs

Linear system

The proposed observer design has been applied to a 5-tray distillation column model taken from the literature [7]. The model has seven states and the top product of the column is measured. The reduced-order observer has four states. Figure1 shows the response of the seventh state for the plant and the observer. Analyzing the response of proposed reduced-order observer and full-order observer, it can be observed that for this case the performance of the reduced-order observer is comparable to the full-order observer.

In the second example, a 30-tray distillation column model is considered [3]. This model has 32 states and the 6th state is measured. A reduced-order observer with four states has been designed. Figure2 shows the response of the 5th state for the full-order and reduced order observer based upon a model with 4 states. The results clearly show that the observer performance for states with good observability is comparable for the full-order and the reduced-order observer. However, for states having poor observability it can be noted that full-order observer result in high gains and reconstruction of these state via a full-order observer may result in very large overshoot. As a result, a full order observer is highly sensitive to noise. The performance of the reduced observer for poorly observed states is not as good as for the strongly observable states but it does not exhibit the problem of large overshoot and converges to the same steady state.

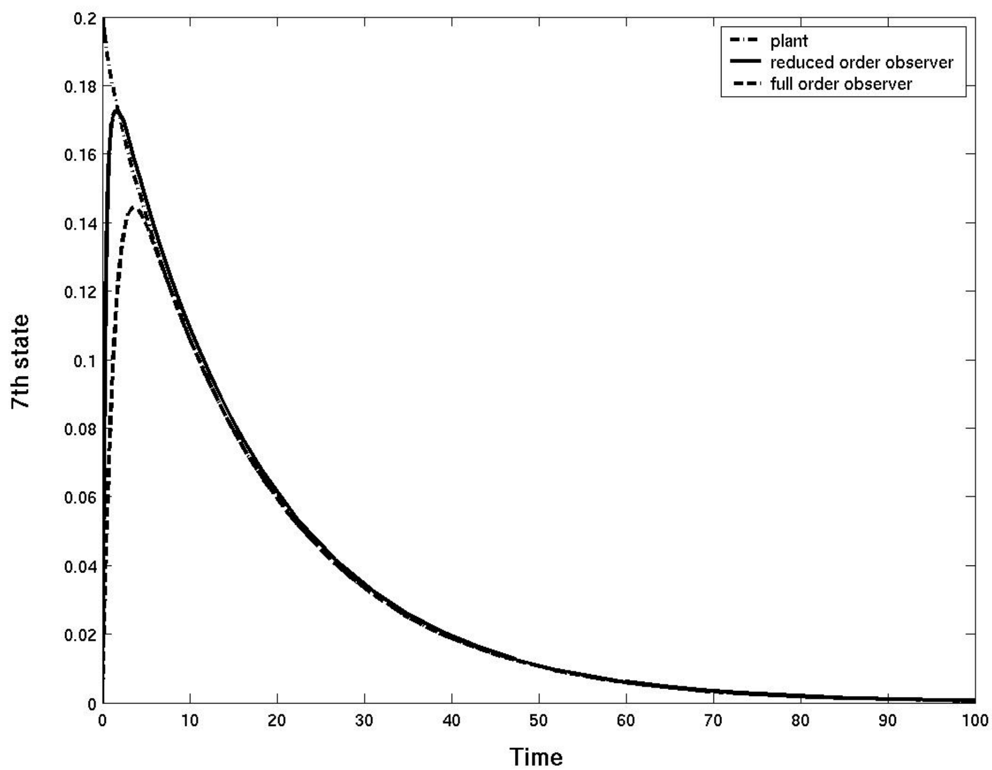

Figure 1. Response of the 7^{th} state for reduced order and full order observer.

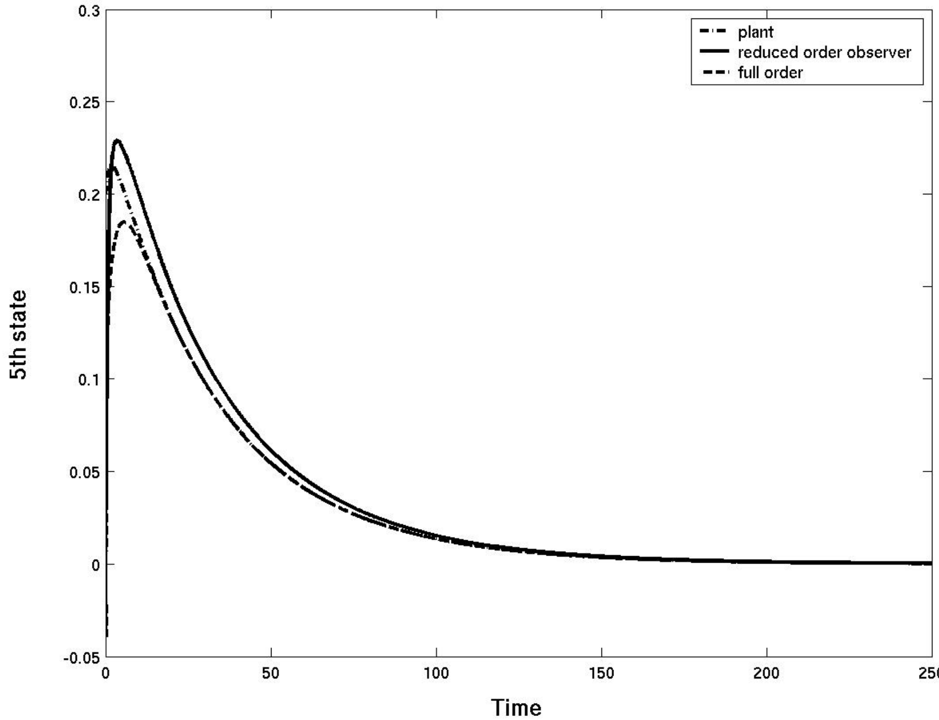

Figure 2. Response of the 5th state for reduced-order and full-order observers.

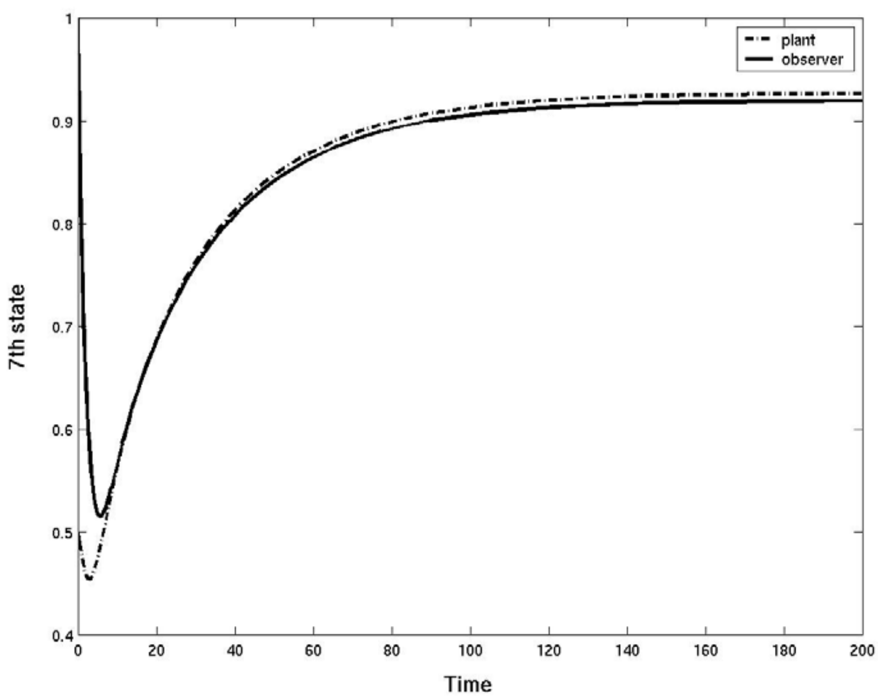

Figure 3. 7th state response for reduced order observer.

Nonlinear system

The designed reduced-order observer for the 32 state nonlinear distillation model [3] with one measurement has six states. From Figure 3 it can be seen that the response of the 7th state reconstructed by the reduced order observer is in good agreement with the plant.

3.2 Reduced-order design for systems with inputs

Linear system

The reduced-order observer for a 5-tray distillation column has 4 states. From Figure 4, it can be observed that the reduced-order observer performs as good as the full-order observer for input changes and shows good tracking of the dynamic behavior of the plant.

In the second example, a fifth order, reduced observer is designed for the 30-tray distillation column model. Figure 5 shows the response of the 7th state for the observer and the plant. It can be seen that the reduced-order observer performs similar to the full order observer.

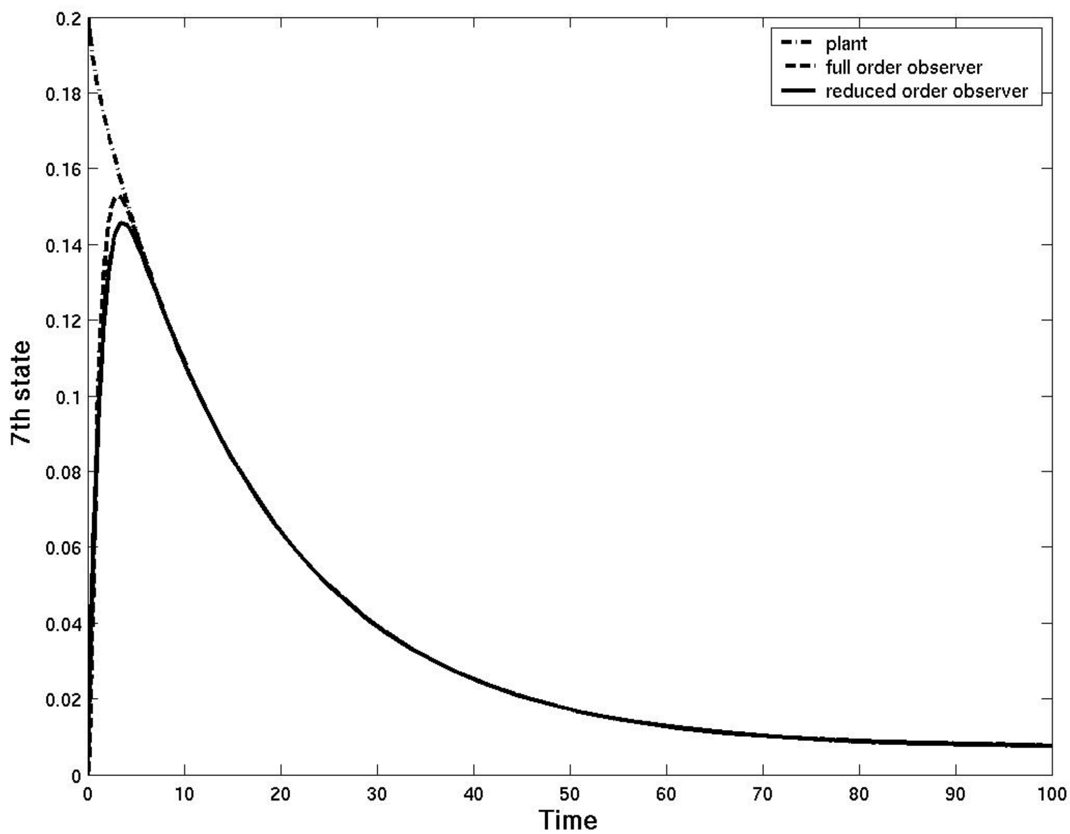

Figure 4. Response of the 7th state for reduced order observer and full order observer

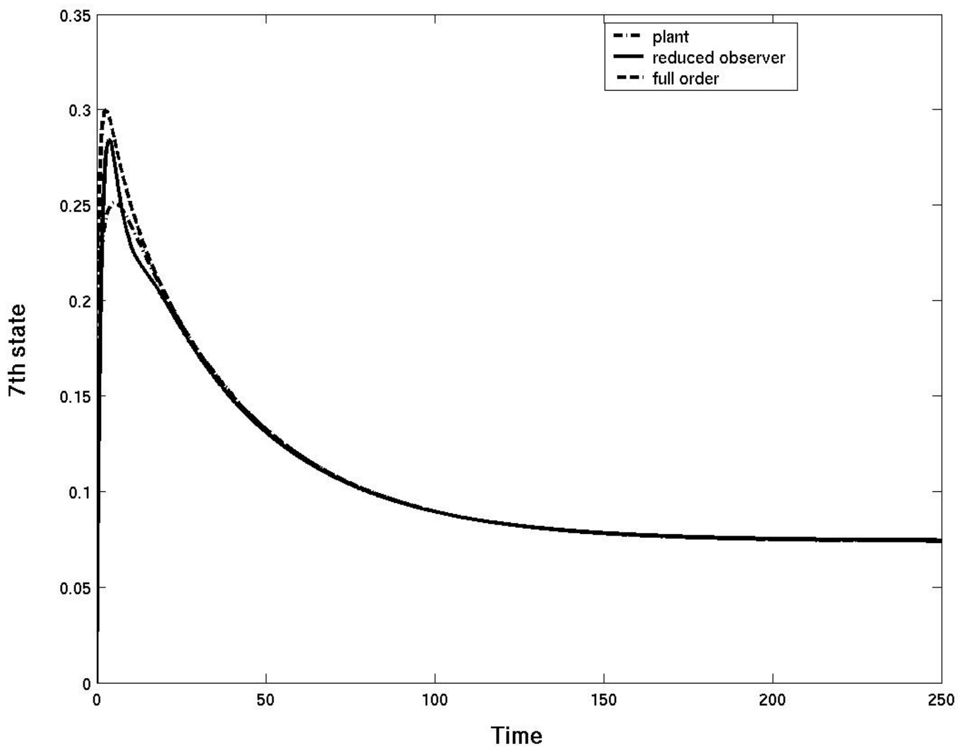

Figure 5. Response of 7^{th} state for reduced-order observer and full order observer

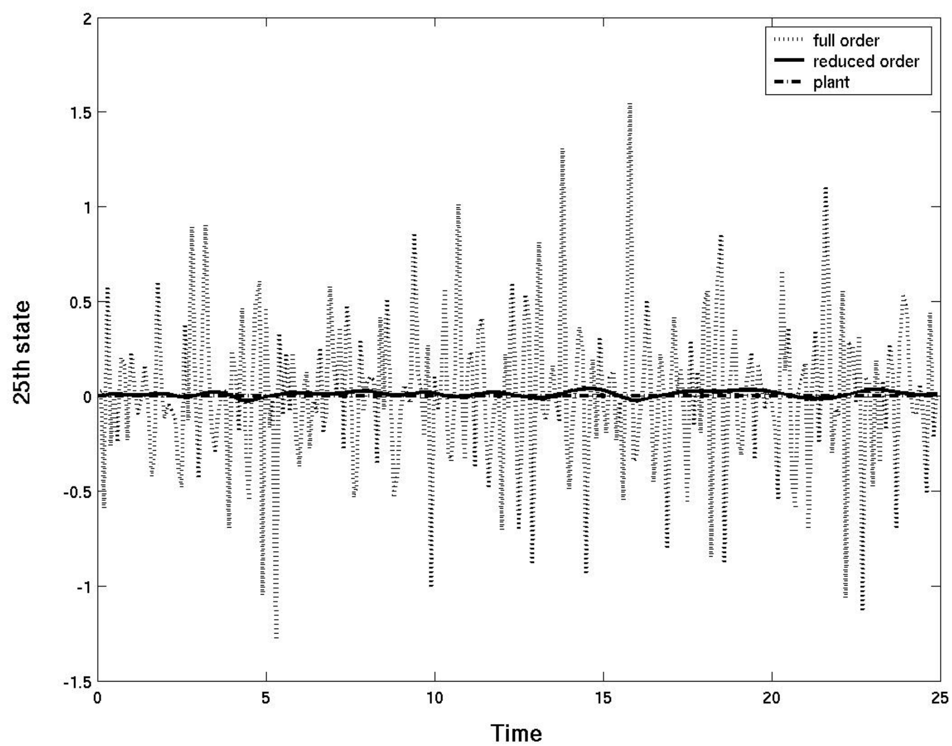

Figure 6. Response of 25th state for reduced order and full order observer with measurement noise.

When the state reconstruction under the influence of noise is considered then the reduced-order observer shows advantages as it does not require high gains. This is illustrated in Figure 6 where the state response of the reduced and the full-order observer for the 25th state are shown under the influence of measurement noise. It can be concluded that the measurement noise is amplified by the high gain in the full order observer whereas the reduced-order observer is insensitive to measurement noise.

Nonlinear system

The reduced order observer for the nonlinear 30-tray distillation column has 8 states. Figure 7 shows that the observer tracks the dynamic behavior of the plant for input changes very well.

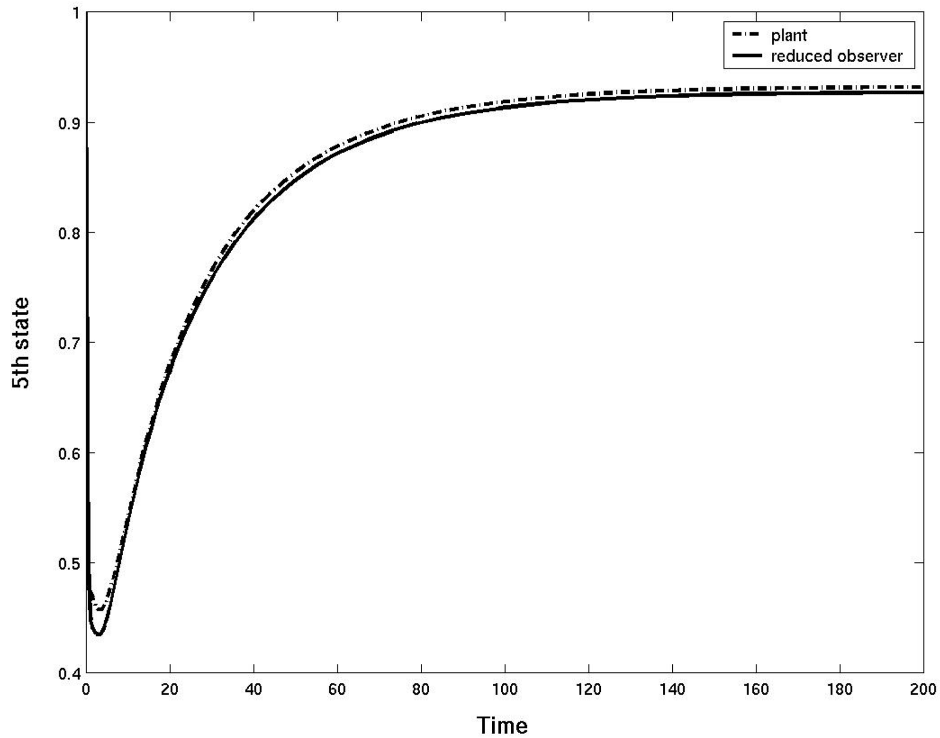

Figure 7. Response of 5^{th} state for reduced-order observer

4. Conclusion

The presented work proposes two types of observer design, one for systems with and one for model without inputs. Both reduced-order observer designs can be applied to linear as well as nonlinear systems. While performance of the presented observers is comparable to full-order observers, the reduced-order observers have certain advantages for high dimensional systems:

1) They are simpler to design as only states which contribute to the observable behavior of the process are included in the model.

2) The reduced order observer is not sensitive to noise, unlike the full-order observer for large-scale system.

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