Branching analysis of T-periodic solutions in nonlinear biochemical systems using reductive perturbation method

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Oscillations occur commonly in physiochemical and biochemical systems. Many biochemical systems are nonlinear in nature and exhibits oscillatory behavior in some parameter space. Such oscillatory behavior may include a variety of patterns such as multi-peak periodic oscillations, bursting, quasiperiodicity, and aperiodicity or chaotic oscillations. Often, oscillations can be practically unavoidable and therefore a systematic bifurcation and stability analysis of such solutions is warranted. Although considerable work on oscillations, multiplicity of T-periodic solutions, and chaos exists in the literature, there seems to be a paucity of literature on the application of reductive perturbation method on these problems.

Karimi and Inamdar (2002) applied a mathematical method called Reductive Perturbation Method (RPM) to analyze nonlinear multi-equation systems, as is, to seek static branching in a system of first order ordinary differential equations from a simple zero eigenvalue and derived explicit analytical conditions both for the occurrence of various static branches and their stability. In Inamdar and Karimi (2002) they analyzed a multi-equation system of partial delay-differential equations at a Hopf point and derived explicit analytical conditions that characterize the Hopf point as subcritical or supercritical. From this work, it appears that the method may have some advantages such as the ability to deal with multiple equations as such without reducing to a single equation, analyticity of expressions, etc. and may not have been put to full use so far in the literature. The aim of this paper is to explore further the application of reductive perturbation method towards a detailed analysis of the occurrences and bifurcations of various oscillatory patterns in general multi-equation systems, with application to biochemical systems in view.

In this investigation, we focus on the use of RPM to construct a complete bifurcation diagram without resorting to any extensive numerical tracing or analysis. We develop a systematic methodology to use the analytical results of RPM to do this for general multi-equation systems and successfully demonstrate this method on a biochemical system (Ajbar and Ibrahim, 1997) in the literature. We use optimization

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technique which would give the connections between various branches in the parameter space subject to the constraints determined by bifurcation theory. However, to be complete, this methodology still needs the analysis of higher-order bifurcations of T-periodic solutions using the RPM, and that is another focus of this paper. Therefore our second aim is to the use the RPM to derive analytical conditions for the bifurcation of T-periodic oscillations that emanate from a Hopf point into higher order bifurcations such as turning point, sub-harmonic oscillations; period doubling, period tripling, quasi-periodic oscillations and bifurcation of tori. Applying Floquet theory and multiple time scales, we find that RPM can give us the perturbation equations that equip us with the ability to determine the parameter conditions and to best analyze the bifurcation of T-periodic solutions.

Keywords: T-periodic solution, bifurcation, perturbation method, hopf point, Floquet, Torus, Period doubling.

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