Economic Design of Stateless Control Charts for Deteriorating Systems

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Control charts are a family of process monitoring techniques grounded in the theory of mathematical statistics. Originally developed for use in the field of quality control in a manufacturing environment [1], their application has expanded to the detection of abrupt changes in a variety of manmade and natural systems [2].

Typical control charts output a single signal called an alarm upon detection of an abrupt change. This alarm may serve a variety of functions depending on the context. Alarms for mechanical, chemical, or electrical systems often drive maintenance actions. The timing and nature of these maintenance actions may in turn have a significant impact on the long-term cost or profitability of the system, and this impact can be assessed given a reliability model, cost model, and process model for the system.

From the inception of control chart theory, it was recognized that control charts could provide tangible economic benefits [3], and there were efforts early-on to explicitly optimize control chart design to maximize economic benefit [4].

Previous research on the economic design of control charts for deteriorating systems is found largely in two bodies of literature: the economic design of quality control charts for a manufacturing environment, and the economic design of replacement policies for deteriorating machines [5]. Both problems have a relatively small action space, with only two or three actions possible given any state of the system.

The literature on the economic design of quality control charts largely derives from the early work of Duncan [4]; References [6] and [7] are surveys of much of the subsequent work. A major limitation on the applicability of this work to a broader range of deteriorating systems is the assumption that the system has only two states: "in-control" and "out-of-control." In contrast, the results presented in this paper apply to systems with any finite number of states.

The economic importance of maintenance scheduling for deteriorating systems has led to a broad operations research literature on replacement policies for these systems [8]. Though much of the literature assumes that the state of the policy can be perfectly known, a portion of the literature develops replacement policies for partially observed systems (see [5, 9, 10]). Application of the results of the theory of partially observed Markov decision processes has been limited because of the computational intractability of solution methods for even moderately sized problems [11].

A stateless control chart is an algorithm that produces an alarm at a given time based only on system measurements at that time. In this paper, we achieve an approximate solution to a partially observed Markov decision process by limiting the solution search to those solutions that take the form of stateless control charts, and thereby arrive at solution techniques that are computationally tractable and results that are easily implemented in an automatic monitoring system.

Two practical control chart design methods are presented. Both methods utilize a state space formulation with a reliability model, action space, cost model, and process model of the deteriorating system of interest. Each state represents a different degree of system degradation. The reliability model is a Markov chain that describes how the system transitions stochastically from one state to another. The action space consists of two actions: 1) inaction or, 2) replacement or repair of system components such that the system is restored to its undeteriorated state. The cost model assigns costs to taking each action in a given state. The process model takes the form of probability mass functions that describe how system measurements reflect the state of the system.

The first control chart design method produces a control chart through constrained local minimization of the infinite horizon discounted cost functional. Local minimizers of the cost functional can be found through widely implemented techniques such as sequential quadratic programming or interior point methods, and it is shown that, while this optimization problem can have large numbers of local minimizers, the objective functions for the local minimizers are close in value to a global minimizer for this particular class of optimization problems.

The second design method derives from the generalized likelihood ratio test used in mathematical statistics, and relies on dynamic programming techniques to achieve a classification of system states.

A comparison of the two design methods shows that the second method is generally preferable because it requires less computation, is insensitive to convergence problems associated with computation of the gradient of the cost functional, and always produces a deterministic control chart.

The concepts and results of the paper are illustrated with a model problem that is rich in behavior but easily described and analyzed. A fictitious company, LumenHour Light Company, sells light from light bulb arrays to its customers. LumenHour operates and maintains light bulb arrays consisting of a series of rows of light bulbs in parallel. Because the failure rate of a light bulb scales with applied voltage, the failure rate of bulbs in the array is affected by the location of failed bulbs throughout the array. LumenHour's revenue from a bulb array is proportional to the light output of the array, but it incurs costs to replace burnt-out bulbs. Furthermore, LumenHour knows the state of a bulb array only through noisy voltage measurements across each row of bulbs. For a test case with seventy system states and one-hundred sixty measurement values, economical control chart designs are produced on a personal computer in less than two hundred seconds of computation using the local minimizer design method, and less than two seconds of computation using the generalized likelihood ratio method.

References

[1] W. A. Shewhart. Quality control charts. *The Bell System Technical Journal* V:593–603, 1926.

[2] M. Basseville and I. Nikiforov. *Detection of Abrupt Changes: Theory and Application*. Prentice-Hall, 1993.

[3] W. A. Shewhart. Economic quality control of manufactured product. *The Bell System Technical Journal* IX:364–389, 1930.

[4] A. J. Duncan. The economic design of $\bar{X}$ charts used to maintain current control of a process. *Journal of the American Statistical Association* 51(274):228–242, 1956.

[5] G. E. Monahan. A survey of partially observable Markov decision processes: Theory, models, and algorithms. *Management Science* 28(1):1–16, 1982.

[6] D. C. Montgomery. The economic design of control charts: A review and literature survey. *Journal of Quality Technology* 12:75–87, 1980.

[7] C. Ho and K. Case. Economic design of control charts: A literature review for 1981-1991. *Journal of Quality Technology* 26:39–53, 1994.

[8] H. Wang. A survey of maintenance policies of deteriorating systems. *Europrean Journal of Operational Reserach* 139:469–489, 2002.

[9] W. Pierskalla and J. Voelker. A survey of maintenance models: The control and surveillance of deteriorating systems. *Naval Research Logistics Quarterly* 23:353–388, 1976.

[10] V. Makis and X. Jiang. Optimal replacement under partial observations. *Mathematics of Operations Research* 28(2):382–294, 2003.

[11] W. S. Lovejoy. A survey of algorithmic methods for partially observed Markov decision processes. *Annals of Operations Research* 28:47–66, 1991.